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Anisotropic Fiber with Cylindrical Polar Axes

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Abstract. The newly proposed anisotropic fiber structures with cylindrical polar principal axes appear to be an interesting novel class of special lightguides. In this paper, some interesting results relating to such fibers are derived which, to the knowledge of this author, have not yet been reported in the literature. It is found that, if $n_{\phi c0}(\Delta_\phi)^{1/2} > n_{zc0}(\Delta_r)^{1/2}$, TE_{01} will be the fundament mode with a range of single-mode operation given by $2.61 n_{zc0}(2\Delta_r)^{1/2} < \lambda/a < 2.61 n_{\phi c0}(2\Delta_\phi)^{1/2}$. On the other hand, if $n_{zc0}(\Delta_r)^{1/2} > n_{\phi c0}(\Delta_\phi)^{1/2}$, then TM_{01} becomes the fundamental mode whose single-mode operation range is $2.61 n_{\phi c0}(2\Delta_\phi)^{1/2} < \lambda/a < 2.61 n_{zc0}(2\Delta_r)^{1/2}$.

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This paper is concerned with the special class of anisotropic fibers introduced by Black et al. [1]. Such fibers have cylindrical polar axes $\hat{r}$, $\hat{\phi}$, $\hat{z}$ for the anisotropic refractive indices, which are z-independent but otherwise arbitrary functions of position. For anisotropic fibers, we consider HE_{mn}, EH_{mn}, TE_{mn}, and TM_{mn} modes, most of which contain all three electrical field components E_r, E_ϕ, and E_z, each "seeing" the refractive index in its respective direction. A mode becomes leaky when its effective index β/k is smaller than any of the three principal refractive indices of the cladding. Some special modes do not contain all the three electrical field components. For such modes, index profiles in the zero electrical field directions are irrelevant to the mode cut-off conditions. For example, the TE_{0n} modes have a E_ϕ component only, so that only the refractive index profile in the ϕ direction determines the mode cut-off conditions. Following the approach adopted by [1], one readily obtains the following coupled equations for the transverse field components

$$(\nabla_t^2 + k^2 n_r^2 - \beta^2 - 1/r^2)e_r = \frac{-2}{r^2}\frac{\partial e_\phi}{\partial \phi} + \left(1 - \frac{n_r^2}{n_z^2}\right)\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(re_r)\right] + \left(1 - \frac{n_\phi^2}{n_z^2}\right)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial e_\phi}{\partial \phi}\right), \tag{1a}$$

$$(\nabla_t^2 + k^2 n_\phi^2 - \beta^2 - 1/r^2)e_\phi = \frac{2}{r^2}\frac{\partial e_r}{\partial \phi} + \left(1 - \frac{n_r^2}{n_z^2}\right)\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial e_r}{\partial \phi}\right) + \left(1 - \frac{n_\phi^2}{n_z^2}\right)\frac{1}{r^2}\frac{\partial^2 e_\phi}{\partial \phi^2}. \tag{1b}$$

It is difficult to find an exact solution of the above equations. Under weakly-guiding condition, the mode propagation constant can be derived with the aid of perturbation technique and Green's integral theorem.

The anisotropic fibers are considered as perturbed isotropic fibers with $n_0^2(\bar{r}) = n_z^2(\bar{r})$. Then the index profile of the anisotropic fibers can be expressed as $\bar{\bar{n}}^2(\bar{r}) = n_0^2(r)(\bar{\bar{I}} + \bar{\bar{D}}(\bar{r}))$, where $\bar{\bar{I}}$ is unit dyadic, the perturbation parameter $\varepsilon = (2\Delta_z)^{1/2}$, and the dyadic $\bar{\bar{D}}$ is

$$\bar{\bar{D}} = \begin{pmatrix} -(2/\Delta_z)^{1/2}\delta_{rz} & 0 & 0 \\ 0 & -(2/\Delta_z)^{1/2}\delta_{\phi z} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

We now substitute the above refractive index components into (1) and expand β^2 and the electrical field components in terms of ε, such that

$$\beta^2 = \beta_0^2 + \varepsilon\beta_1^2 + \varepsilon^2\beta_2^2 + \dots, \tag{3}$$

$$\bar{e} = \bar{e}_0 + \varepsilon\bar{e}_1 + \varepsilon^2\bar{e}_2 + \dots . \tag{4}$$

Then, in the resulting equations, we equate the coefficients of the zero and first order of ε to obtain

$$(\nabla_t^2+k^2n_0^2-\beta_0^2-1/r^2)e_{0r}=\frac{-2}{r^2}\frac{\partial e_{0\phi}}{\partial\phi},\tag{5a}$$

$$(\nabla_t^2+k^2n_0^2-\beta_0^2-1/r^2)e_{0\phi}=\frac{2}{r^2}\frac{\partial e_{0r}}{\partial\phi},\tag{5b}$$

$$\begin{aligned}(\nabla_t^2+k^2n_0^2-\beta^2-1/r^2)e_{1r}&=\frac{-2}{r^2}\frac{\partial e_{1\phi}}{\partial\phi}+\left(\beta_1^2+k^2n_0^2\frac{2\delta_{rz}}{(2\Delta_z)^{1/2}}\right)e_{0r}+\frac{2\delta_{rz}}{(2\Delta_z)^{1/2}}\\&\quad\times\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}re_{0r}\right)+\frac{2\delta_{\phi z}}{(2\Delta_z)^{1/2}}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial e_{0\phi}}{\partial\phi}\right),\end{aligned}\tag{6a}$$

$$\begin{aligned}(\nabla_t^2+k^2n_0^2-\beta^2-1/r^2)e_{1\phi}&=\frac{2}{r^2}\frac{\partial e_{1r}}{\partial\phi}+\left(\beta_1^2+k^2n_0^2\frac{2\delta_{\phi z}}{(2\Delta_z)^{1/2}}\right)e_{0\phi}+\frac{2\delta_{rz}}{(2\Delta_z)^{1/2}}\frac{1}{r^2}\\&\quad\times\frac{\partial}{\partial r}\left(r\frac{\partial e_{0r}}{\partial\phi}\right)+\frac{2\delta_{\phi z}}{(2\Delta_z)^{1/2}}\frac{1}{r^2}\frac{\partial^2 e_{0\phi}}{\partial\phi^2},\end{aligned}\tag{6b}$$

where $\delta_{ij}=\frac{1}{2}(1-n_i^2/n_j^2)$.

With the help of Green's integral theorem, integration of the expression $[e_{0r}(6a)-e_{1r}(5a)+e_{0\phi}(6b)-e_{1\phi}(5b)]$ over the core and cladding regions, respectively, yields

$$\begin{aligned}&\lim_{\varepsilon_p\to 0}\oint_l\left[\frac{\partial e_{1r}}{\partial r}e_{0r}-\frac{\partial e_{0r}}{\partial r}e_{1r}+\frac{\partial e_{1\phi}}{\partial r}e_{0\phi}-\frac{\partial e_{0\phi}}{\partial r}e_{1\phi}\right]_{r=a+\varepsilon_p}^{r=a-\varepsilon_p}dl\\&=\int_{s_\infty}\left[\left(\beta_1^2+k^2n_0^2\frac{2\delta_{rz}}{(2\Delta_z)^{1/2}}\right)e_{0r}^2+\left(\beta_1^2+k^2n_0^2\frac{2\delta_{\phi z}}{(2\Delta_z)^{1/2}}\right)e_{0\phi}^2+\frac{2\delta_{rz}}{(2\Delta_z)^{1/2}}\right.\\&\quad\times\left\{e_{0r}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(re_{0r})\right]+e_{0\phi}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial e_{0r}}{\partial\phi}\right)\right\}\\&\quad\left.+\frac{2\delta_{\phi z}}{(2\Delta_z)^{1/2}}\left\{e_{0r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial e_{0\phi}}{\partial\phi}\right)+e_{0\phi}\frac{1}{r^2}\frac{\partial^2 e_{0\phi}}{\partial\phi^2}\right\}\right]ds.\end{aligned}\tag{7}$$

In the weakly-guiding case, one has $\bar{e}_{r=a+\varepsilon_p}=\bar{e}_{r=a-\varepsilon_p}$ and the line integral on the left-hand side of (7) vanishes. A first approximation of the perturbed propagation constant can thus be written as

Zero-order field solutions of (5) for the lowest-mode LP_{01} are

$$e_{0r}=\begin{cases}\dfrac{J_0(ur/a)}{J_0(u)}\begin{Bmatrix}\cos\phi\\\sin\phi\end{Bmatrix} & r<a\\[2ex]\dfrac{K_0(wr/a)}{K_0(w)}\begin{Bmatrix}\cos\phi\\\sin\phi\end{Bmatrix} & r>a,\end{cases}\tag{9}$$

$$e_{0\phi}=\begin{cases}\dfrac{J_0(ur/a)}{J_0(u)}\begin{Bmatrix}-\sin\phi\\\cos\phi\end{Bmatrix} & r<a\\[2ex]\dfrac{K_0(wr/a)}{K_0(w)}\begin{Bmatrix}-\sin\phi\\\cos\phi\end{Bmatrix} & r>a,\end{cases}\tag{10}$$

where $w=a(\beta_0^2-k^2n_{cl}^2)^{1/2}$, $u=a(k^2n_{c0}^2-\beta_0^2)^{1/2}$; a is the radius of the fiber and β_0 is the solution of equation $w\,K_1(w)/K_0(w)=u\,J_1(u)/J_0(u)$ with J_n and K_n denoting, as usual, the Bessel and modified Bessel functions.

Substituting (9 and 10) into (8), one has

$$\begin{aligned}\beta_{LP_{01}}^2\approx\beta_0^2+\frac{u^2w^2}{V^2\Lambda^2}&[(\delta_{\phi z}^{c0}-\delta_{\phi z}^{cl}-\delta_{rz}^{c0}+\delta_{rz}^{cl})/a^2\\&-(\delta_{rz}^{c0}\beta_0^2+\delta_{\phi z}^{c0}k^2n_{zc0}^2)(1+\Lambda^2/u^2)\\&+(\delta_{rz}^{cl}\beta_0^2+\delta_{\phi z}^{cl}k^2n_{zcl}^2)(1-\Lambda^2/W^2)],\end{aligned}\tag{11}$$

where $V^2=u^2+W^2$ and $\Lambda=w\,K_1(w)/K_0(w)$. As a result, if $\beta_{LP_{01}}$ is larger than $\max(kn_{rcl}, kn_{\phi cl}, kn_{zcl})$, the fundamental mode will be HE_{11}; while in the case of $\beta_{LP_{01}}$ smaller than $\max(kn_{rcl}, kn_{\phi cl}, kn_{zcl})$, one has TE_{01} or TM_{01} as the fundamental mode. A fuller discussion about this result is as follows.

For the TE_{01} mode having a field component e_ϕ only, which is independent of the ϕ coordinate, we see from (1b) that the differential equations satisfied by the non-zero field component reduces to a form exactly the same as the equations for isotropic fibers. Therefore, in the absence of HE_{11} and TM_{01} modes, the guide will be single-mode with a range determined by the TE_{01} and TE_{02} cut-offs, i.e., $2.405<V<5.52$, a result already given in [1].

However, if the TM_{01} mode does exist, the whole event will be different. Thus, substituting $\partial/\partial\phi=0$ into (1a), one finds a non-zero self-coupling term on the right-hand side of (1a), which can be simplified as

$$[\nabla_t^2+(k^2n_r^2-\beta^2)n_z^2/n_r^2-1/r^2]e_r=0\tag{12}$$

$$\varepsilon\beta_1^2=\frac{-2\int_{s_\infty}\left[k^2n_0^2(\delta_{rz}e_{0r}^2+\delta_{\phi z}e_{0\phi}^2)+\delta_{rz}\left[e_{0r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}re_{0r}\right)+e_{0\phi}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial e_{0r}}{\partial\phi}\right)\right]+\delta_{\phi z}\left[e_{0r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial e_{0\phi}}{\partial\phi}\right)+\frac{e_{0\phi}}{r^2}\frac{\partial^2 e_{0\phi}}{\partial\phi^2}\right]ds\right.}{\int_{s_\infty}(e_{0r}^2+e_{0\phi}^2)ds}.\tag{8}$$

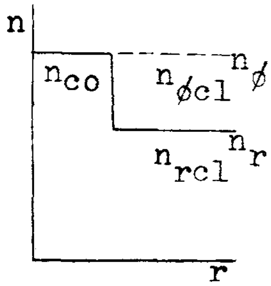

Fig. 1. Anisotropic refractive index profile for azimuthal birefringent cladding

whose solution is

$$e_r = \begin{cases} \dfrac{A}{n_r^2} \dfrac{J_1(u\varrho n_z/n_r)}{J_1(un_z/n_r)} & \varrho < 1 \\ \dfrac{A}{n_r^2} \dfrac{K_1(w\varrho n_z/n_r)}{K_1(wn_z/n_r)} & \varrho > 1 \end{cases} \tag{13}$$

with $\varrho = r/a$, $u = a(k^2 n_{rc0}^2 - \beta^2)^{1/2}$, $w = a(\beta^2 - k^2 n_{rcl}^2)^{1/2}$. The boundary condition gives

$$\frac{u J_0(un_{zc0}/n_{rc0})}{J_1(un_{zc0}/n_{rc0})} = -\frac{n_{zc0} n_{rc0}}{n_{zcl} n_{rcl}} \frac{w K_0(wn_{zcl}/n_{rcl})}{K_1(wn_{zcl}/n_{rcl})} \tag{14}$$

and at cut-offs, one has

$$J_0(un_{zc0}/n_{rc0}) = 0 . \tag{15}$$

Zeros of $J_0(x)$ are 2.405, 5.52, 8.654 .... Thus, in the absence of LP_{01} and TE_{01} modes in a fiber whose refractive index profile is shown in Fig. 1, the guide is single-mode in the range

$$2.405 n_{rc0}/n_{zc0} < V < 5.52 n_{rc0}/n_{zc0} \tag{16}$$

we also note that, if the HE_{11} mode is cut-off, but $\Delta_r \neq 0$, then in the case of $n_{\phi c0}(\Delta_\phi)^{1/2} > n_{zc0}(\Delta_r)^{1/2}$, the fundamental "bound" mode will be TE_{01} with the single-mode range

$$2.61 n_{zc0}(2\Delta_r)^{1/2} < \lambda/a < 2.61 n_{\phi c0}(2\Delta_\phi)^{1/2} . \tag{17}$$

Conversely, if $n_{zc0}(\Delta_r)^{1/2} > n_{\phi c0}(\Delta_\phi)^{1/2}$, then the fundamental "bound" mode will be TM_{01} whose single-mode range is

$$2.61 n_{\phi c0}(2\Delta_\phi)^{1/2} < \lambda/a < 2.61 n_{zc0}(2\Delta_r)^{1/2} . \tag{18}$$

Conclusion

Quite different from anisotropic fibers with $\hat{x}$, $\hat{y}$, and $\hat{z}$ as principal axes, the degenerate TE_{0n} and TM_{0n} modes in a radially anisotropic fiber do not have a leaky-mode effect, and their cut-off conditions are determined only by the profiles with respect to those principal axes along which the modes have non-zero electrical components.

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Reference

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