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Relativistic Filamentation of Laser Beams in Plasma

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Abstract. We describe the results of a numerical code which models the relativistic self-focussing of high-intensity laser beams in plasmas by the nonlinear relativistic dependence of the optical constants on laser intensity. The plasma dynamics of 10^{13} W Nd glass lasers of 30 μm initial beam diameter in nearly cut-off density plasmas is followed for a few picoseconds interaction time and 25 μm depth into the plasma. Rapid relativistic self-focussing down to a beam diameter of one micron in a distance of the order of the original beam diameter is observed, as well as the production of GeV ions moving against the laser light.

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The theory of the relativistic self-focussing of a laser beam in a homogeneous plasma was derived from a consideration of the dependence of the optical constants on the relativistic change of mass of the electrons during their oscillations in a high-intensity laser field [1]. The electron mass will undergo a relativistic change if the oscillation energy of the electrons, given by

$$E_{\mathrm{osc}} = m_0 c^2 \left\{ \left[1 + 3A(I)\frac{I}{I^*} \right]^{1/2} - 1 \right\} \tag{1}$$

is close to or above m_0c^2, where m_0 is the rest mass of the electron, c the velocity of light, I the laser intensity and I^* the relativistic threshold intensity

$$I^* = \frac{3m_0^2\omega^2c^2}{8\pi e^2}. \tag{2}$$

The factor $A(I)$ is a monotonic function rising from 1 for $I \ll I^*$ to 1.061 for $I \gg I^*$. It is interesting that this relativistic self-focussing becomes significant even for intensities one thousand times lower than I^* (which means intensities exceeding 4×10^{15} W/cm^2 for a Nd glass laser). The relativistic self-focussing occurs almost instantaneously (the time taken is of the order of one period of the laser light) and results in focussing down to a diameter of the order of one wavelength of the laser light, provided that the electron density is very close to the critical density [2] (i.e. that value of the electron density at which the plasma frequency is equal to the laser frequency. For Nd glass lasers this is 1.0×10^{21} cm^{-3}).

Another self-focussing mechanism for laser beams in plasmas is due to the nonlinear ponderomotive force [3]. This force is proportional to the gradient of the square of any electric field in the plasma. The Gaussian radial dependence of the electric field in a laser beam results in a radial nonlinear force which lowers the electron density near the centre of the beam, which changes the optical constants and again leads to self-focussing. This movement of the plasma by the nonlinear pondermotive force destroys the homogeneity of the plasma required for the observation of relativistic self-focussing. Fortunately the nonlinear force self-focussing is delayed by the inertia of the plasma, so by looking at sufficiently short times (a few picoseconds) we are able to see the self-focussing due to relativistic effects.

The gradient in electric field strength along the axis of the laser beam produced by the relativistic self-focussing also results in a strong nonlinear force which accelerates ions to high energies. Approximate calculations by Hora et al. have shown that these energies can reach the GeV range for appropriate laser powers and plasmas [4].

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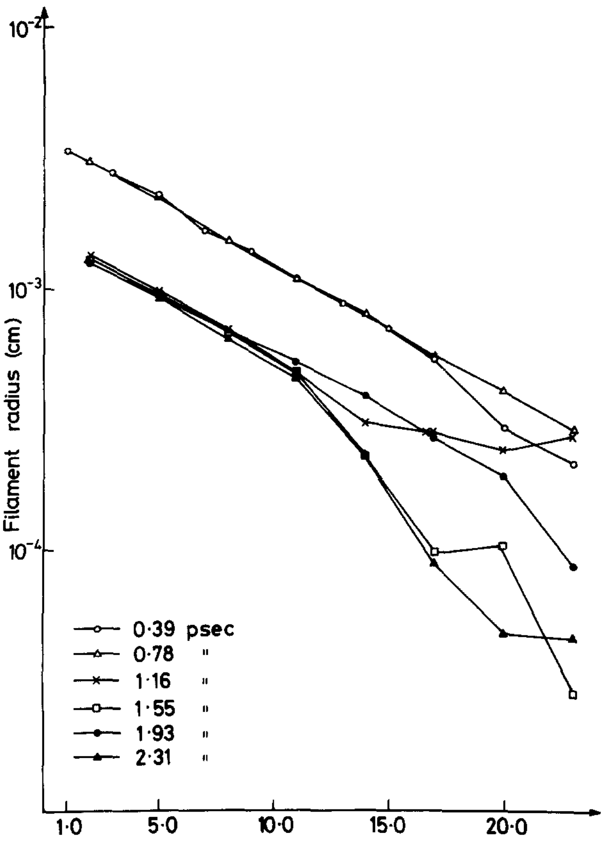

Fig. 1. Filament radius as a function of axial distance into the plasma for various time steps

A two-dimensional time dependent code has been written which accurately models the propagation of a high intensity laser beam in a plasma, including relativistic self-focussing and the thermokinetic and nonlinear forces of plasma motion [5]. In this paper we describe recent results from this programme for the highest laser powers presently attainable (10^{13} W). These results illustrate the occurrence of strong relativistic self-focussing and acceleration of ions by the non-linear force to energies of the order of 5 GeV.

The programme combines Maxwell's laws for the electromagnetic field and the two fluid conservation equations for the plasma. The electric field $\mathbf{E}$ satisfies

$$\nabla^2\mathbf{E}-\nabla(\nabla\cdot\mathbf{E})-\frac{1}{c^2}\frac{\partial^2(n^2)}{\partial t^2}=0, \qquad (3)$$

where the refractive index n is given by

$$n^2=1-\frac{\omega_p^2}{\omega^2+\nu^2}\left(1+\frac{i\nu}{\omega}\right). \qquad (4)$$

Here $\omega_p=(4\pi e^2 n_e/m_0\gamma^R)^{1/2}$ is the generalised plasma frequency which takes into account the relativistic mass change resulting from the high electron quiver velocity in the strong laser field. The electron collision frequency ν also includes a relativistic generalization to allow for a nonlinear dependence on the electric field amplitude.

By writing

$$\mathbf{E}=\mathrm{Re}\{\psi(\mathbf{r},t)e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})}\} \qquad (5)$$

and separating out the rapidly varying space and time dependent terms, (3) can be written

$$\frac{\partial\psi}{\partial z}+\frac{i}{2k_0 r}\frac{\partial}{\partial r}\left(\frac{r\partial\psi}{\partial r}\right)+\frac{ik_0}{2}\left(\frac{\alpha(t)}{k_0^2}-1\right)+\frac{i\beta(t)}{2k_0}\frac{\partial\psi}{\partial t}=0, \qquad (6)$$

where cylindrical coordinates have been used, and $\alpha(t)$ and $\beta(t)$ contain the intensity dependent nonlinear and relativistic optical constants. For computational purposes the time derivative in (6) is removed using an independent variable transformation.

The plasma dynamics is described by using the continuity and momentum conservation equations for the electron and ion fluids. The electron momentum equation contains the relativistic factor γ^R and the nonlinear pondermotive force, i.e.

$$m_0\frac{\partial}{\partial t}(\gamma^R\mathbf{v}_e)+m_0\mathbf{v}_e\cdot\nabla(\gamma^R\mathbf{v}_e)=-\frac{m_0c^2}{2}\nabla\gamma^R-e\mathbf{E}-\frac{\nabla p_e}{n_e}. \qquad (7)$$

The ion momentum equation is similar but does not contain relativistic or ponderomotive effects. By eliminating the electric field between the two equations, using the continuity equations, and setting $n_e=Z_I n_I$ an equation for $N=n_e/n_0$ can be written,

$$\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\ln N}{\partial\eta}\right)-\Gamma\frac{\partial^2\ln N}{\partial\tau^2}-\sqrt{\Gamma}\frac{\partial}{\partial\tau}\left(\frac{v_R}{c_I}\frac{\partial\ln N}{\partial\eta}\right)=G(\eta,z,\tau), \qquad (8)$$

where η and τ are dimensionless radial and time coordinates and G is the electrodynamic source term. Derivatives in the axial direction have been neglected and several small nonlinear terms have been dropped. Equation (8) is solved by the combination of an iteration technique and a Hankel transformation over the variable η. Solution of the coupled set of equations, i.e. ion-acoustic (8) and electrodynamic (6), proceeds in a self-consistent stepwise fashion.

For the computation, we used a five picosecond pulse from a Nd glass laser having a maximum power of 10^{13} W and an initial beam diameter of 30 μm interacting with a 38 times ionized tin plasma with an electron density equal to the critical value. The laser parameters were chosen to model currently available powers and pulse lengths. The laser system at the Australian

National University is now operating with five to six picosecond pulses at 0.5 TW [6], and other laser systems have been operating with powers of five terrawatts per beam for several years [7].

Figure 1 shows the filament radius as a function of distance into the plasma for various times. The filament radius is defined as that value of the radial coordinate which contains 90% of the beam power, which explains why the initial filament radius of 33 μm is larger than the beam radius in vacuum, as this is defined in terms of the half intensity points. The sudden decrease in filament radius between 0.78 and 1.16 ps is caused by the steep rise in laser power (by one hundred) during this interval. The effect is not seen when more smoothly varying envelope functions are used. Decrease of the filament radius to approximately one micrometer at a depth into the plasma of 22 μm is clearly seen. The oscillation of the filament radius with time beyond a depth of 15 μm is consistent with a detailed application of the paraxial-ray approximation to a generalized self-focussing situation [5]. Another factor contributing to these oscillations is the breakdown of the uniform plasma approximation. After only one ps interaction time, an examination of the electron density profile shows depression of the n_e/n_{ec} ratio to approximately 0.5 near the plasma interface. As the self-focussing length is critically dependent on this ratio some deviations in the self-focussing length are to be expected.

Figure 2 shows the centreline ion energy as a function of depth into the plasma for different times. The highest energy of approximately 5 GeV is consistent with earlier approximate theories of this effect. An approximate expression from Hora et al. [4] gives, for the maximum ion energy,

$$\mathscr{E}_I^{\text{trans}} \simeq Z_I \frac{m_0 c^2}{4} \frac{I}{I_{\text{rel}}}. \tag{9}$$

We estimate the intensity from the filament radius at a previous time step and a few μm deeper into the plasma. This gives $I \simeq 3 \times 20^{21}\ \text{W/cm}^2$ and hence $\mathscr{E}_I^{\text{trans}} \simeq 5$ Gev.

It is important to note that we are nearing the limit of the applicability of Newtonian mechanics to the ion motion. For consideration of much lighter ions a relativistic treatment of the ion equations will be required.

The numerical study included a general plasma dynamics and dominating processes counteracting relativistic self-focussing. Since we found the conditions for relativistic self-focussing by using very short pulses, the use of present-day high-power single-beam lasers to produce very high energetic photons, ions and subsequent reactions can open new instrumental methods for nuclear and elementary particle research.

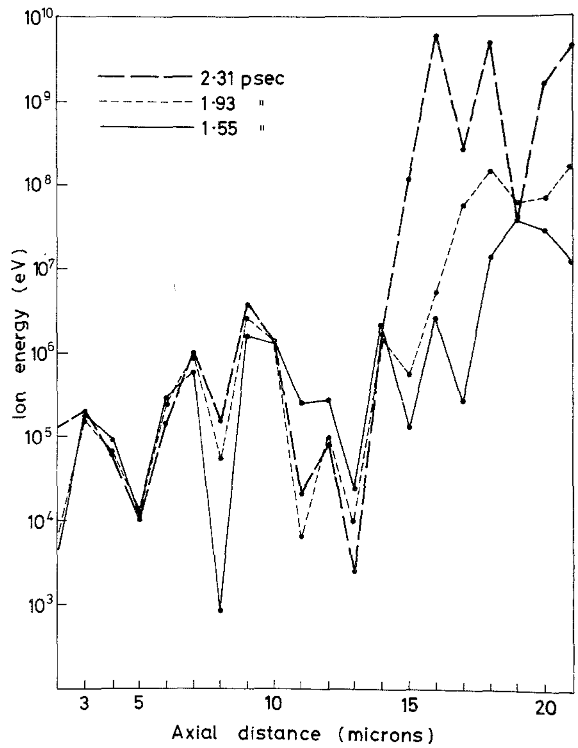

Fig. 2. Centreline ion energy as a function of axial distance into the plasma for various time steps

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