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Generation of Nearly Diffraction-Free Laser Beams

K. Uehara and H. Kikuchi*

Department of Physics, Faculty of Science and Technology, Keio University, Hiyoshi, Kohoku-ku, Yokohama 223, Japan

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Abstract. Laser beams which behave like the so-called diffraction-free beams have been generated by an argon ion laser with a new type of cavity. They have almost the same intensity distributions and propagation characteristics as those predicted for the Bessel-Gauss beams which were introduced recently. Beams corresponding not only to the lowest-order but also to higher-order Bessel-Gauss beams have been obtained.

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Recently, new classes of coherent light beams, called diffraction-free beams, have been studied by Durnin and coworkers [1, 2]. These beams conserve their transverse intensity distribution while they propagate in free space. The diffraction-free beams can be formed by arbitrary superpositions of plane waves whose wave vectors lie on a cone. The simplest diffraction-free beam, called the J_0 beam, has a circularly symmetric amplitude distribution expressed by the zeroth-order Bessel function. The J_0 beam has a central bright spot whose diameter can be very narrow, on the order of one wavelength. However, any ideal diffraction-free beam is experimentally unrealizable because it carries an infinite amount of energy. It was theoretically shown, however, that a J_0 beam of finite width behaves like a diffraction-free beam over a certain propagation distance [1]. Subsequently, this prediction was confirmed by an experiment [2]. In that experiment, a J_0 beam of finite width was generated by illuminating with laser light a circular slit of 10-μm width placed at the focal plane of a focusing lens. Each point source along the slit was transformed by the lens into a plane wave of finite width. As predicted, the central spot of the finite-width J_0 beam thus formed was retained over the propagation distance where the component plane waves overlapped each other.

In the present paper we report the direct generation of nearly diffraction-free beams by an argon ion laser which has a new type of cavity. This active method enables us to generate much more intense nearly diffraction-free beams than the passive method mentioned above. The beams obtained here had almost the same intensity distributions and propagation characteristics as those of the Bessel-Gauss beams which were introduced recently by Gori et al. [3]. Beams corresponding not only to the zeroth-order but also to the first-order Bessel-Gauss beams have been obtained.

1. Expressions for the Diffraction-Free Beams

The expressions for the diffraction-free beams are summarized here for use in the following discussions.

The J_0 beam is formed by an equal-weight superposition of all the plane waves having the same wavelength λ and with wave vectors lying on a cone [1]. The field distribution of a J_0 beam propagating along the z-axis is given by

$$E = E_0 \exp\{\mathrm{i}(\beta z - \omega t)\} J_0(\alpha r), \tag{1}$$

where J_0 is the zeroth-order Bessel function of the first kind and r is the radial distance. The parameters α and β are related to the wavelength λ and the half apex angle θ of the cone by

$$\alpha = 2\pi \sin\theta/\lambda \tag{2}$$

and

$$\beta = 2\pi \cos\theta/\lambda . \tag{3}$$

* Present address: NTT Electrical Communication Laboratories, Yokosuka 238-03, Japan

The intensity distribution is proportional to $J_0^2(\alpha r)$. The diameter of the central spot is given by $4.8/\alpha$, which is equal to 0.76λ in the extreme case of $\theta=\pi/2$. More generally, if the superposition is carried out with a weight function $\exp(in\phi')$, where n is an integer and ϕ' is the azimuthal angle of the wave vector, the resultant field distribution is proportional to the nth-order Bessel function:

$$E=E_0\exp\{i(\beta z-\omega t-n\phi)\}J_n(\alpha r), \quad (4)$$

where ϕ is the azimuthal angle. The intensity distributions of the Bessel beams expressed by (4) are all circularly symmetric. A non-circularly symmetric beam is formed with, for example, a weight function $\cos(n\phi')$. The Bessel beams are experimentally unrealizable because (4) is not square integrable. Similarly, all ideal diffraction-free beams are unrealizable. The finite-width J_0 beam mentioned above is a realizable good approximation to the ideal diffraction-free beams. Further discussions about the properties of the diffraction-free beams have been given by several authors [4–8].

More recently, Gori et al. [3] introduced another new type of beam called the Bessel-Gauss beam whose field distribution at $z\approx 0$ is given by

$$E=E_0\exp\{i(\beta z-\omega t)\}J_0(\alpha r)\exp\{-(r/w_0)^2\}. \quad (5)$$

Eq. (5) is square integrable owing to the last factor. Therefore, the Bessel-Gauss beams can be realized experimentally. The Bessel-Gauss beam expressed by (5) is formed by an equal-weight superposition of gaussian beams which have waists of spot size w_0 at $z=r=0$ and their symmetry axes lying on a cone. The intensity distribution of the Bessel-Gauss beam is not conserved. However, if the diffraction angle of the component gaussian beams $\lambda/\pi w_0$ is smaller than the half apex angle $\theta(=\sin^{-1}(\alpha\lambda/2\pi))$ of the cone, the Bessel-Gauss beam behaves like a diffraction-free beam over the propagation distance where the component gaussian beams overlap each other. At larger distances an annular intensity distribution with a mean radius θz is formed. In the opposite case of $\lambda/\pi w_0>\theta$, on the other hand, the Bessel-Gauss beam behaves almost like an ordinary gaussian beam. Analytic expressions for the field distribution of the Bessel-Gauss beam for arbitrary values of α, w_0, and z are given in [3]. It is easy to show that if the gaussian beams are superposed with a weight function $\exp(in\phi')$, then the nth-order Bessel-Gauss beam is formed. The field distribution at $z\approx 0$ is then

$$E=E_0\exp\{i(\beta z-\omega t-n\phi)\}J_n(\alpha r)\exp\{-(r/w_0)^2\}. \quad (6)$$

2. Experiment and Results

Figure 1 shows a schematic drawing of the new laser cavity used in the present experiment and the contour of a cavity mode in it. The cavity consists of a focusing lens L of focal length f, an output mirror M_1, and an annular mirror M_2 of mean diameter d. The distances l_1 and l_2 of the mirrors from the lens are nearly equal to f. When the width δ of the annular mirror is chosen properly, the cavity will have low losses only for the light waves whose propagation angle θ on the right-hand side of the lens is equal to $\tan^{-1}(d/2f)$. Each such wave returns to the same position on M_2 after two round trips between the mirrors as shown in Fig. 1. If all such waves oscillate simultaneously, their relative amplitudes, including phase factors, are likely to be fixed rather than random because of their mutual overlap. Therefore, it is expected that their superposition forms a beam which behaves like a diffraction-free beam. If individual component waves are gaussian beams of equal amplitudes, the resultant superposed beam becomes a Bessel-Gauss beam as expressed by (5). The combination of a circular slit and a lens employed in the above-mentioned experiment by Durnin et al. [2] was a clue to the present cavity configuration.

An argon plasma tube (Lexel 95) of 5-mm inner diameter and 50-cm discharge length with Brewster-angle windows was inserted between the lens L($f=120$ cm) and the output mirror M_1 (flat, 50% reflectivity). The annular mirror M_2 was composed of a flat reflector with a 1.3-mm diameter hole and a variable iris placed close to the reflecting surface. The mirror distances l_1 and l_2 were set to a few cm longer than f. The output beam was detected at various propagation distances z from the output mirror by an image pick-up tube (Hamamatsu N2634, 20-μm spatial resolution) connected to a monitor television. At the same time, the video signal was fed into an oscilloscope to display the radial intensity profile on it. The image pick-up tube used here has a nominal γ-value of 0.6, i.e., the signal output is proportional, approximately, to $I^{0.6}$, where I is the incident light intensity. The beam was attenuated by filters to prevent saturation of the video signal.

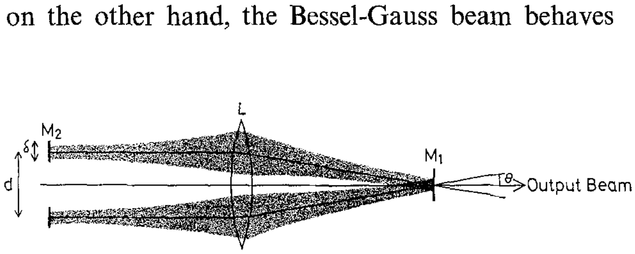

Fig. 1. Schematic of the new laser cavity and the contour of a cavity mode. M_1: output mirror, M_2: annular mirror of mean diameter d and width δ, L: focusing lens of focal length f. The distances l_1 and l_2 are nearly equal to f

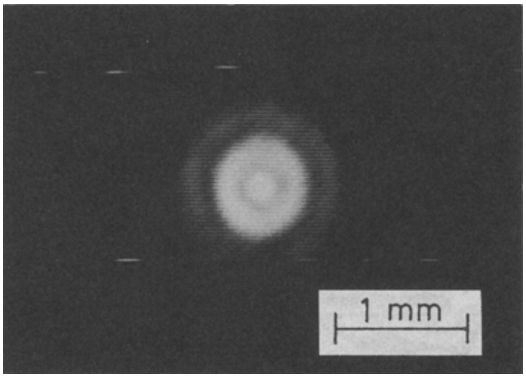

Fig. 2. A circularly symmetric beam pattern with a central bright spot monitored at $z=11$ cm from the output mirror

A laser oscillation was achieved at $\lambda=488$ nm at which the argon plasma tube has the highest gain. A variety of beam patterns have been observed depending on the mirror positions and angles, the iris diameter, the discharge current and the distance z. Figure 2 shows a circularly symmetric beam pattern observed at $z=11$ cm. (The image pick-up tube could not placed at $z<11$ cm for instrumental reasons.) This beam consists of a central bright spot and a few weak concentric rings. The oscilloscope traces in Fig. 3 show the radial intensity profiles of this beam monitored at $z=11$ cm, 50 cm, and 1 m. The half-width (HWHM) at $z=11$ cm is 100 µm. Although the peak intensity decreases with z, the central spot is retained at least up to $z=1$ m, without appreciable spreading. This diffraction-free behavior should be compared with the fact that a single gaussian beam with a 100-µm half-width becomes 5.5 times broader after propagating 1 m.

The profile in Fig. 3a has the first and second intensity minima at 180 µm (r_1) and 410 µm (r_2), respectively. Their ratio r_2/r_1 is very close to the ratio 2.30 of the first and second zeros of the J_0 function, implying that the radial field distribution of this beam at $z=0$ has the J_0 function as a factor. From the values of r_1 and r_2, the beam parameter α is calculated to be $\alpha=14\ \mathrm{mm}^{-1}$. Then, θ can be determined from (2) as $\theta=1.1\times10^{-3}$ rad. This value of θ is consistent with the observation that the outer peak in Fig. 3c ($z=1$ m) is located at $r=1.1$ mm.

Figure 4 shows the theoretical radial intensity profiles of the zeroth-order Bessel-Gauss beam calculated from [Ref. 3, Eq. 2.7] with the beam parameters $\alpha=14\ \mathrm{mm}^{-1}$ and $w_0=0.55$ mm. This value of w_0 was chosen to obtain the best fit to the observed profiles in Fig. 3. The criterion $\lambda/\pi w_0<\theta$ for the diffraction-free behavior of a Bessel-Gauss beam is satisfied by these values of α and w_0. We can conclude

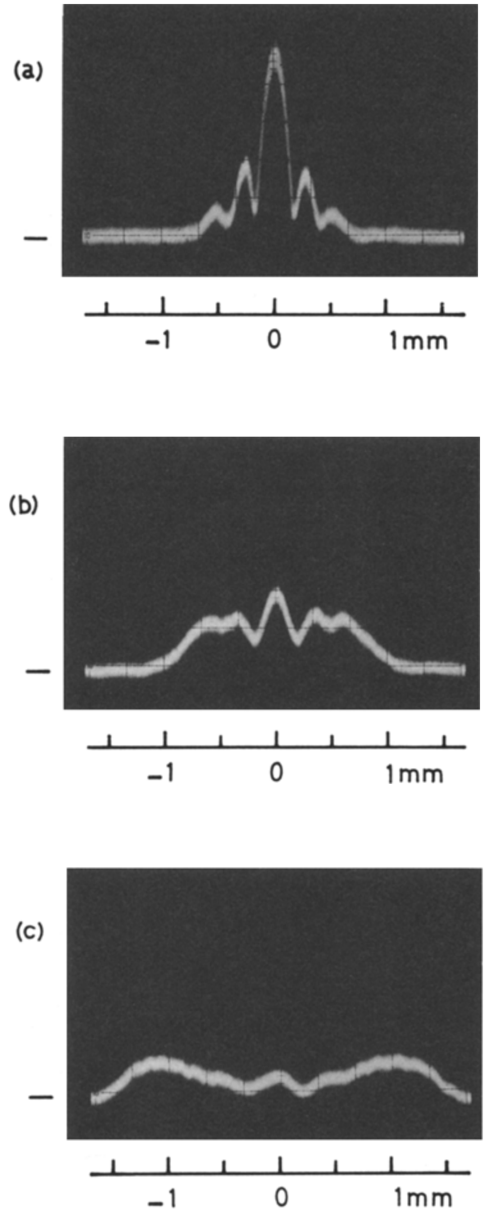

Fig. 3a–c. Radial intensity distributions of a circularly symmetric beam shown in Fig. 2 monitored at $z=11$ cm **a**, 50 cm **b**, and 1 m **c**. The vertical scales are different in the three traces

that the agreement between the observed intensity profiles in Fig. 3a–c and the calculated profiles in Fig. 4b–d is very good by taking account of the possible deviation of the output characteristic of the image pick-up tube from the nominal $I^{0.6}$ dependence.

The adjustment of the annular mirror was especially critical for obtaining highly symmetric beams. An iris diameter of 2.5 mm was the best to obtain the simplest beam shown in Fig. 2. Too much excitation current through the plasma tube also degraded the beam symmetry. So far, output powers of up to several

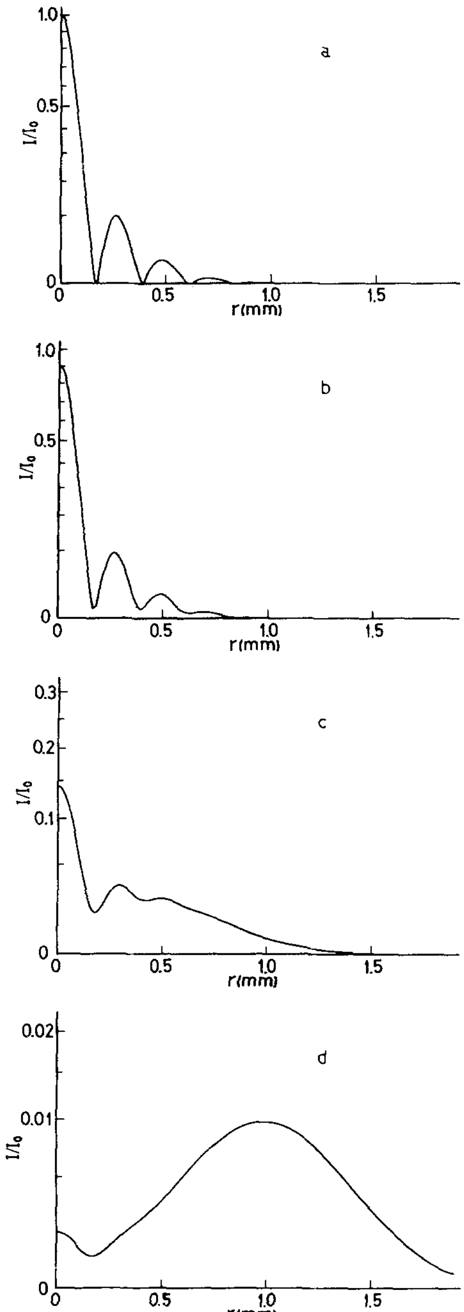

Fig. 4a–d. Calculated radial intensity distributions of the zeroth-order Bessel-Gauss beam at $z=0$ cm **a**, 11 cm **b**, 50 cm **c**, and 1 m **d**. The beam parameters are $\alpha=14\ \mathrm{mm}^{-1}$ and $w_0=0.55$ mm. The intensity is normalized by I_0, the central-peak intensity at $z=0$. Note that the vertical scale is proportional not to I but to $I^{0.6}$, in accordance with the characteristic of the image pick-up tube

milliwatts have been obtained in the simplest beam. Sometimes, another circularly symmetric beam which had a dark central spot was observed. Its radial intensity distribution at $z=11$ cm is shown in Fig. 5. The first and second outer intensity minima of this beam are located at 270 μm and 510 μm, respectively,

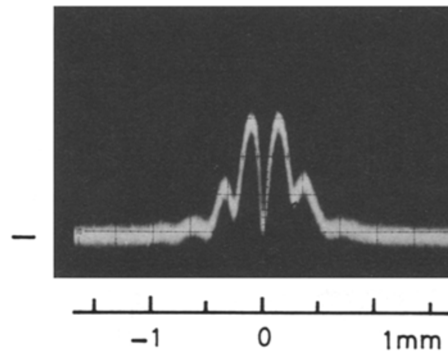

Fig. 5. Radial intensity distribution of a circularly symmetric beam with a dark central spot; $z=11$ cm

which give $\alpha r=3.8$ and 7.1, respectively, using the value of α determined above. These values of αr agree well with the zeros of the first-order Bessel function J_1, implying that the field distribution of this beam has the J_1 function as a factor. Actually, this beam can be fitted well by the first-order Bessel-Gauss function given by (6) with $n=1$.

3. Concluding Remarks

It has been shown that some of the laser beams obtained in the present experiment have almost the same intensity distributions and propagation characteristics as the Bessel-Gauss beams. However, the next step of our study should perhaps be to calculate theoretically the exact forms of the eigenmodes of the new laser cavity used here and to investigate how these eigenmodes are related to the Bessel-Gauss beams. It is easy to show that the lens L and the flat mirror M_1 in Fig. 1, when viewed from the mirror M_2, are equivalent to a single spherical mirror M_3 of the radius of curvature $f^2/(f-l_1)$ placed at the distance $l_2+fl_1/(f-l_1)$ from M_2. If an eigenmode of this virtual two-mirror cavity composed of M_2 and M_3 is known, the corresponding field distribution on the right-hand side of the lens in the actual cavity can be calculated by a proper transformation of this eigenmode. A theoretical study along these lines is now in progress.

An experimental study of the laser oscillation in a ruby rod having a parallel-plate cavity with a central hole in one of the reflector coatings was reported by Yajima et al. [9]. They observed annular intensity distributions of a very high azimuthal mode order (≈ 225) both in the near field and in the far field.

Radial field distributions expressed by Bessel functions also appear in some of the electromagnetic waves propagating through a cylinder. Therefore, as pointed out in [6], the diffraction-free beams have the same mathematical forms as Lord Rayleigh's solutions [10] to classical problems in electromagnetism. An electron accelerator using the intense light field at $r=0$ in a cylindrical laser cavity has been proposed by Shimoda [11].

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