Appl. Phys. B 28, 77–80 (1982)

Applied Physics B Photophysics and Laser Chemistry

Limits of Self-Impedance of a Conductor with Eccentric External Return

J. Bajorek

Department of Electrical Engineering, I. Lukasiewicz Technical University, W. Pola 2, PL-35-959 Rzeszow, Poland

Received 6 July 1981/Accepted 15 January 1982

Abstract. High-and low-frequency limits of the harmonic electromagnetic fields and of the self impedances for parallel conductors are presented. The distribution of current density in the wire and in the eccentric external return is determined analytically with two opposite assumptions for the magnetic field strength on the conductor surface.

PACS: 41, 84

For a simulation of switching processes and other electrical transients on power transmission lines, it is desirable to know the frequency dependence of the line parameters. The problem of this dependence is approached by electromagnetic fields theory. The frequency of the sinusoidally varying field is assumed to be so slow that the wavelength of the oscilations of the alternating currents in the conductors is very much larger than the dimensions of the conductor cross-section. Thus, the electromagnetic field need only be investigated in the cross-section as a two-dimensional boundary value problem.

The electromagnetic theory of wave propagation along parallel conductors is very old. Much important work has been done in developing and extending this theory. The case of the wire with external coaxial return was described by Schelkunoff [1], but for the problems with an eccentric external return, a bifilar lead and a multiconductor line we have only approximate solutions, obtained with relatively drastic assumptions [2–5]. The aim of the paper is to show two limits of the above problem. Note that for the current density problem in the conductors, the tangential component of the magnetic field strength on the conductor surface must be found as the boundary condition. The first boundary condition is obtained in the paper with the assumption that direct current (dc) is flowing in the conductors. Hence, the quickest way to determine the magnetic field strength of the conductors is to use the principle of linear superposition. It is the so-called low-frequency behaviour, and we get the dc approximation of the problem. In the second estimation we assume as the boundary condition that the current density corresponds to the tangential component of the magnetic field strength for a system of lossless conductors. It is the so-called high-frequency behaviour, and we get the LL (loss-less) approximation of the problem.

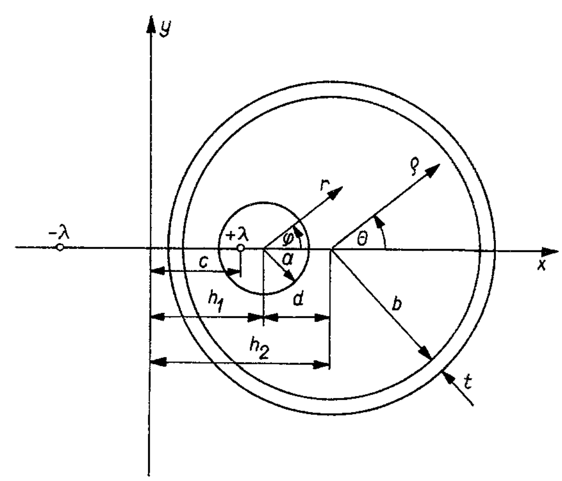

Fig. 1. Configuration of wire with external eccentric return and surfaces of eccentric cable as equipotential surfaces of two charged filaments

Current Density in the Conductors

Consider a wire with an external eccentric return, as in Fig. 1. We assume that the permeability of the conductors is $\mu=\mu_0$ (permeability of free space) and their

0721-7269/82/0028/0077/$01.00

conductivity is σ. We also assume that the current density vector has only an axial component, $\mathbf{J}=J_z\mathbf{1}_z$. The current in the wire is I, and in the external return is $-I$. When using cylindical coordinates (r, φ, z) and considering a sinusoidal variation of the currents in the conductors with time, the equation for the current density in the wire can be written from Maxwell's equations in the form [3]

$$\frac{\partial^2 J_z}{\partial r^2}+\frac{1}{r}\frac{\partial J_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 J_z}{\partial \varphi^2}=\mathrm{j}\omega\mu\sigma J_z=k^2 J_z, \tag{1}$$

where $\omega=2\pi f$ is frequency in rad/s. For the tangential component of the magnetic field strength H_φ on the conductor surface we have from Maxwell's equation

$$H_\varphi(r,\varphi)=\frac{1}{k^2}\frac{\partial J_z(r,\varphi)}{\partial r}. \tag{2}$$

For the low-frequency behaviour we assume that the tangential component of the magnetic field strength on the wire surface is the same as for the conductors with direct current

$$H_\varphi(r,\varphi)|_{r=a}=I/(2\pi a). \tag{3}$$

Thus, the current density inside the wire is

$$J_z(r,\varphi)=\frac{Ik}{2\pi a}\frac{\mathrm{I}_0(kr)}{\mathrm{I}_1(ka)}, \tag{4}$$

where $k=(\mathrm{j}\omega\mu\sigma)^{1/2}$, and I_0, I_1 are the modified Bessel functions of the first kind and order zero and one, respectively.

The tangential component of the static magnetic field on the inner surface of the pipe, in the pipe cylindrical coordinates (ϱ, θ, z), is

$$H_\theta(\varrho,\theta)|_{\varrho=b}=\frac{I}{2\pi b}\left[1+\sum_{n=1}^{\infty}(-d/b)^n\cos(n\theta)\right] \tag{5}$$

and on the outer surface of the pipe, obtained with the help of a linear superposition, is

$$H_\theta(\varrho,\theta)|_{\varrho=b+t}=\frac{I}{2\pi(b+t)}\sum_{n=1}^{\infty}[-d/(b+t)]^n\cos(n\theta). \tag{6}$$

In practice the condition $t \ll b$ can be accepted for the pipe, and (1) may be written in cartesian coordinates

$$\frac{\partial^2 J_z}{\partial \varrho^2}+\frac{1}{b^2}\frac{\partial^2 J_z}{\partial \theta^2}=\mathrm{j}\omega\mu\sigma J_z. \tag{7}$$

The solution of (7) that satisfies the boundary conditions (5) and (6) is

$$J_z(\varrho,\theta)=\frac{Ik_0}{2\pi b}\left\{\frac{\exp[k_0(\varrho-b)]+\exp[-k_0(\varrho-b-2t)]}{1-\exp(2k_0 t)}\right.$$
$$\left.+k_0\sum_{n=1}^{\infty}(-d/b)^n\frac{\cos(n\theta)}{k_n}\,\frac{\exp[k_n(\varrho-b)]-\exp[-k_n(\varrho-b-t)]}{1+\exp(k_n t)}\right\}, \tag{8}$$

where $k_n=[(n/b)^2+\mathrm{j}\omega\mu\sigma]^{1/2}$.

If the frequency of the current is so high that the current is practically a surface current, we assume that the tangential component of the magnetic field strength on the conductor surface is the same as in a lossless system with a TEM wave. The axial component of the electric field strength is assumed to be only a disturbance of the TEM field [6]. Our problem is to find the electromagnetic field between the conductors. Since the TEM field obeys the laws of a static field in the two-dimensional transverse plane, the problem can be solved as the electrostatic problem of two charged filaments. We assume that the surface of the wire and the inner surface of the pipe are equipotential surfaces of these charged filaments $+\lambda$ and $-\lambda$, as shown in Fig. 1. In the figure λ is a uniform linear density of charge [c/m], and

$$h_1=(b^2-a^2-d^2)/(2d), \qquad c=(h_1^2-a^2)^{1/2}. \tag{9}$$

The electrostatic potential of the filaments is

$$V(x,y)=\frac{\lambda}{2\pi\varepsilon}\ln(r_1/r_2), \tag{10}$$

where $r_1=[(x+c)^2+y^2]^{1/2}$ and $r_2=[(x-c)^2+y^2]^{1/2}$. The normal component of the electric field strength in cylindrical coordinates of the wire is

$$E_r(r,\varphi)=-\partial V(r,\varphi)/\partial r \tag{11}$$

and the tangential component of the magnetic field strength between the conductors, as the component of TEM field, is proportional to $E_r(r,\varphi)$ [6]. As the additional condition for $H_\varphi(r,\varphi)$, we notice that

$$a\int_0^{2\pi}H_\varphi(r,\varphi)|_{r=a}\,d\varphi=I. \tag{12}$$

From (10)–(12) we get

$$H_\varphi(r,\varphi)|_{r=a}=\frac{I}{2\pi a}\left\{1+2\sum_{n=1}^{\infty}[-a/(h_1+c)]^n\cos(n\varphi)\right\}. \tag{13}$$

The current density in the wire, as the solution of (1) that satisfies the boundary condition (2) and (13), is

$$J_z(r,\varphi)=\frac{Ik}{2\pi a}\left\{\frac{\mathrm{I}_0(kr)}{\mathrm{I}_1(ka)}+4\sum_{n=1}^{\infty}[-a/(h_1+c)]^n\cdot\frac{\mathrm{I}_n(kr)\cos(n\varphi)}{\mathrm{I}_{n-1}(ka)+\mathrm{I}_{n+1}(ka)}\right\}, \tag{14}$$

where I_n is modified Bessel function of the first kind and order n.

For the inner surface of the pipe we get

$$H_\theta(\varrho,\theta)|_{\varrho=b} = \frac{I}{2\pi b}\left\{1+2\sum_{n=1}^{\infty}[-b/(h_2+c)]^n\cos(n\theta)\right\}, \tag{15}$$

where $h_2=(b^2-a^2+d^2)/(2d)$.

For the outer surface of the pipe in the LL approximation we assume that

$$H_\theta(\varrho,\theta)|_{\varrho=b+t}=0. \tag{16}$$

The solution of (7) that satisfies the boundary conditions (15) and (16) is

$$J_z(\varrho,\theta)=\frac{Ik_0}{2\pi b}\left\{\frac{\exp[k_0(\varrho-b)]+\exp[-k_0(\varrho-b-2t)]}{1-\exp(2k_0t)} + 2k_0\sum_{n=1}^{\infty}[-b/(h_2+c)]^n\frac{\cos(n\theta)}{k_n}\frac{\exp[k_n(\varrho-b)]+\exp[-k_n(\varrho-b-2t)]}{1-\exp(2k_nt)}\right\}. \tag{17}$$

Self Impedances of the Conductors

The self impedance of the conductor is defined with the help of Poynting's theorem as the ratio of electromagnetic power flow into the conductor per unit length and the square of the current modulus [7]. For the wire of radius a, for instance, we have

$$Z_s=\frac{1}{|I|^2}\int_A(\mathbf{E}\times\mathbf{H}^*)\cdot d\mathbf{A} = \frac{a}{|I|^2}\int_0^{2\pi}E_z(r,\varphi)|_{r=a}H_\varphi^*(r,\varphi)|_{r=a}d\varphi,\ [\Omega/\mathrm{m}], \tag{18}$$

where $E_z(r,\varphi)=J_z(r,\varphi)/\sigma$, and the asterisk denotes complex conjugation.

The self-impedance consists of the resistance $R=\mathrm{Re}\{Z_s\}$, representing the amount of energy dissipated in heat, and the reactance $X=\mathrm{Im}\{Z_s\}$, due to the magnetic flux in the conductor itself. The reactance may be written as a product of frequency and self-inductance L, thus the self-inductance of the conductor is $L=\mathrm{Im}\{Z_s\}/\omega$.

The self-impedance of the wire in the dc approximation, obtained from (3), (4), and (18), is

$$Z_{\mathrm{dc}}=\frac{k}{2\pi a\sigma}\frac{\mathrm{I}_0(ka)}{\mathrm{I}_1(ka)}. \tag{19}$$

We notice that, if $\omega\to 0$, the resistance and the inductance of the wire become

$$R_0=1/(\pi a^2\sigma),\qquad L_0=\mu/(8\pi). \tag{20}$$

Substitution of (5), (6), and (8) into Poynting's theorem shows that the self-impedance of the thin pipe in the dc approximation is given by

$$Z_{\mathrm{dc}}=\frac{k_0}{2\pi b\sigma}\left[\frac{\exp(2k_0t)+1}{\exp(2k_0t)-1} + k_0\sum_{n=1}^{\infty}(d/b)^{2n}\frac{1}{k_n}\frac{\exp(k_nt)-1}{\exp(k_nt)+1}\right] \tag{21}$$

and, if $\omega\to 0$, the resistance and the inductance of the thin pipe become

$$R_0=1/(2\pi bt\sigma),\qquad L_0=\mu t[2/3+d^2/(b^2-d^2)]/(4\pi b). \tag{22}$$

The LL approximation of the self impedance for the wire follows from (13) and (14)

$$Z_{\mathrm{LL}}=\frac{k}{2\pi a\sigma}\left\{\frac{\mathrm{I}_0(ka)}{\mathrm{I}_1(ka)} + 4\sum_{n=1}^{\infty}[a/(h_1+c)]^{2n}\frac{\mathrm{I}_n(ka)}{\mathrm{I}_{n-1}(ka)+\mathrm{I}_{n+1}(ka)}\right\} \tag{23}$$

and for the pipe follows from (15)–(17)

$$Z_{\mathrm{LL}}=\frac{k_0}{2\pi b\sigma}\left\{\frac{\exp(2k_0t)+1}{\exp(2k_0t)-1} + 2k_0\sum_{n=1}^{\infty}[b/(h_2+c)]^{2n}\frac{1}{k_n}\frac{\exp(2k_nt)+1}{\exp(2k_nt)-1}\right\}. \tag{24}$$

As an illustration of the derived expressions for the self-impedance of the wire with external eccentric return, Fig. 2 for the wire and Fig. 3 for the pipe are presented. In these figures we introduce so-called skin-depth

$$\delta=[2/(\omega\mu\sigma)]^{1/2}. \tag{25}$$

Figures 2 and 3 illustrate the frequency dependence of the relative resistance R/R_0 and the relative inductance L/L_0 for the case, when the eccentricity of the cable is "large". If the eccentricity of the cable is smaller, then the difference between the curves is also smaller.

Conclusions

Applying the method of the dc approximation and the lossless approximation for magnetic field strength on the conductor surface, it is possible to derive two limits

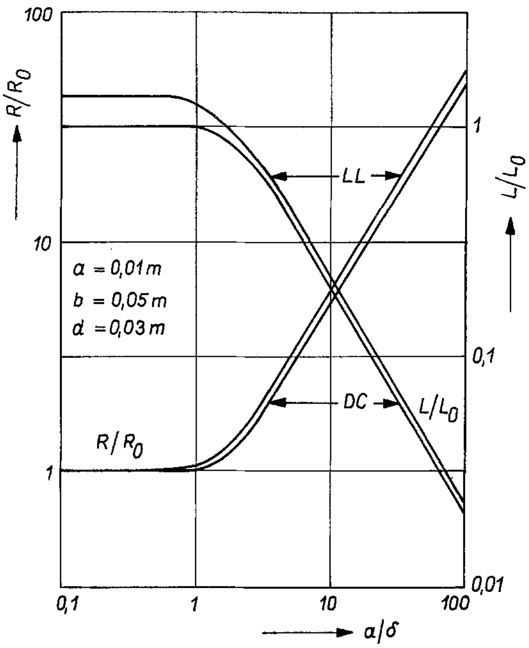

Fig. 2. Ratio of resistance to R_0 and ratio of inductance to L_0 for wire of eccentric cable

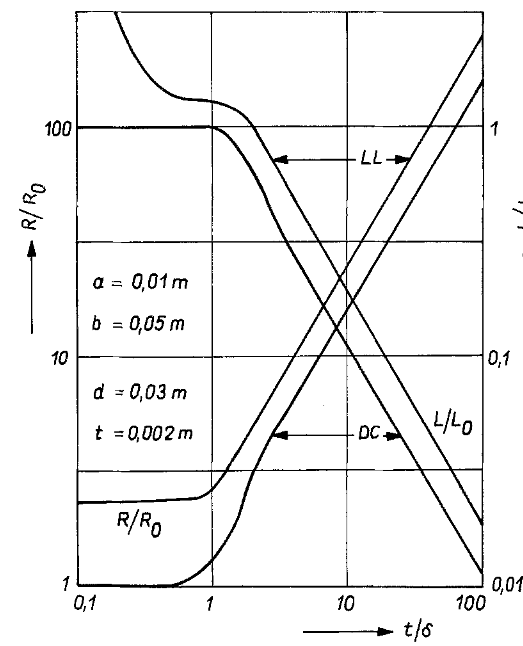

Fig. 3. Ratio of resistance to R_0 and ratio of inductance to L_0 for pipe of eccentric cable

for the self-impedance of the transmission line conductors. The dc-solution is correct if the product $\omega\sigma$ is small and the LL solution is correct if the product $\omega\sigma$ is large. If $\omega\sigma$ is of intermediate value, we can interpolate the curve for the self impedance.

The method may be extended for propagation constant and for multiconductor line as, for instance, three-phase cable (including the shield, if it is present). The results obtained by this method may be utilized for comparing various methods with different assumptions.

References

1. S.A.Schelkunoff: Bell Syst. Techn. J. **13**, 532–578 (1934)
2. R.W.P.King: *Transmission-Line Theory* (Dover, New York 1965)
3. J.A.Tegopoulos, E.E.Kriezis: IEEE Trans. PAS-**90**, 1278–1294 (1971)
4. T.A.Lenahan: Bell Syst. Techn. J. **56**, 597–625 (1977)
5. R.Schinzinger, A.Ametani: IEEE Trans. PAS-**97**, 1680–1687 (1978)
6. R.E.Collin: *Field Theory of Guided Waves* (McGraw-Hill, New York 1960)
7. L.M.Magid: *Electromagnetic Fields, Energy and Waves* (Wiley, New York 1972) p. 588

Responsible for Advertisements: E. Lückermann, G. Probst, Kurfürstendamm 237, D-1000 Berlin 15, Telephone: (030) 8821031, Telex 01-85411
Springer-Verlag Berlin–Heidelberg–New York. Printers: Brühlsche Universitätsdruckerei, Giessen, Printed in Germany © by Springer-Verlag GmbH & Co. KG Berlin · Heidelberg 1982
Beihefterhinweis: Dieser Ausgabe ist ein Prospekt des Springer-Verlages beigeheftet.