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Nonlinear Waves Guided by Graded-Index Films

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Abstract. We derive both the dispersion relation and the power flow for waves guided by a linear graded-index film in contact with an arbitrary nonlinear cladding. For an exponential-like graded-index profile and a Kerr-like nonlinearity we present numerical results and compare them with those familar from the step-index profile.

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Since a few years attention is directed to the unique properties predicted for waves guided by thin films, when at least one of the bounding media exhibits a field-dependent refractive index [1, 2].

These nonlinear guided waves (NGWs), recently observed experimentally [3, 4], lead to a number of novel phenomena.

The dispersion relations were derived for Kerr-like media [1, 2] and for more general nonlinearities [5, 6, 8].

Self-focusing media lead to both multiple new branches with power thresholds and field patterns whose maxima cross the film boundaries towards the adjacent nonlinear media with increasing power.

These NGWs may have a great potential as all-optical devices, e.g., limiters, bistable switches, power thresholds, in integrated optics [7]. Up to now, numerous different implementations with respect to the waveguiding geometry (film or interface), to the linear materials (dielectrics or metals), to the kind of nonlinearities and to the state of polarisation have been explored in the literature.

But the current state-of-art is restricted to step-like linear profiles. On the other hand, some waveguide fabrication techniques as, e.g., diffusion, field-assisted diffusion and ion exchange provide graded-index profiles.

Such graded-index profiles will surely modify the NGW properties in comparison to step-index films. In this letter we present a formalism that permits to derive dispersion relations as well as the guided power flow of TE–NGWs in graded-index films with general field-dependent claddings. As an example we apply our results to exponential-like profiles.

1. Dispersion Relation and Power Flow Dependence of the Propagation Constant

We characterise the waveguide by the following arbitrary spatial and field-dependent dielectric coefficient

$$\varepsilon(z,|E|)=\begin{cases}\bar{\varepsilon}_1+\varepsilon_{\mathrm{NL}}^{(1)}(|E|), & z<0\\ \bar{\varepsilon}_2+\varepsilon_{\mathrm{IH}}^{(2)}(z), & z\geqq 0\end{cases} \tag{1}$$

with $\varepsilon_{\mathrm{NL}}^{(1)}(|E|)>0$.

For TE-polarised fields we get from Maxwells equations with the ansatz

$$E(x,z,t)=\frac{1}{2}E(z)\exp[\mathrm{i}(k\beta x-\omega t)]+\mathrm{c.c.} \tag{2}$$

the nonlinear wave equation

$$d^2E(z)/dz^2+k^2[\varepsilon(z,|E|)-\beta^2]E(z)=0, \tag{3}$$

where β is the normalised propagation constant, ω is the frequency and $k^2=\omega^2/c^2$.

For the nonlinear region, labelled by the subscript 1, it is easy to find a first integral [5]

$$(dE_1/dz)^2-k^2\left[\alpha_1^2E_1^2-2\int^{E_1(z)}\varepsilon_{\mathrm{NL}}^{(1)}(|\tilde{E}|)\tilde{E}d\tilde{E}\right]=0, \tag{4}$$

where $\alpha_1=(\beta^2-\bar{\varepsilon}_1)^{1/2}$, that must be zero for vanishing fields and its derivatives in the limit $z\to-\infty$.

This relation holds for an arbitrary dielectric function $\varepsilon_{\mathrm{NL}}^{(1)}(|E|)$.

Furthermore, we introduce the solution of (1) in the inhomogeneous, linear medium to be $E_2(z)=F(z)$, where $F(z)$ tends to zero for z tending to infinity. Applying Maxwell's continuity requirements and taking advantage of (4) we arrive at

$$E_2(0)=E_1(0) \quad \text{or} \quad F(0)=E_0 \tag{5}$$

and

$$dE_2/dz|_0=dE_1/dz|_0$$

or

$$dF/dz|_0 \equiv F'_0=\mathrm{sgn}(dE_1/dz)kE_0\left[\alpha_1^2-2/E_0^2 \times \int^{E_0} \varepsilon_{\mathrm{NL}}^{(1)}(|\tilde{E}|)\tilde{E}d\tilde{E}\right]^{1/2}.$$

With the formal abbreviations

$$\alpha_{1\mathrm{NL}}=(-1)^M\left[\alpha_1^2-2/E_0^2\int^{E_0}\varepsilon_{\mathrm{NL}}^{(1)}(|\tilde{E}|)\tilde{E}d\tilde{E}\right]^{1/2} \tag{6}$$

and $\alpha_{2\mathrm{IH}}=-F'_0/(kE_0)$, we find from (5 and 6) the NGW-dispersion relation

$$\alpha_{1\mathrm{NL}}+\alpha_{2\mathrm{IH}}=0 \tag{7}$$

where $M=1$ applies for NGWs with a field maximum situated within the nonlinear cladding, and $M=0$ otherwise.

The familar limiting case of an arbitrary nonlinear cladding in contact with an homogeneous linear medium leads to

$$\alpha_{2\mathrm{IH}}=-F'_0/(kE_0)=\alpha_2E_0/E_0=\alpha_2$$

with $\alpha_2=(\beta^2-\bar{\varepsilon}_2)^{1/2}$ and eventually (7) becomes $\alpha_{1\mathrm{NL}}+\alpha_2=0$.

This is the well-known dispersion relation of nonlinear surface polaritons investigated in detail in [8].

The guided power flow can easily be evaluated, using (4), by [5, 6]

$$\begin{aligned}P&=\frac{\beta k}{2\mu_0\omega}\int_{-\infty}^{\infty}E^2(z)dz\\&=\frac{\beta k}{2\mu_0\omega}\left[\int_0^{\hat{E}_1}E_1^2/(|dE_1/dz|)dE_1\right.\\&\left.\quad+(-1)^M\int_{\hat{E}_1}^{E_0}E_1^2/(|dE_1/dz|)dE_1+\int_0^{\infty}F^2(z)dz\right].\end{aligned} \tag{8}$$

$\hat{E}_1$ is the maximum attainable field strength within the nonlinear cladding to be calculated from (4) for $dE_1/dz=0$. Note that only for the linear graded-index region the field pattern must be known explicitely.

The combination of (7 and 8) offers the option to calculate the important power dependence of the NGW-propagation constant, familar from the literature as the nonlinear dispersion curve $\beta(P)$.
We want to point out that the procedure is valid for general nonlinearities including, e.g., saturation properties of the nonlinearity and power flow dependence $|E|^p$ $(p\neq 2)$ arising in semiconductor materials as well as for arbitrary graded-index profiles. For the linear region it is necessary to find an analytical solution or the first two coefficients of a power expansion around $z=0$, see (5).

2. Numerical Results: Exponential-Like Film in Contact with a Nonlinear Cladding

As an example we derive and investigate the dispersion relation and the power flow for a geometry where an exponential-like film is in contact with a Kerr-like cladding.

Now (1) reads as

$$\varepsilon(z,|E|^2)=\begin{cases}\bar{\varepsilon}_1+a|E|^2, & z<0\\ \bar{\varepsilon}_2+\Delta\varepsilon_2\exp(-z/\delta), & z\geqq 0\end{cases},$$

$a>0$.

Together with (5, 6) and $F(z)$ from [9] we write down the dispersion relation as

$$\alpha_{1\mathrm{NL}}+(\Delta\varepsilon)^{1/2}[\mathrm{J}_{h-1}(l_0)-\mathrm{J}_{h+1}(l_0)]/\mathrm{J}_h(l_0)=0 \tag{9}$$

where we used (J_h: Bessel function)

$$F(z)=(E_0/\mathrm{J}_h(l_0))\cdot\mathrm{J}_h[l(z)]$$

with

$$h=2\delta k(\beta^2-\bar{\varepsilon}_2)^{1/2}, \quad l(z)=2\delta k(\Delta\varepsilon)^{1/2}\exp(-z/2\delta)$$
$$l_0\equiv l(z=0), \quad d\mathrm{J}_h(l)/dl=\tfrac{1}{2}[\mathrm{J}_{h-1}(l)-\mathrm{J}_{h+1}(l)]$$

Taking advantage of (6) we find $\alpha_1^2-\alpha_{1\mathrm{NL}}^2=a/2E_0^2$ and together with (4 and 8) the power flow is given by

$$\begin{aligned}P&=\frac{\beta}{2\mu_0\omega}\left\{2(\alpha_1-\alpha_{1\mathrm{NL}})/a+(2k/a)\right.\\&\quad\left.\times(\alpha_1^2-\alpha_{1\mathrm{NL}}^2)/\mathrm{J}_h^2(l_0)\int_0^{\infty}\mathrm{J}_h^2[l(z)]dz\right\}\\&=\frac{\beta}{\mu_0\omega a}(\alpha_1-\alpha_{1\mathrm{NL}})\left\{1+k(\alpha_1+\alpha_{1\mathrm{NL}})/\mathrm{J}_h^2(l_0)\right.\\&\quad\left.\times\int_0^{\infty}\mathrm{J}_h^2[l(z)]dz\right\}.\end{aligned} \tag{10}$$

A subsequent elimination of $\alpha_{1\mathrm{NL}}$ by (9) provides an explicit relation between the guided power flow P and the propagation constant β. Note that such an elimination is not possible for step-like film configurations.

$$P=\frac{\beta a_1}{\mu_0\omega a}\{1+\Delta\varepsilon_2/\alpha_1[\mathrm{J}_{h-1}(l_0)-\mathrm{J}_{h+1}(l_0)]/\mathrm{J}_h(l_0)\}$$
$$\times\left\{1+\alpha_1 k(1+\Delta\varepsilon_2/\alpha_1[\mathrm{J}_{h+1}(l_0)\right.$$
$$\left.-\mathrm{J}_{h-1}(l_0)]/\mathrm{J}_h^2(l_0))\cdot\int_0^\infty \mathrm{J}_h[l(z)]dz\right\}. \tag{11}$$

By fixing β we find P allowing for $|\alpha_{1\mathrm{NL}}|<\alpha_1$ with $\alpha_{1\mathrm{NL}}$ from (9). In Fig. 1 we have sketched the propagation constant in dependence on the normalised guided power flow for different values of δ. The fixed parameters are $n_i=1.57$, $n_2=1.56$, $\Delta n_2=\Delta\varepsilon_2/2n_2=0.02$, $\lambda=0.515$ µm, $\bar{a}=a/2n_1$, $P_0=(2\mu_0\omega a)^{-1}$ is the normalisation constant. Due to a comparison the same dependence is depicted in Fig. 2 for a comparable step-like configuration ($n=1.57$, $n_f=1.58$, $n_2=1.56$). The choice of the parameters ensures equal numbers of NGWs in both configurations.

The small arrows indicate the transition of the maximum into the nonlinear cladding.

Comparing Figs. 1 and 2 we find that the dispersion curves of both configurations exhibit a similar shape. The main differences consist in the lowering of power necessary for NGW operation as well as in the exceeding of the upper power threshold of the TE_1–NGW over that of the TE_0–NGW for graded-index films. This is due to a larger penetration depth of the TE field into the linear region.

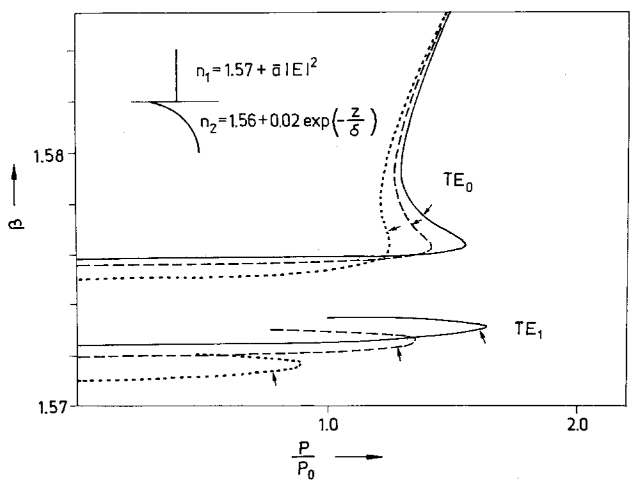

Fig. 1. The normalised guided wave power dependence of the effective index β for TE_0 and TE_1 waves in an exponential-like graded index film. ($\delta=6.5$ µm: full line, $\delta=5$ µm: dashed line, $\delta=4$ µm: dotted line)

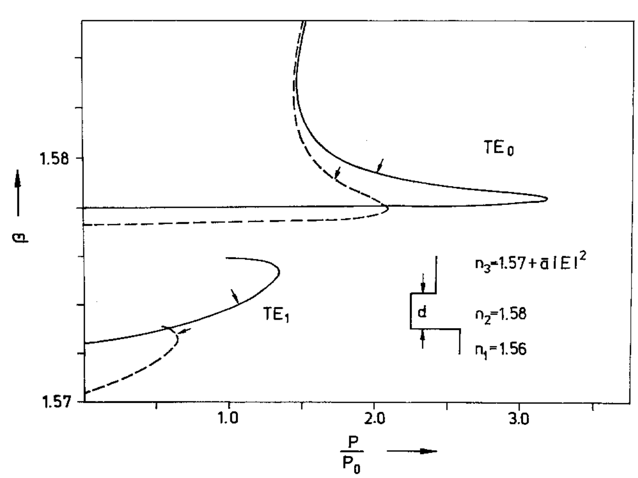

Fig. 2. The normalised guided wave power dependence of the effective index β for TE_0 and TE_1 waves in a step-like configuration ($d=2.5$ µm: full line, $d=2.2$ µm: dashed line)

3. Conclusions

We have presented an approach to derive both the dispersion relation and the power flow for nonlinear waves guided by a linear graded-index film that is in contact with an arbitrary nonlinear cladding. It permits a simultaneous analysis of both a fairly realistic model of the nonlinear cladding as well as non-step-like film configurations. To take into account these both demands is important for a more proper device modelling [7] for all nonlinear media exhibit saturation properties for large intensities and behaves eventually non-Kerr-like. Furthermore semiconductor materials attract an increasing interest as materials with large nonlinear coefficients, but unfortunately due to diffusion and recombination effects they do not behave Kerr-like and hence call also for an approach that is not restricted to the Kerr case. Although the state-of-art in designing step-like configurations is well developed today, in various fabrication techniques graded-index films are produced to serve as integrated-optics devices. All these situations have been covered.

A mathematical advantage over step-like configurations consists in the opportunity to find an explicit analytical relation between the power flow and the propagation constant without a numerical matching of the dispersion relation and the power-flow dependence. The calculated curves show that also in graded-index configurations NGWs do exist and that the typical features are preserved. These features are, e.g., power thresholds, power limits and multibranched curves.

Differences arises in details as the maximum guided power flow and the different behaviour of the TE_1 waves in comparison to the TE_0 waves. Finally, we want to point out that much additional numerical work is in order to investigate other graded-index profiles (e.g., sech profile) and to include into the numerical work arbitrary nonlinear dielectric coefficients. Both these points are under current investigation.

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