# Optimal Pollution Taxation in a Cournot Duopoly

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Abstract. It is well known that the optimal pollution tax in a competitive industry is equal to the marginal damage inflicted by the pollution. It has also been shown that the optimal pollution tax on a monopoly is less than the marginal damage. In this paper, I derive the optimal pollution tax for a Cournot duopoly. If firms have different production costs, the optimal tax rate may exceed the marginal damage. This is so because the tax may be an effective instrument for allocating production from the less to the more efficient firm. It is also shown that, if one firm has a positive most preferred pollution tax, the sum of consumer and producer surpluses will be declining in the tax at this level.

Key words: Cournot duopoly, Pigovian tax, pollution

# Introduction

It has long been known that the optimal tax on a harmful effluent produced by a competitive industry should be set equal to the marginal value of the environmental damage the pollution causes (Pigou, 1932).<sup>1</sup> When a monopolist, rather than a competitive firm, is the producer of the harmful effluent, the second-best<sup>2</sup> policy is to impose a tax lower than the marginal environmental damage (Buchanan, 1969). Imposing a pollution tax further increases the costs of the monopolist and induces it to cut production further. As the monopolist already produces too little output, society may prefer to "subsidize" the monopolist by charging it less than the full value of the marginal environmental damage it inflicts. On can derive a formula relating the optimal tax to the marginal environmental damage in which it may be shown that the tax should be lower (relative to marginal damages) the greater is the difference between price and marginal cost and the greater the rate of change of output with respect to effluent production (Barnett, 1980; Baumol and Oates, 1988).

In this paper I consider optimal pollution taxation in one simple and commonly used oligopoly model, that of Cournot duopoly. The optimal pollution tax in an oligopoly is not necessarily less than the value of marginal environmental damage. In addition to imposing welfare loses on society by producing too little, Cournot duopoly may also result in an inefficient allocation of production between the firms. In some instances, a pollution tax may be an effective instrument for redistributing output from a less efficient firm to its more efficient rival. In some situations, the optimal pollution tax will be greater than marginal damage.

Environmental and Resource Economics 6: 359–369, 1995. © 1995 Kluwer Academic Publishers. Printed in the Netherlands. In addition to filling a missing niche in the literature, there is another motivation for the present inquiry. It might often be assumed that firms will be unanimously opposed to stricter environmental regulation. This is not always the case in oligopoly models. Lobbying for more stringent environmental regulation might constitute a strategy for "raising rivals' costs" (Salop and Scheffman, 1982); that is, a firm may find it to be in its own interest to advocate cost-increasing regulation if its rival's cost increases by even more. The rival may then have to restrict output enough as to allow the regulation advocate's profits to increase on net. It has been suggested that firms can use environmental regulation (Maloney and McCormick, 1982; Barrett, 1991, 1992) to achieve these ends. Conditions under which one – and, it turns out in this model, at most one – firm will prefer higher pollution taxation are derived. It is also demonstrated that, if one firm would prefer higher taxation, it is likely to prefer that taxes be set at a level higher that at which the sum of producers and consumers surpluses are maximized.

The body of the paper is laid out in the four sections following. In the first, the effects of pollution taxation on each firm's output are derived. In the second section the optimal pollution tax is derived. Implications of increased pollution taxation for firm profitability are derived in the third section. A final section presents conclusions.

## I. Pollution Taxes and Firm Outputs

Suppose that each of two Cournot duopolists, firm 1 and firm 2, has a cost function  $c_i(t)q_i$ . *t* is the pollution tax chosen by the regulator<sup>3</sup> and  $q_i$  is firm *i*'s output.<sup>4</sup> The assumption of constant marginal cost, given the tax, may not be innocuous in general, but in many cases it would be reasonable to argue that production processes are replicable. That is, unit costs would remain the same if output, inputs, and releases of pollution were all scaled up by the same factor. The assumption of constant marginal costs allows us to simplify certain expressions involving strategic interaction below; we might expect qualitatively similar results to follow with general cost functions.<sup>5</sup>

We will assume that the two firms compete as Cournot rivals. That is, they maximize profits taking the other's output as fixed. If their products are perfect substitutes, inverse demand is a function of the sum of outputs, and each firm's objective function may be written as

$$p(Q)q_i - c_i(t)q_i,\tag{1}$$

where  $Q = q_1 + q_2$ .

The first-order conditions<sup>6</sup> for profit maximization with respect to outputs,  $q_1$  and  $q_2$ , are

$$p + p'q_1 - c_1 = 0$$
 and  $p + p'q_2 - c_2 = 0$ , (2)

where p' is the derivative of price with respect to combined production.

Differentiating totally with respect to the tax, t, we have

$$\begin{bmatrix} 2p' + p''q_1 & p' + p''q_1 & -\frac{\partial c_1}{\partial t} \\ p' + p''q_2 & 2p' + p''q_2 & -\frac{\partial c_2}{\partial t} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}q_1}{\mathrm{d}t} \\ \frac{\mathrm{d}q_2}{\mathrm{d}t} \\ 1 \end{bmatrix} = 0.$$
(3)

Define

$$\rho_i = -\frac{p' + p'' q_i}{2p' + p'' q_i}.$$
(4)

 $\rho_i$  is the amount by which firm *i* would optimally adjust its output in response to an exogenous increase in the output of firm *j*, all other things remaining constant (i.e.,  $\rho_i$  is the slope of *i*'s best-response function in output space). It is reasonable to bound the  $\rho$ 's strictly between negative one and zero. Sufficient conditions for this to be the case are that demand is downward sloping and marginal revenue slopes down more steeply than demand (Bulow, Geanakoplos, and Klemperer 1986); these conditions are in turn sufficient for the Cournot equilibrium to be stable.

It will be useful in what follows to interpret a firm's production of pollution in a somewhat unusual way: we will regard pollution production as "consumption" of environmental amenities. The cost of each unit of environmental amenity consumption is, then, the tax paid per unit of pollution production.<sup>7</sup> It will be convenient to use Shepherd's Lemma in many places that follow. Thus  $\partial [c_i(t)q_i]/\partial t = e_i$ , firm *i*'s production of effluent.  $\partial c_i/\partial t$  can be regarded as the production of pollution per unit output, and it should be positive.<sup>8</sup>

Using these definitions and rearranging somewhat, (3) may be rewritten as

$$\begin{bmatrix} 1 & -\rho_1 & \frac{-\partial c_1/\partial t}{2p' + p'' q_1} \\ -\rho_2 & 1 & \frac{-\partial c_2/\partial t}{2p' + p'' q_2} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}q_1}{\mathrm{d}t} \\ \frac{\mathrm{d}q_2}{\mathrm{d}t} \\ 1 \end{bmatrix} = 0.$$
(5)

Solving for the derivatives of the quantities with respect to the tax, t, we have

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$$\frac{dq_{1}}{dt} = \frac{\frac{\partial c_{1}/\partial t}{2p' + p''q_{1}} + \rho_{1}\frac{\partial c_{2}/\partial t}{2p' + p''q_{2}}}{1 - \rho_{1}\rho_{2}} \text{ and}$$

$$\frac{dq_{2}}{dt} = \frac{\frac{\partial c_{2}/\partial t}{2p' + p''q_{2}} + \rho_{2}\frac{\partial c_{1}/\partial t}{2p' + p''q_{1}}}{1 - \rho_{1}\rho_{2}} \cdot$$
(6)

Either  $dq_1/dt$  or  $dq_2/dt$  could be positive, but not both. To demonstrate this point, note that

$$\frac{p'}{2p' + p''q_i} = 1 + \rho_i.$$
(7)

Using (7), we can see that for  $dq_1/dt$ , for example, to be positive, we would have

$$\frac{1}{p'}\frac{\mathrm{d}q_1}{\mathrm{d}t} = \frac{(1+\rho_1)\frac{\partial c_1}{\partial t} + \rho_1(1+\rho_2)\frac{\partial c_2}{\partial t}}{1-\rho_1\rho_2} < 0,$$

or

$$\frac{\partial c_1}{\partial t} < \frac{-\rho_1(1+\rho_2)}{1+\rho_1} \frac{\partial c_2}{\partial t}.$$
(8)

Our assumptions assure that  $-\rho_1(1 + \rho_2)/(1 + \rho_1) > 0$ , so if there is sufficient disparity in the costs imposed by the tax, firm 1's output would increase.

It is also easy to demonstrate that

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}q_1}{\mathrm{d}t} + \frac{\mathrm{d}q_2}{\mathrm{d}t} = \frac{(1+\rho_2)\frac{\partial c_1/\partial t}{2p'+p''q_1} + (1+\rho_1)\frac{\partial c_2/\partial t}{2p'+p''q_2}}{1-\rho_1\rho_2} < 0.$$
(9)

Thus, if firm 1's output does increase in response to a tax change, firm 2's output must decrease by enough as to make the total effect on industry output negative.

## **II.** Optimal Pollution Taxation

Consider now the regulator's problem. We will suppose that the regulator chooses a pollution tax to maximize the sum of consumer and producer surpluses net of environmental damage. Write the regulator's objective as

$$W(t) = \int_0^Q p(z) \, dz - c_1 q_1 - c_2 q_2 + t(e_1 + e_2) - D(e_1 + e_2). \tag{10}$$

We assume by making damage a function of the sum of pollution releases that the firms are relatively close together. Differentiation with respect to the pollution tax, t, yields

$$\frac{\mathrm{d}W}{\mathrm{d}t} = p\left(\frac{\mathrm{d}q_1}{\mathrm{d}t} + \frac{\mathrm{d}q_2}{\mathrm{d}t}\right) - c_1\frac{\mathrm{d}q_1}{\mathrm{d}t} - c_2\frac{\mathrm{d}q_2}{\mathrm{d}t} - \frac{\partial c_1}{\partial t}q_1 - \frac{\partial c_2}{\partial t}q_2 + (e_1 + e_2) + (t - D')\left(\frac{\partial e_1}{\partial q_1}\frac{\mathrm{d}q_1}{\mathrm{d}t} + \frac{\partial e_2}{\partial q_2}\frac{\mathrm{d}q_2}{\mathrm{d}t} + \frac{\partial e_1}{\mathrm{d}t} + \frac{\partial e_2}{\partial t}\right).$$
(11)

Using Shepherd's Lemma,  $(\partial c_i/\partial t)q_i = e_i$ . Thus the fourth, fifth, and sixth terms cancel (as they must, of course: this is the transfer from producers to the government).

The quantity in the final set of parentheses should be negative. It is the total derivative of combined pollution production with respect to the tax rate. when the pollution tax goes up, pollution should go down. From Shepherd's Lemma,

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \left( \frac{\partial c_1}{\partial t} q_1 + \frac{\partial c_2}{\partial t} q_2 \right) = \frac{\partial^2 c_1}{\partial t^2} q_1 + \frac{\partial^2 c_2}{\partial t^2} q_2 + \frac{\partial c_1}{\partial t} \frac{\mathrm{d}q_1}{\mathrm{d}t} + \frac{\partial c_2}{\partial t} \frac{\mathrm{d}q_2}{\mathrm{d}t} \cdot (12)$$

The first two terms to the right of the second equal sign are negative by the concavity of the cost function in input price (of which, recall, t is considered to be one). Using (6) and (7), the last two terms can be rewritten as

$$\frac{\partial c_1}{\partial t} \frac{dq_1}{dt} + \frac{\partial c_2}{\partial t} \frac{dq_2}{dt}$$
$$= \frac{1}{p'} \frac{(1+\rho_1) \left(\frac{\partial c_1}{\partial t}\right)^2 + \left[\rho_1(1+\rho_2) + \rho_2(1+\rho_1)\right] \frac{\partial c_1}{\partial t} \frac{\partial c_2}{\partial t} + (1+\rho_2) \left(\frac{\partial c_2}{\partial t}\right)^2}{1-\rho_1\rho_2}.$$

While this expression might conceivably be positive, we should generally expect it to be negative. To illustrate with a benchmark case, when the  $\rho$ 's are the same (implying that the q's are the same, demand is linear, or both), the above expression reduces to

$$\frac{\frac{\partial c_1}{\partial t}\frac{\mathrm{d}q_1}{\mathrm{d}t} + \frac{\partial c_2}{\partial t}\frac{\mathrm{d}q_2}{\mathrm{d}t}}{\frac{1}{p'}} = \frac{1}{p'}\frac{\left(\frac{\partial c_1}{\partial t}\right)^2 + 2\rho\frac{\partial c_1}{\partial t}\frac{\partial c_2}{\partial t} + \left(\frac{\partial c_2}{\partial t}\right)^2}{1-\rho} < \frac{1}{p'}\frac{\left(\frac{\partial c_1}{\partial t} - \frac{\partial c_2}{\partial t}\right)^2}{1-\rho} \le 0.$$

Recall that the first two terms in (12) were unambiguously negative. It is reasonable to assume that dE/dt < 0, then, and we will do so.

Returning to (11), it may be reduced to

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -p'\left(\frac{\mathrm{d}q_1}{\mathrm{d}t}q_1 + \frac{\mathrm{d}q_2}{\mathrm{d}t}q_2\right) + (t - D')\frac{\mathrm{d}E}{\mathrm{d}t} \cdot \tag{13}$$

One of the questions we wish to answer here concerns the sign of the first term in (13). Recall that, in the monopoly case, the optimal corrective tax will be below the Pigouvian level (Buchanan, 1969; Barnett, 1980; Baumol and Oates, 1988). An enlightened regulator might offset the losses due to imperfect competition by imposing a less stringent pollution tax; the regulator in effect "subsidizes" production by underpricing the damage arising from effluents. While deadweight losses due to imperfect competition also arise in Cournot duopoly, there is another effect that may argue for stronger, rather than weaker pollution taxes. Cournot duopolists will not, in general, produce their outputs efficiently. To the extent that a pollution tax may shift production from the less efficient firm to its more efficient rival, higher tax rates may be called for.

As we have already seen that  $dq_1/dt + dq_2/dt < 0$ , the first term in (13) cannot be positive if the two firms are of the same size, and *a fortiori*, cannot be positive if the firms are identical (that is, if their cost functions are the same as well).<sup>9</sup> Using the definitions derived above, dW/dt may be restated as

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\left[\frac{(1+\rho_1)(q_1+\rho_2q_2)\frac{\partial c_1}{\partial t} + (1+\rho_2)(q_2+\rho_1q_1)\frac{\partial c_2}{\partial t}}{1-\rho_1\rho_2}\right] + (t-D')\frac{\mathrm{d}E}{\mathrm{d}t}.$$
 (14)

While we might expect them to be exceptional, it is easy to generate examples in which the optimal tax would exceed marginal damages.<sup>10</sup> If, for example,  $\partial c_1/\partial t = 0$  (suppose that firm 1 uses a proprietary process that releases no effluents) and  $q_1/q_2 > -\rho_1$ , the first term of (14) will be positive, implying that the optimal tax should exceed the marginal value of environmental damage. More generally, the first term of (14) will be positive if the output of one of the firms is sufficiently greater than that of the other and the larger firm gains a greater relative advantage<sup>11</sup> as a result of the increased tax.

## **III.** Pollution Taxation and Profitability

Firm 1 will find the imposition of a higher tax profit enhancing if

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}t} = (p + p'q_1 - c_1)\frac{\mathrm{d}q_1}{\mathrm{d}t} + p'q_1\frac{\mathrm{d}q_2}{\mathrm{d}t} - \frac{\partial c_1}{\partial t}q_1 > 0.$$

The expression in parentheses vanishes by presumed satisfaction of the firstorder condition, so we can write

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}t} = p' q_1 \frac{\mathrm{d}q_2}{\mathrm{d}t} - \frac{\partial c_1}{\partial t} q_1. \tag{15}$$

Using (6) and expanding terms, (15) may be rewritten as

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}t} = \frac{(1+\rho_2)\frac{\mathrm{d}c_2}{\mathrm{d}t} - [1-\rho_2(1+2\rho_1)]\frac{\mathrm{d}c_1}{\mathrm{d}t}}{1-\rho_1\rho_2}q_1.$$
 (16)

It is easy to verify that  $d\pi_1/dt$  cannot be positive if firm 1 does not gain a relative cost advantage. If  $\partial c_1/\partial t = \partial c_2/\partial t$ , (16) could be written as

$$\frac{\mathrm{d}\pi_1}{\mathrm{d}t} = \frac{2\rho_2(1+\rho_2)\frac{\partial c_1}{\partial t}}{1-\rho_1\rho_2}q_1 < 0.$$

It also follows that *at most* one firm will profit from a pollution tax increase. To establish this proposition, note that  $\frac{d\pi_1}{dt} > 0$  iff  $(1 + \rho_2) \frac{\partial c_2}{\partial t} - [1 - \rho_2(1 + 2\rho_1)] \frac{\partial c_1}{\partial t} > 0$ , and  $\frac{d\pi_2}{dt} > 0$  iff  $(1 + \rho_1) \frac{\partial c_1}{\partial t} - [1 - \rho_1(1 + 2\rho_2)] \frac{\partial c_2}{\partial t}$ > 0; but  $\frac{d\pi_1}{dt} + \frac{d\pi_2}{dt}$  will have the sign of  $(1 + \rho_2) \frac{\partial c_2}{\partial t} - [1 - \rho_2(1 + 2\rho_1)] \frac{\partial c_1}{\partial t} + (1 + \rho_1) \frac{\partial c_1}{\partial t} - [1 - \rho_1(1 + 2\rho_2)] \frac{\partial c_2}{\partial t} = [\rho_1(1 + \rho_2) + \rho_2(1 + \rho_1)] \left(\frac{\partial c_1}{\partial t} \frac{\partial c_2}{\partial t}\right) < 0.$ 

While one must be careful in interpreting an expression with as many implicit relationships as are embodied in (16), some general intuitions can be confirmed by considering its form. First, if  $\partial c_1/\partial t = 0$  (equivalently,  $\partial e_1/\partial q_1 = 0$ ; see footnote 8), firm 1 will find the tax increase profitable. In this case, firm 1 is not directly affected by the tax. In essence, firm 2's best-response function is shifted inward as a result of the tax being imposed while firm 1's best-response function remains unchanged. Thus firm 1's output and profit are increased. Second, if  $\rho_2$  approaches  $^-1$ , firm 1 cannot find the tax increase profitable. The reasoning behind this result becomes clear on inspection of (4); the slope of the best response function approaches negative one as the demand curve becomes highly elastic. In such circumstances, the firms would have few strategic incentives as behavior would be approximately competitive anyway.

Recall from the preceding section that a regulator may want to increase the effluent tax if the tax results in an increased cost advantage for a firm that is already dominant. Thus a regulator who knows the relative sizes of both firms and can infer – possibly from the statements of one of the firms – which will enjoy an enhanced cost advantage as a result of a pollution tax increase may be able to make a wiser decision. It is now clear that a firm that supports higher pollution taxation will achieve a relative cost advantage.<sup>12</sup> The next result suggests that caution should be employed in interpreting regulation-advocating firms' suggestions, however.

We have already noted that at most one firm will favor an increase in the

pollution tax. Let us suppose that firm 1 does favor an increase in the pollution tax. From (16), it must be true that

$$(1+\rho_2)\frac{\partial c_2}{\partial t} \ge [1-\rho_2(1+2\rho_1)]\frac{\partial c_1}{\partial t}.$$
(17)

If (17) holds as an equality – that is, if the tax is set at that level that maximizes firm 1's profits, we would have, substituting into (14),

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \left[ (1+2\rho_1)q_1 + q_2 \right] \frac{\partial c_1}{\partial t} + (t-D') \frac{\mathrm{d}E}{\mathrm{d}t} \,. \tag{18}$$

Note that  $1 + 2 \rho_1$  is

$$1 - \frac{2(p' + p''q_1)}{2p' + p''q_1} = \frac{-p''q_1}{2p' + p''q_1} = \frac{-p''q_1}{2p' + p''q_1}$$

so the term in square brackets will be unambiguously positive if demand is convex. More generally, this term will be positive except in instances in which demand is strongly concave and/or  $q_1$  is much greater than  $q_2$ . Thus, we should expect that, under "normal" conditions, the first term in (18) – the sum of consumer and producer surplus – is declining in the pollution tax at the point most preferred by a firm that finds a tax increase attractive.<sup>13</sup>

# **IV.** Conclusion

As was noted in the introduction, this paper has been motivated by two considerations. The first is to fill a niche in the literature concerning optimal pollution taxation. The second motive is to determine when a firm may prefer stricter environmental regulation.

Several factors limit the applicability of the findings. Among these are the assumptions of constant marginal cost, perfectly substitutable products, Cournot rivalry between firms, the choice of pollution taxation as an environmental policy instrument, and the proximity of firms, which implies that pollution from each is "perfectly substitutable" in generating environmental damage. The point of this paper is not so much that the model developed generates important and meaningful policy advice – it is almost certainly too simple and specific to be applied in any particular policy context – but rather that simple presumptions may not be valid when strategic interactions are taken into account. In particular, imperfect competition does not necessarily imply that lower-than Pigouvian taxes are optimal. In addition, the model demonstrates another circumstance under which one firm might advocate higher taxes to raise a rival's cost.

While the many special assumptions noted above argue for caution in drawing policy implications, two conclusions might be offered. The first is that this paper is yet another in a growing number of contributions suggesting that, as a practical matter, Pigouvian taxation may yet be the best public policy.

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By making different assumptions about market structure, strategic interactions, and other factors, one can generate optimal environmental tax rules above or below Pigouvian levels. Given the near-impossibility of determining empirically what circumstances apply in particular industries, the "central case" of Pigouvian taxes may well be the best rule in practice.

The second conclusion is that policy makers may glean some useful information from the opinions of interested parties. Again, the model presented here is too simple to offer strong policy prescriptions. There are also potentially formidable problems with truthful revelations of information if it is known that firms' preferences will be considered in designing environmental policy. This simple analysis does suggest, however, that fruitful research might be done to extend and broaden the analysis of firms' incentives in communicating with environmental policy makers.

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### Notes

<sup>1</sup> This simple prescription has been subject to some qualifications to reflect the effects of Pigouvian taxes on entry-exit conditions of competitive firms. See, e.g., Kohn (1994) and the literature cited there, and Carlton and Loury (1980, 1986), who argue that Piouvian taxes augmented by subsidies will adequately address externalities.

 $^2$  Second-best policies will be discussed throughout this paper: it is assumed that structural impediments prevent the regulator from addressing imperfect competition issues directly.

<sup>3</sup> It is assumed throughout that regulators cannot impose different taxes on the same effluent coming from different sources. This restriction seems reasonable on the basis of practical concerns (e.g., political acceptability and prevention of sham transactions). The subsequent analysis suggests another motivation for uniform policy, however: by inducing different responses from different firms, a regulator may be able more easily to elicit information from firms concerning the effects of proposed regulations.

<sup>4</sup> The assumptions of constant and unequal marginal costs imply that the industry would be more profitably organized as a monopoly. It is not necessarily the case that this would be the outcome of a dynamic strategic interaction between firms, however. From a theoretical perspective, there is no agreed-upon solution to the problem of splitting monopoly rents between competing duopolists (even notions such as the Nash bargaining solution are based on *ad hoc*, albeit plausible, assumptions concerning the properties a "reasonable" solution should have). From a practical perspective, antitrust authorities might require an industry to remain a duopoly and punish either unilateral (predation) or mutual (merger) attempts to monopolize. Finally, cost advantages would likely be temporary, and might be eliminated by investment. Thus, the description offered here might be justified as a description of period-by-period equilibrium in which firms make investment decisions and regulators choose pollution taxes.

<sup>5</sup> We might suppose also that an effluent tax would induce a trade off between fixed and variable costs (e.g., process innovations to reduce effluent emissions). We will assume that we are dis-

cussing the firm's long-run cost curve here, however. For a model in which an increase in effluent taxation induces innovation, see Simpson and Bradford (forthcoming).

 $^{6}$  While the second-order conditions for profit maximization with respect to quantity choices are straightforward – and will, in fact, be used in bounding the slope of the best response functions – second-order conditions in the remainder of the paper will be intractable. We simply assume that they are satisfied, and could appeal to heuristic arguments for this assumption.

<sup>7</sup> Alternatively, one can consider the firm as a producer of the good it markets at price p and effluent, which it "sells" at price -t. Under this interpretation, Hotelling's Lemma may be substituted for Shepherd's in what follows, with the same results.

<sup>8</sup> That is, if firm *i*'s cost function depends on  $q_i$ , *t*, and a vector of other factor prices that have been suppressed as they are assumed to remain unchanged, then  $\partial^2[c_i)t)q_i]/\partial t\partial q_i = \partial c_i/\partial t = \partial e_i/\partial q_i$ , where  $e_i$  is firm *i*'s production of effluent.

<sup>9</sup> If the firms were identical, we could have rewritten (13) as

$$\frac{\partial W}{\partial t} = -p'q \frac{\mathrm{d}Q}{\mathrm{d}t} + (t - D') \frac{\mathrm{d}E}{\mathrm{d}t},$$

or, substituting back in p - c for p'q,

$$\frac{\partial W}{\partial t} = (p-c)\frac{\mathrm{d}Q}{\mathrm{d}t} + (t-D')\frac{\mathrm{d}E}{\mathrm{d}t} \,.$$

Rearrangement then shows that the optimal tax is

$$t^* = D' - (p - c)\frac{\mathrm{d}E}{\mathrm{d}Q}.$$

This is the formula derived by Barnett (1980) and Baumol and Oates (1988). The wedge between marginal damage and the optimal tax rate will, of course, be smaller in this instance than in their example as the duopoly price will be lower than that chosen by the monopolist.

<sup>10</sup> Strictly speaking, what we are discussing here is situations in which the tax should be set higher than its level at some *status quo ante*; if the social objective function, (10), is continuous and concave in the effluent tax rate, however, a global optimum may exist in which the tax rate exceeds the Pigouvian level.

<sup>11</sup> Note that the larger firm *must* have a cost advantage in this model:

$$p + p'q_1 - c_1 = 0 = p + p'q_2 - c_2$$
, so  $q_1 - q_2 = \frac{c_1 - c_2}{p'}$ .

<sup>12</sup> I have largely side-stepped the question of how such support might be expressed. There exists a literature on optimal mechanism design for inducing the revelation of firms' preferences for environmental regulation (see, e.g., Spulber 1988), but the present inquiry is motivated in part by the difficulty of implementing such mechanisms in practice. For present purposes it is sufficient to assume that a firm's public statements will be credible if it would not have been profit-enhancing to have made the statement unless the firm were of the type that actually achieves a greater relative cost advantage (this is, in essence, Cho and Kreps (1987) "intuitive criterion" refinement in the game theory literature).

<sup>13</sup> Again, we assume the satisfaction of second-order conditions: at some point the direct effects on production costs outweigh the indirect effects through weakening the rival.

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