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# Behaviour of Polarization Beam Splitters Made from Different Types of Fiber

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**Abstract.** In the paper, polarization beam splitters are studied. Our theoretical study reveals that the coupling length and modulation period of a polarization beam splitter made from the depressed cladding fiber are shorter compared with that made from the matched fiber. Thus the former is characterized by a short interaction length. On the other hand, a polarization beam splitter made from the raised cladding fiber has a longer coupling length and a longer modulation period than those made from the matched fiber.

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Optical fiber polarization beam splitters, which have been realized in several laboratories [1–4], possess a wider range of potential applications, such as in the optical time domain reflectometer (OTDR), optical circulator and various sensors. A polarization beam splitter made from conventional single-mode fibers usually requires a long interaction length (up to 600 mm) compared with that made from a birefringence fiber. It was reported that the interaction length of the polarization beam splitter made from the conventional single-mode fiber had been reduced to about 10 mm [4], an acceptable geometrical size for practical utilization.

The mechanism of fused polarization beam splitters, investigated by several researchers [5–7], is similar to that of the fused coupler. The transfer of light power between two arms is explained by the beating or interference of two lowest-order modes of a composite waveguide. The difference between them lies in the length of the devices. The slight difference in the modal propagation constants of two orthogonally polarized modes, mainly resulting from the asymmetry of the coupler cross section and stress birefringence, will exert tremendous influence on a long-length beam splitter while the asymmetry of the coupler cross section has little impact on a short-length beam splitter. The form birefringence of the couplers, a main source of birefringence, make the output power of the long-length fused coupler exhibit a rapid sinusoidal variation of intensity versus wavelength and elongation, displaying a varied modulation period.

With the help of the rectangular model, Love and Hall [7] presented the theoretical power distribution at two outputs of a polarization beam splitter made from the matched fiber, assuming a linearly polarized light to be incident. Their theoretical result of the output power versus wavelength confirms the experimental measurement quantitatively. Yet, in a general case, the input light of the polarization beam splitter may be elliptically polarized, and the fiber sometimes used for fabricating the beam splitters has the raised or depressed cladding feature. Here we report on the effect of the raised or depressed cladding on the behaviour of the polarization beam splitter and some other relevant problems.

## Polarization Beam Splitter Made from a Matched Fiber

The geometry of the neck section of a beam splitter is shown in Fig. 1a, approximated by the rectangular waveguide illustrated in Fig. 1b, with an equivalent cross section. This equivalence yields the similar transmission characteristics of the circular and rectangular cross-section.

Letting an elliptically polarized light

$$\mathbf{E} = E_{x0} \cdot \mathbf{a}_x + E_{y0} \mathrm{e}^{-j\phi} \cdot \mathbf{a}_y \tag{1}$$

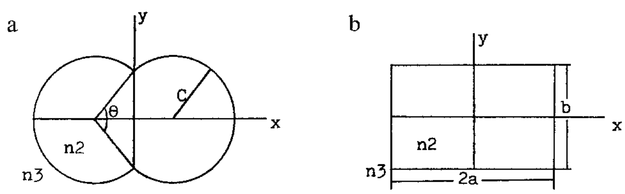


Fig. 1. (a) Cross-section of coupler. (b) Rectangular model $b=\sqrt{\pi}c$, $a=\sqrt{\pi}c[1-(\theta-\sin\theta)/2\pi]$

with unit power enter one of the input ends, supposing $f_1(x,y)$ and $f_2(x,y)$ which are normalized by $\iint_{S\infty} f_i^2(x,y)dxdy=1$ $(i=1,2)$ are the lowest- and second-lowest-mode fields of the composite waveguide, and identifying the powers $P_1$ and $P_2$ with the total power in the region $\chi>0$ and $\chi<0$, respectively, then the powers at distance $L$ from the input ends are given

$$P_1=\left(1+2Q\iint_{x>0} f_1(x,y)f_2(x,y)dxdy\right)\Big/2, \tag{2a}$$

$$P_2=\left(1+2Q\iint_{x<0} f_1(x,y)f_2(x,y)dxdy\right)\Big/2, \tag{2b}$$

$$\begin{aligned}Q=&\cos[(C_x+C_y)L]\cos[(C_x-C_y)L]\\&-\cos(2\psi)\sin[(C_x+C_y)L]\sin[C_x-C_y)L],\end{aligned} \tag{2c}$$

where $\psi=\tan^{-1}E_{y0}/E_{x0}$, and the sum and difference of the coupling strength of the $x$ polarization and $y$ polarization are described by

$$\begin{aligned}C_x+C_y=&\frac{3\lambda}{16n_2c^2[1-(\theta-\sin\theta)/2\pi]^2}\\&+\left[\left(1+\frac{1}{V}\right)^{-2}+\left(1+\frac{n_3^2}{n_2^2}\frac{1}{V}\right)^{-2}\right],\end{aligned} \tag{3a}$$

$$C_x-C_y=\frac{3\lambda^2(n_2^2-n_3^2)^{1/2}}{32n_2^3c^3\pi^{3/2}[1-(\theta-\sin\theta)/2\pi]^3} \tag{3b}$$

with $\lambda$ being the free-space wavelength, and

$$V=2\pi a\sqrt{n_2^2-n_3^2}/\lambda.$$

The term $[1-(\theta-\sin\theta)/2\pi]$ that has not appeared in the previous publication is due to the overlapping of two fibers. If this term is neglected, the coupling length of the beam splitter obtained from (3a) will be larger than its exact value. For instance, if taking $C=16$ µm, $\theta=102.6°$, $n_2=1.458$ and $n_3=1.4$, the evaluation of (3a) in which the term $[1-(\theta-\sin\theta)/2\pi]$ is left out gives the coupling length $Z_c=4.8$ mm at $\lambda=1.3$ µm. Taking into account the term $[1-(\theta-\sin\theta)/2\pi]$, (3a) gives the coupling length $Z_c=3.82$ mm at $\lambda=1.3$ µm which is in good agreement with the value 3.93 mm obtained by the variational method. The integration involving $f_1(x,y)$ and $f_2(x,y)$ in (2) is $8/3\pi$ if employing rectangular waveguide-mode fields, resulting in an incomplete power transfer. This discrepancy with experimental results, introduced by the artificially modelling, can be circumvented by determining the cross integration of $f_1(x,y)$ and $f_2(x,y)$ with the aid of the field at input ends of the beam splitter. Thus we have

$$\iint_{x\gtrless 0} f_1(x,y)f_2(x,y)dxdy=\pm 1/2. \tag{4}$$

With the help of (2) and the principle of equivalent volume before and after fusing, the point at which the power transferring in $a$ through arm reaches maximum can be written as

$$L_n=L_0\sqrt{n},\quad n=1,2,3\ldots, \tag{5a}$$

$$L_0=4C_i\left\{\frac{\pi n_2L_i}{3\lambda}[1-(\theta-\sin\theta)/2\pi]\right\}^{1/2}, \tag{5b}$$

where $C_i$ and $L_i$ are an initial fiber cladding radius and length, respectively. The coupling length, the peak-to-peak distance of the power transfer are expressed as

$$Z_c=L_{n+1}-L_n=L_0(\sqrt{n+1}-\sqrt{n}). \tag{6}$$

It is obviously that $Z_c$ gradually diminishes with increasing peak order $n$, namely with increasing distance $L$, just as reported in the experimental measurement [4]. The modulation period, the peak-to-peak distance of an envelope, is described by

$$Z_n=L_p[(m+1)^{2/5}-m^{2/5}], \tag{7a}$$

$$L_p=\pi\left\{\frac{64n_2^3c_i^3L_i^{3/2}}{3\lambda^2(n_2^2-n_3^2)^{1/2}}[1-(\theta-\sin\theta)/2\pi]^{3/2}\right\}^{2/5} \tag{7b}$$

decreasing as the order $m$ increases, i.e., as the elongation of the waist region increases, and behaving similarly as the reported experimental result [4]. The modulation depth is determined by $\psi$, turning out to be unity for an equal excitation $E_{x0}=E_{y0}$ and zero for $\psi=0$ or $\psi=\pi/2$.

So far we have discussed the power distribution at the output ports under polarized-light illumination. If unpolarized light illuminates one of the inputs of the polarization beam splitter, the power distribution at the output ports can be obtained by taking the statistical average with respect to $\psi$ over the range of $2\pi$

$$\langle P_1\rangle=\{1+\cos[(C_x+C_y)L]\cos[(C_x-C_y)L]\}/2, \tag{8a}$$

$$\langle P_2\rangle=\{1-\cos[(C_x+C_y)L]\cos[(C_x-C_y)L]\}/2 \tag{8b}$$

having a constant modulation depth of 1.

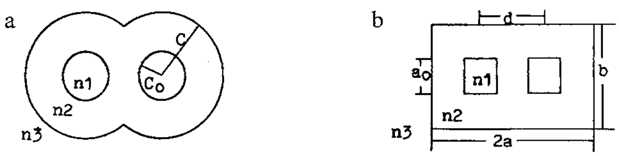


Fig. 2. (a) Cross-section of coupler made from fiber with disturbed cladding. (b) Rectangular model $a_0=\sqrt{\pi}C_0$, $d=2a-b$

## Polarization Beam Splitter Made from a Raised or Depressed Cladding Fiber

Now we turn our attention to the polarization beam splitters made from a raised or depressed cladding fiber, especially their coupling length and modulation period, as their other characteristics are similar to those made from a matched fiber. The refractive index profile of the cross section in the waist region together with its equivalent rectangular model is shown in Fig. 2. Without loss of generality, we restrict our analysis to the weakly guiding fiber; therefore, the difference between $n_1$ and $n_2$ is small and the vector perturbation theory may be employed to find the propagation constants of the symmetric and asymmetric modes of the composite waveguide as well as coupling coefficients.

The vector perturbation theory is based on the stationary expression [9]

$$\delta\beta^i_{mn}=\bar{\beta}^i_{mn}-\beta^i_{mn}=\frac{\omega\varepsilon_0\iint(n_1^2-n_2^2)\bar{\mathbf{e}}^i_{mn}\cdot\mathbf{e}^{i*}_{mn}ds}{\iint(\bar{\mathbf{e}}^i_{mn}x\mathbf{h}^*_{mn}+\mathbf{e}^{i*}_{mn}x\bar{\mathbf{h}}^i_{mn})\cdot\hat{z}ds},\tag{9}$$

where $i$ takes $x$ or $y$, the unbarred quantities stand for the rectangular waveguide without the perturbation, and the barred quantities represent the rectangular waveguide having the raised or depressed index profile near the center of two arms. The integrals extend over an infinite cross section, and $n_1-n_2$ is non-zero only in the region marked by $n_1$ in Fig. 2. In the case of a small difference between $n_1$ and $n_2$, $\bar{\mathbf{e}}$ and $\bar{\mathbf{h}}$ can be substituted by the unbarred quantities. Employing the field distribution of the rectangular waveguide, we have

$$\delta\beta^x_{11}=\left(\frac{n_1^2}{n_2^2}-1\right)k_0^2n_2^2F_+(k^x_y,a_0)\,[(\beta^x_{11})^2F^c_+(k^x_{1x},a_0)+(k^x_{1x})^2F^c_-(k^x_{1x},a_0)]/p^x_{11}\beta^x_{11},\tag{10a}$$

$$\delta\beta^x_{21}=\left(\frac{n_1^2}{n_2^2}-1\right)k_0^2n_2^2F_+(k^x_y,a_0)\,[(\beta^x_{21})^2F^c_-(k^x_{2x},a_0)+(k^x_{2x})^2F^c_+(k^x_{2x},a_0)]/p^x_{21}\beta^x_{21},\tag{10b}$$

$$\delta\beta^y_{11}=\left(\frac{n_1^2}{n_2^2}-1\right)\{[k_0^2n_2^2-(k^y_y)^2]^2F^c_+(k^y_{1x},a_0)F_+(k^y_y,a_0)+(k^y_y)^2F_-(k^y_y,a_0)\,[(\beta^y_{11})^2F^c_+(k^x_{1x},a_0)+(k^y_{1x})^2F^c_-(k^y_{1x},a_0)]\}/p^y_{11}\beta^y_{11},\tag{10c}$$

$$\delta\beta^y_{21}=\left(\frac{n_1^2}{n_2^2}-1\right)\{[k_0^2n_2^2-(k^y_y)^2]^2F^c_-(k^y_{2x},a_0)F_+(k^y_y,a_0)+(k^y_y)^2F_-(k^y_y,a_0)\,[(\beta^y_{21})^2F^c_-(k^y_{2x},a_0)+(k^y_{2x})^2F^c_+(k^y_{2x},a_0)]\}/p^y_{21}\beta^y_{21},\tag{10d}$$

$$p^i_{11}=[k_0^2n_2^2-(k^i_y)^2]F_+(k^i_x,2a)F_+(k^i_y,b)+\frac{2n_2^2}{q_{y_i}n_3^2}(k_0^2n_3^2+q^2_{y_i})\cos^2(\tfrac{1}{2}k^i_yb)F_+(k^i_{1x},2a)+\frac{2}{q_{1x_i}}\frac{n_2^2}{n_3^2}\eta_i[k_0^2n_3^2-(k^i_y)^2]\cos^2(k^i_{1x},a)F_+(k^i_y,b),\tag{10e}$$

$$p^i_{21}=[k_0^2n_2^2-(k^i_y)^2]F_-(k^i_{2x},2a)F_+(k^i_y,b)+\frac{2}{q_{y_i}}\frac{n_2^2}{n_3^2}(k_0^2n_3^2+q^2_{y_i})\cos^2(\tfrac{1}{2}k^i_yb)F_-(k^i_{2x},2a)+\frac{2}{q_{2x_i}}\frac{n_2^2}{n_3^2}\eta_i[k_0^2n_3^2-(k^i_y)^2]\cos^2(k^i_{2x},a)F_+(k^i_y,b),\tag{10f}$$

$$F^c_\pm(x,y)=y\pm\frac{1}{x}\sin(xy)\cos(xa),\tag{10g}$$

$$F_\pm(x,y)=y\pm\frac{1}{x}\sin(xy)\tag{10h}$$

where

$$k_0=2\pi/\lambda,\qquad k^x_{mx}=m\pi/2a\left(1+\frac{n_3^2}{n_2^2V}\right),$$

$$k^y_{mx}=m\pi/2a\left(1+\frac{1}{V}\right),\qquad k^x_y=\pi/b\left(1+\frac{a}{bV}\right),$$

$$k^y_y=\pi/b\left(1+\frac{n_3^2}{n_2^2}\frac{a}{bV}\right),$$

$$\beta^i_{mn}=[k_0^2n_2^2-(k^i_{mx})^2-(k^i_y)^2]^{1/2},$$

$$q_{y_i}=[k_0^2(n_2^2-n_3^2)-(k^i_y)^2]^{1/2},$$

$$q_{mx_i}=[k_0^2(n_2^2-n_3^2)-(k^i_{mx})^2]^{1/2},\qquad \eta_i=\begin{cases}n_2^2/n_3^2 & i=x\\ 1 & i=y.\end{cases}$$

The perturbation of the coupling coefficients stemming from the refractive-index perturbation inside the rectangular are expressed as

$$\delta C_x=\bar{C}_x-C_x=(\delta\beta^x_{11}-\delta\beta^x_{21})/2,\tag{11a}$$

$$\delta C_y=\bar{C}_y-C_y=(\delta\beta^y_{11}-\delta\beta^y_{21})/2.\tag{11b}$$

It is impossible to evaluate quantitative effects of the raised or depressed cladding on the coupling length and modulation period of the polarization beam splitter by (10 and 11) without resorting to numerical calculations. Assuming a large $V$ value that occurs in most cases, we can have the simplified expressions for the perturbed terms of the sum and difference of the coupling coefficients of two orthogonal modes

$$\delta(C_x+C_y)\simeq\frac{-1}{2}\left(1-\frac{n_2^2}{n_1^2}\right)\left(\frac{C_0}{C}+\frac{1}{\pi}\sin\pi\frac{C_0}{C}\right)\times\left\{(C_x+C_y)\frac{a_0}{2a}+\frac{n_2}{\lambda}\left[\sin\pi\frac{a_0}{a}\cos\pi\alpha-2\sin\frac{\pi a_0}{2a}\sin\frac{\alpha\pi}{2}\right]\right\},\tag{12a}$$

$$\begin{aligned}\delta(C_x-C_y)\simeq&\frac{-1}{2}\left(1-\frac{n_2^2}{n_1^2}\right)\left(1-\frac{n_3^2}{n_2^2}\right)\frac{n_2}{V\lambda}\times\left\{\left(\frac{C_0}{C}+\frac{1}{\pi}\sin\pi\frac{C_0}{C}\right)\right.\\&\times\left[\pi(1-\alpha)\left(\cos\frac{\pi}{2}\alpha\sin\frac{\pi}{2}\frac{a_0}{a}+\sin\pi\alpha\sin\pi\frac{a_0}{a}\right)\right.\\&\left.+\pi\frac{a_0}{a}\left(\cos\pi\alpha\cos\pi\frac{a_0}{a}-\sin\frac{\pi}{2}\alpha\cos\frac{\pi}{2}\frac{a_0}{a}\right)\right]\\&+2\sin\frac{\pi}{2}\frac{a_0}{a}\left(\sin\frac{\pi}{2}\alpha-\cos\frac{\pi}{2}\frac{a_0}{a}\cos\pi\alpha\right)\\&\left.\times\left(\frac{C_0}{C}-\frac{1}{\pi}\sin\pi\frac{C_0}{C}+\frac{2C_0}{C}\cos\pi\frac{C_0}{C}\right)\right\}\end{aligned}\tag{12b}$$

with $\alpha=b/a-1$. Since $\alpha$ is generally small even for a strongly fused cross section, we can approximate it by zero. Then it is easy to prove that the terms in braces of (12) always exceed zero. Thus the quantitative relation between $n_1$ and $n_2$ will determine $\delta(C_x+C_y)$ and $\delta(C_x-C_y)$, whether to be positive or negative. If $n_1>n_2$ for the case of the raised cladding, the quantities $\delta(C_x+C_y)$ and $\delta(C_x-C_y)$ will be less than zero. On the contrary, the quantities $\delta(C_x+C_y)$ and $\delta(C_x-C_y)$ for the depressed cladding will be greater than zero. Because the coupling length and modulation period are inversely proportional to the sum and difference of the coupling coefficients, thus we may conclude that the polarization beam splitter made from the raised cladding has the longer coupling length and modulation period than that made from the matched fiber; and the polarization beam splitter made from the depressed cladding possesses the shorter coupling length and the shorter modulation period than those made from the matched fiber. This means that the polarization beam splitter made from the depressed cladding requires the shortest interaction length and the one made from the raised cladding needs the longest interaction length among three kinds of conventional single-mode fiber. In reaching these conclusions, a tacit assumption has been born in mind that all the parameters of the fibers are the same except the difference in the cladding index profile. Our prediction has been confirmed by the experiment engaged in our laboratory.

As seen, the rectangular model has been employed in the analysis. Similar conclusions can be obtained if we resort to elliptical model, which may be more realistic for practical couplers [10], after undergoing somewhat complex derivation.

## Closing Remark

The behaviour of the polarization beam splitter versus elongation has been discussed theoretically. Our theoretical analysis agrees with the experimental measurement quantitatively. Employing the vector perturbation theory, the coupling coefficients of the polarization beam splitters made from the raised and depressed cladding fibers have been derived. Finally, the useful prediction for practical fabrication is presented.

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