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Birefringence and Polarization Mode Dispersion in Graded-Core Stress-Applied Polarization-Maintaining Fibers

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Abstract. We present a simple analytical method to study birefringence and polarization mode dispersion (PMD) of graded-core stress-applied polarization-maintaining fibers. It is based on the equivalent step-index-fiber method. It is shown that the agreement between present results and the results obtained using the finite-element method is very good. Polarization characteristics of stress-applied polarization-maintaining fibers having a dip in the refractive index at the center of the core are also investigated.

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Optical fibers composed of highly birefringent material are presently of great interest for use in a number of sensor applications and coherent optical communications [1]. To realise dispersion-shifted polarization-maintaining fibers, stress-applied polarization-maintaining fibers with a graded-core profile have been proposed [2]. On the other hand, when manufacturing imperfections occur, we can be left with a graded-core polarization-maintaining fiber. Therefore, it is important to study the birefringence and polarization dispersion of such fibers. A finite-element method has been used to obtain the polarization characteristics of graded-core stress-applied polarization-maintaining fibers [2]. However, the numerical method provides little insight into physical parameter changes. No simple analytical method seems to exist for the study of such fibers.

In this paper, we propose an analytical method that provides simple, closed-form expressions for the birefringence and PMD of graded-core stress-applied polarization-maintaining fibers without significant profile depression. Our objective here is to obtain an acceptable trade-off between the accuracy and the simplicity of the results. It is shown that in the latter, the present results agree very well with the results obtained using the finite-element method [2].

1. Geometry of the Problem

Figure 1 shows the cross section of a graded-core stress-applied polarization-maintaining fiber, where a and b are the radii of core and cladding, respectively, and r_1 and r_2 are the inner and outer radii of stress-applying parts (SAPs), respectively, and θ is the angle of the SAPs. Here the bulk refractive indices of the core, cladding, and SAPs are $n(r)$, n_{cl} and n_s, respectively. For simplicity we assume the refractive index in the SPA is matched with that of the cladding, i.e., $n_s = n_{cl}$. The relative refractive-index difference between the core and the cladding is defined by $\Delta = (n_{co}^2 - n_{cl}^2)/(2n_{cl}^2)$, where n_{co} is the maximum value of the core index.

We use the representation [5] of the index profile of graded-core:

$$n^2(r) = n_{cl}^2[1 + 2\Delta h(r/a)] \tag{1a}$$

with

$$\max\{h(r/a)\} = 1 \qquad 0 \leqq r \leqq a\,, \tag{1b}$$

$$h(r/a) = 0 \qquad r \geqq a\,, \tag{1c}$$

where r is the radial coordinate, $h(r/a)$ denotes the profile variation, and $n(0) = n_{co}$.

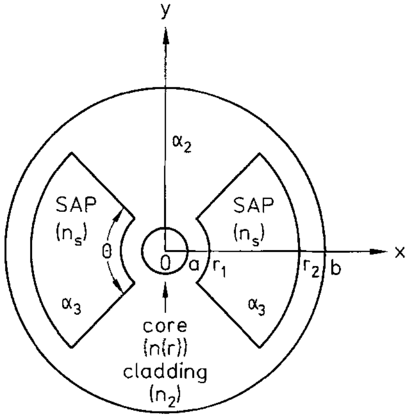

Fig. 1. Cross section of graded-core polarization-maintaining fiber. The parameter value were the following: $2a=5\,\mu m$, $2b=125\,\mu m$, $r_2/b=0.76$, $\theta=90°$, $E=7830\,kg/mm^2$, $v=0.186$, $\alpha_2=5.4\times10^{-7}°C^{-1}$, $\alpha_3=1.554\times10^{-6}°C^{-1}$, $T=-800°C$

For an α-power profile:

$$h(r/a)=1-(r/a)^{\alpha}, \tag{1d}$$

where α is a parameter between 1 and ∞ describes the index profile in the core.

For the dip in refractive index at the centre of the core:

$$h(r/a)=1-q(1-r/a)^{g},$$

where q denotes the relative dip depth and g is a parameter between 1 and ∞ which describes the dip width.

2. Equivalent-Material Birefringence Profiles

Material birefringence B_m in stress-applied optical fibers is produced through the photoelastic effect, and is given by

$$B_m=n_x-n_y, \tag{2a}$$

where n_x and n_y denote the refractive indice for x and y directions, respectively.

It has been shown in [8] that the material birefringence hardly depends on the refractive-index profile in the core. In fact, the material birefringence distribution in stress-applied fibers is complex and varies with position. For simplicity we calculate the equivalent-material birefringence distributions by averging the material birefringence profile in each region. For example,

$$B_{mco}=\frac{\int_0^a\int_0^{2\pi} B_m ds}{\int_0^a\int_0^{2\pi} ds} \qquad 0\leqq r\leqq a, \tag{2b}$$

$$B_{mne}=\frac{\int_a^{r_1}\int_0^{2\pi} B_m ds}{\int_a^{r_1}\int_0^{2\pi} ds} \qquad a\leqq r\leqq r_1, \tag{2c}$$

$$B_{mcl}=\frac{\int_{r_1}^{b}\int_0^{2\pi} B_m ds}{\int_{r_1}^{b}\int_0^{2\pi} ds} \qquad r_1\leqq r\leqq b. \tag{2d}$$

We approximately let

$$B_{mcl}=0 \tag{3a}$$

because it is much smaller than that in the core and the vicinity of the core-cladding interface [9]. We also let

$$B_{mne}=B_{mco} \tag{3b}$$

$$=\frac{E}{1-v}(\alpha_2-\alpha_3)T\frac{C}{\pi}\times\{2\ln(r_2/r_1)-1.5[(r_2/b)^4-(r_1/b)^4]\}\sin\theta \tag{3c}$$

by using (2b) and the concise expression for the materal birefringence in [7]. The parameters C, E, and v denote the stress-optic coefficient, the Young's modulus and the Poission's ratio, respectively, α_2 and α_3 denote the thermal expansion coefficients of a cladding and stress-applying parts, respectively, and T denotes the temperature change in the drawing process. The feasibility of such an approximation will be confirmed in the following.

3. Modal Birefringence and PMD

The analysis is limited to weakly guiding and weakly anisotropic fibers. First, a stress-applied fiber with step-index profile in the core is considered. Since the equivalent-material birefringence n_x-n_y in the fiber is known, the modal birefringence B of such fiber can be derived from [4], where $B=(\beta_x-\beta_y)/k$ (β_x and β_y are the propagation constants for the HE_{11}^x and HE_{11}^y modes, respectively, k is the free-space wave number).

The result is

$$B=B_{mco}S(V)+B_{mne}H(V)+(B_{mne}-B_{mcl})2D(R)/V^2, \tag{4a}$$

with

$$S(V)=(U^2+U^2J_1^2(U)/J_0^2(U)-2UJ_1(U)/J_0(U))/V^2, \tag{4b}$$

$$H(V)=1-S(V), \tag{4c}$$

$$D(R)=\frac{(WR)^2}{2K_0^2(W)}[K_1^2(WR)-K_0(WR)k_2(WR)], \tag{4d}$$

where $R=r_1/a$, $V=kan_{co}(2\Delta)^{1/2}$.

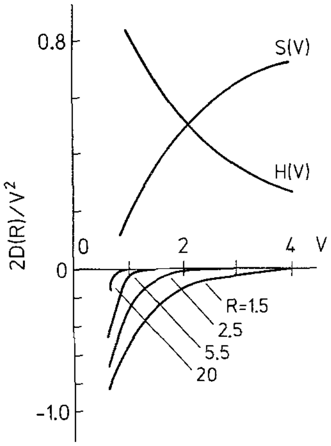

Fig. 2. Normalized frequency dependence of parameters required to calculate birefringence

Given V and its relation to W, then U can be found from [6]:

$$UJ_1(U)/J_0(U)=WK_1(W)/K_0(W) \tag{5a}$$

together with the condition:

$$V^2=U^2+W^2. \tag{5b}$$

The symbols $S(V)$, $H(V)$, and $D(R)/V^2$ indicate dependence on the normalized frequency. The dependence of the birefringence on normalized frequency is plotted in Fig. 2.

Polarization mode dispersion, that is, the group delay difference between the two polarization modes would be

$$\tau=\frac{1}{c}\frac{d(kB)}{dk}, \tag{6a}$$

where c is velocity of light.

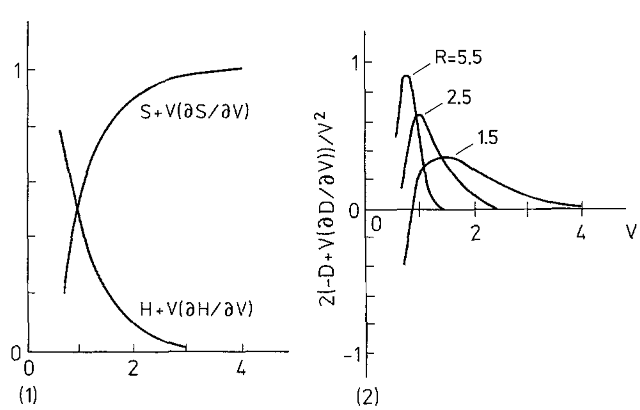

Fig. 3. Normalized frequency dependence of parameters required to calculate polarization dispersion: (1) $S(V)+V\frac{\partial S(V)}{\partial V}$, $H(V)+V\frac{\partial H(V)}{\partial V}$ as a function of V; (2) $2\left[-D(R)+V\frac{\partial D(R)}{\partial V}\right]/V^2$ as a function of V with different R

Results can be derived to be

$$\tau=\frac{1}{c}\left\{B_{\mathrm{mco}}\left[S(V)+V\frac{\partial S(V)}{\partial V}\right]+B_{\mathrm{mne}}\left[H(V)+V\frac{\partial H(V)}{\partial V}\right]+(B_{\mathrm{mne}}-B_{\mathrm{mcl}})\times 2\left[-D(R)+V\frac{\partial D(R)}{\partial V}\right]/V^2\right\} \tag{6b}$$

by substituting (4) into (6a).

The dispersion of the stress-optic coefficient C has been neglected in the derivation of (6b) because it is small compared with the other terms in the useful region [10]. The normalized frequency dependence of the polarization dispersion is also plotted in Fig. 3.

Now, we consider a graded-core stress-applied polarization-maintaining fiber as shown in Fig. 1. When α or g is infinite, the fiber is reduced to the step-core stress-applied fiber.

It has been observed [3] that the fields of the HE_{11} mode on $n_{\mathrm{co}}\cong n_{\mathrm{cl}}$ graded-core fibers without significant profile depression look like the fields of the HE_{11} mode on some step-index fiber. One can find an equivalent step-index fiber (ESF) whose HE_{11} fields closely approximate those of the given graded fiber by using the variational method.

To estimate the polarization characteristics of the graded-core stress-applied fiber, we approximate it by an ESF with anisotropically, azimuthally, radially perturbed refractive-index profiles. Obviously, the results [4] for the anisotropic step-index fiber apply to the anisotropic graded-core fiber provided we replace radius a and normalized frequency V by $\bar{a}$ and $\bar{V}$, where $\bar{a}$ and $\bar{V}$ can be found in [3, 5]. The parameters $\bar{a}$ and $\bar{V}$ are the radius and the normalized frequency of the ESF, respectively. It is assumed that the equivalent-material birefringence profile B_{m} in each region is given by

$$B_{\mathrm{m}}\cong B_{\mathrm{mco}} \quad 0\leqq\bar{r}\leqq\bar{a}, \tag{7a}$$

$$\cong B_{\mathrm{mne}} \quad \bar{a}\leqq\bar{r}\leqq\bar{r}_1, \tag{7b}$$

$$\cong B_{\mathrm{mcl}} \quad \bar{r}_1\leqq\bar{r}\leqq\bar{b}, \tag{7c}$$

where $\bar{r}$, $\bar{r}_1$, and $\bar{b}$ are the radial coordinate, the inner radius of the SAPs and the outer ridius of the ESF with anisotropically, radially perturbed refractive-index profiles, respectively. It is also assumed that $\bar{r}_1/\bar{a}\cong r_1/a$. After using these crude approximations, (4)–(6) also apply to the graded-core stress-applied fiber provided we replace those parameters (V, U, W) with parameters ($\bar{V}$, $\bar{U}$, $\bar{W}$) of the ESF. The parameter $\bar{V}/V$ can be found by the rigorous approach described in [3]. Although this approach requires complicated calculations, it is

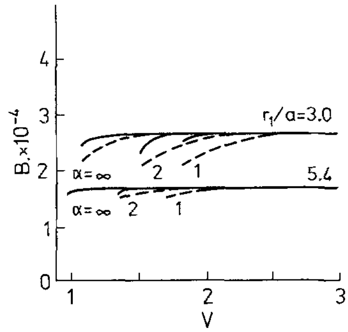

Fig. 4. Variation of the modal birefringence B as a function of normalized frequency V: ——— Reference [2], ------ this paper

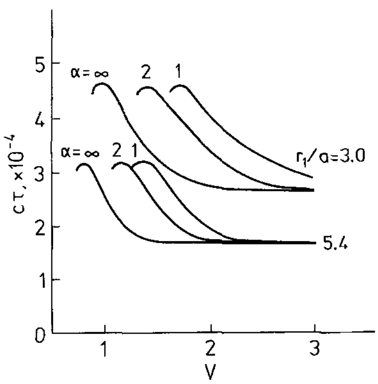

Fig. 5. Variation of the polarization dispersion with V in Fig. 4

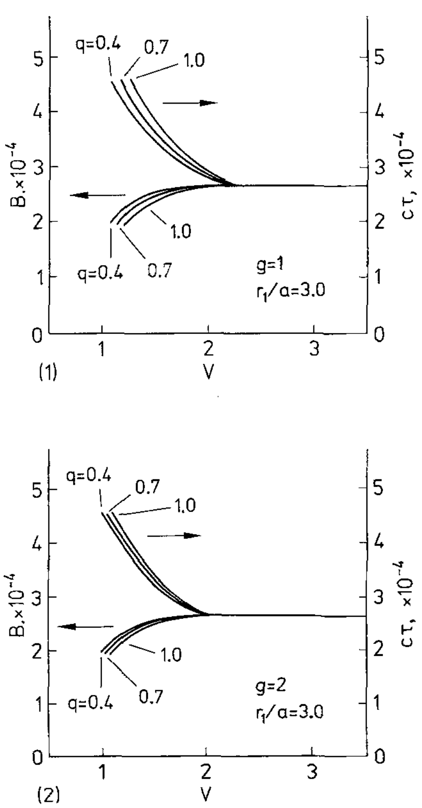

Fig. 6. Variation of birefringence and PMD with V for the fiber having different dip depth q: (1) $g=1$ case, (2) $g=2$ case

very accurate. If we focus on a limited, but useful range (e.g., $1.6 \leqq \bar{V} \leqq 3$), the simple expression for $\bar{V}/V$ is obtained as follows [5]:

$$\bar{V}/V = [\alpha/(\alpha+2)]^{1/2} \quad \text{for} \quad h(r/a) = 1-(r/a)^{\alpha} \tag{8a}$$

$$= [1-2q/(g+1)(g+2)]^{1/2} \quad \text{for} \quad h(r/a) = 1-q(1-r/a)^{g}. \tag{8b}$$

Eq. (4a) will reduce to

$$B = B_{\mathrm{mco}}[1+2D(r_1/a)/\bar{V}^2] \tag{9}$$

by using (3). The value of B approaches B_{mco} as V becomes large. After using (3) and (4), (6b) reduces to

$$\tau = \frac{B_{\mathrm{mco}}}{c}\left\{1+2\left[-D(r_1/a)+\bar{V}\frac{\partial D(r_1/a)}{\partial \bar{V}}\right]/\bar{V}^2\right\}. \tag{10}$$

The value of PMD approaches B_{mco}/c for larger V-values. From (8), $\bar{V}=V$ when α or g is infinite.

4. Results and Discussion

To test the feasibility of the present formulation, we first consider a graded-core stress-applied polarization-maintaining fiber studied by Hayata et al. in [2]. The parameters are chosen to be the same as in [2], which are presented in the caption to Fig. 1.

Figure 4 shows the variation of the modal birefringence B as a function of normalized frequency V. Solid curves correspond to the finite-element calculations [2] and broken curves correspond to the analysis obtained by using (8) and (9). As can be seen from the figure, the two curves agree very well in the region of interest ($1.6 \leqq \bar{V} \leqq 3$), where most of the modal field power of the fiber is confined in the core region. However, the agreement between the two sets of results is not so good in the small-V region, where the fields extend deep into the cladding. When the normalized SAP distance (r_1/a) is large ($=5.4$), the error remains acceptable for the usual applications. It can also be seen from the figure that the smaller the parameter α is, the larger the error. Therefore, the present formulation can be used for estimating the birefringence quite accurately for graded-core stress-applied polarization-maintaining fibers with larger α-values and large normalized SAP distance (r_1/a).

Figure 5 shows the variation of the polarization dispersion with V. As can be seen from the figure, the PMD becomes large in the small-V region. Therefore, we have to pay careful attention to design.

Next, we consider stress-applied polarization-maintaining fibers having a dip in the refractive index at the center of the core. All parameters used in the calculation are the same as in the first example, except for the refractive index distribution in the core. Figure 6 shows the variation of birefringence and PMD with V for a polarization-maintaining fiber having different dip depth q for $g=1$ and 2, respectively. It is known from Fig. 6 that central index dips have little influence on the B and PMD in the large-V region. As shown in Fig. 6, the birefringence in the small-V region gradually decreases as the degree of the dip increases. However, the PMD gradually increases with increase of the degree of the dip in the same V region. It can be seen from Fig. 5 and Fig. 6, the value of B and PMD becomes nearly constant as the V-value becomes large, and then B and PMD can be approximated with the material birefringence B_{mco} in the core and B_{mco}/c, respectively.

In conclusion, we have given a derivation for the modal birefringence and the PMD of graded-core stress-applied polarization-maintaining fibers using the equivalent step-index-fiber method. The equivalent-material birefringence profile has been obtained to analytically deal with the polarization characteristics of the fiber. It has been shown that the present formulation can be used to analyse the birefringence and PMD of graded-core stress-applied fiber with ease.

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