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# WHERE IS AN OBJECT BEFORE YOU LOOK AT IT?

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According to the standard interpretation of quantum mechanics (QM), no meaning can be assigned to the statement that a particle has a precise value of any one of the variables describing its physical propertes before having interacted with a suitable measuring instrument. On the other hand, it is well known that QM tends to classical statistical mechanics (CSM) when a suitable classical limit is performed. One may ask therefore how is it that in this limit, the statement, meaningless in QM, that a given variable has always a precise value independently of having been measured, gradually becomes meaningful. In other words, one may ask how can it be that QM, which is a theory describing the intrinsically probabilistic properties of a quantum object, becomes a statistical theory describing a probabilistic knowledge of intrinsically well determined properties of classical objects.

In the present paper we try to answer to this question and show that an inconsistency arises between the conventional interpretation of CSM which presupposes objectively existing Newtonian trajectories, and the standard interpretation of QM. We conclude that the latter needs revisiting unnless we wish to adopt a strictly subjective conception of the world around us, implying that macroscopic objects as well are not localized anywhere before we look at them.

Key words: Quantum Mechanics, probabilistic properties, classical limit.

There was a young man who said,'God
Must think it exceedingly odd
If he finds that this tree
Continues to be
When there's no one about in the Quad.'
REPLY
Dear Sir:
your astonishment's odd:
I am always about in the Quad.
And that's why the tree
Will continue to be,
Since observed by
Yours faithfully
GOD.

(*Richard Knox,* quoted by *Bertrand Russell*.)

## 1. INTRODUCTION

Before the conceptual revolution introduced by quantum mechanics (QM), physical reality was conceived as a collection of objects each one endowed with properties completely determined by its physical state. In fact, one could define the latter by giving the values of all the independent variables required by its degrees of freedom. The time evolution of the state was determined completely with absolute precision by the universal laws of motion, once the initial conditions and the external forces were given.

Only imperfect knowledge of these data could lead to uncertainties in the future (or past) evolution of a given system. Therefore the introduction of a probabilistic description for this evolution was invented as a means of obtaining the maximum information about a given system compatible with an incomplete knowledge of all the conditions which specify its state. In other words, statistical mechanics does not deny that a given particle in a gas actually has at any moment a well determined position and velocity, but simply works with quantities which do not depend on the detailed values of the variables of each particle.

Quantum mechanics had however to face two new facts: the occurrence of different events in spite of apparently identical initial and external conditions, and the impossibility of assigning simultaneously precise values to all the variables of a given system. The solution was found by loosening the connection between the state of a system and its variables. The former was still considered to be completely determined by the initial conditions and the laws of motion, but the latter were allowed to acquire, with different probabilities depending on the given state, any value within a set of values typical of the variable in question.

Furthermore, it was found that the mathematical entity describing the state was not a probability density of the type introduced in classical probabilistic descriptions, because no joint probability distributions exist for the variables whose precise determination is not simultaneously possible.

All this has led to the standard interpretation of QM which not only denies that a particle may have a precise, although unknown, value of both position and momentum, but moreover affirms that, generally speaking, no physical meaning can be assigned to the statement that a particle has a precise value of the position (or momentum, or any other variable) before having interacted with a suitable measuring instrument. More precisely, a basic axiom of QM is that, if the state of a particle is the superposition of two states belonging to different values of a given dynamical variable, it is only in the act of measurement that the variable acquires, at random, one of these two values.

These conceptual premises of QM are part of the cultural background of the great majority of physicists, which is transmitted by textbooks to each new generation. They are not only a common belief, however, because direct experimental evidence of the impossibility of assigning simultaneously precise values to different components of the spin seems to be provided by the experiments which also confirm the validity of QM in tests of the EPR paradox.

In spite of this general consensus, which has never been seriously challenged by the many attempts made to restore a more "realistic" picture of the atomic world, the answers that the standard interpretation gives to some questions still remain controversial. A typical issue is, from this point of view, the problem of the wave packet collapse in the act of measurement, where no solution has gained as yet unanimous support. One of the reasons of this situation is, in our opinion, that a proper and exhaustive understanding of the relation between quantum mechanics and classical statistical mechanics (CSM) is still lacking.

We will therefore try to contribute to this understanding by asking a very simple question. One may ask how is it that, in the limit when QM tends to CSM, the statement that a given variable of a physical system has always a precise value independently of having been measured or not - a meaningless statement in QM - gradually becomes meaningful. In other words, how can it be that QM, which is a theory describing the intrinsically probabilistic properties of quantum objects, becomes, in this limit, a statistical theory describing a probabilistic knowledge of intrinsically well determined properties of classical objects? To be fair these questions have been asked in a particularly challenging manner by

Schroedinger in his famous 1935 paper.[1] Our answer, however, as we shall see, is different from his one.

The first thing to do, of course, is to define properly what we mean by the "limit when QM tends to CSM." This procedure is not as simple as letting $h$ go to zero. In fact, $h$ is a constant of nature and one should rather take the correct limit for the appropriate dynamical variables when they become large compared with the relevant atomic units. For this purpose, however, one should realize that there are at least two different cases in which one expects that QM should approach CSM.

The first one refers to the limit of large "quantum numbers" for a given quantum system, in the sense of Bohr's correspondence principle. This is the limit in which the variation $\Delta S$ of the phase of the wave function within the region where it is substantially different from zero is much larger than $h$, or, if the state is stationary, the energy $E_n$ is much larger than the ground state energy $E_0$. The question is now: When the particle is driven in a state of this kind, can one still maintain that its variables are not defined before they have been measured? Or, more precisely, if the state vector of the particle is a superposition of two states corresponding to macroscopically different values of a given variable, can one still maintain that this variable acquires, at random, one of these values only when it is measured? This would be in contrast with the classical statistical picture which supposes that the macroscopic variable does have one of the two possible values independently of whether it has been measured or not.

The second case to be considered, namely the case of a macroscopic body, is even more puzzling. The system is now made up of an enormous number N of elementary quantum systems and has a correspondingly large number of degrees of freedom. For such a system it is generally possible to define at least a pair of collective (pseudo)conjugate variables (e.g., the center of mass coordinate and its velocity) that satisfy two conditions: (a) the commutator of these collective variables vanishes as N goes to infinity; (b) they are decoupled from the variables of the individual particles, and their Heisenberg equations of motion tend, in this limit, to the classical equations of motion.

One might therefore think that, in this case, property (b) gives an answer to our question. Since the classical equations of motion can be solved with precisely given initial values for both these variables, their value is completely determined at all later times. This means that these variables "have" precisely determined values whether they are measured or not.

This statement is, however, only a partial answer, because it is true only for the particular choice of initial conditions just considered. In fact, a quantum mechanical state corresponds, as we shall see in more detail in a moment, to a statistical distribution in the phase space of the collective variables that does not generally reduce to a single point. It may happen, for example, that it yields a limiting classical statistical distribution, having the form of two delta functions centered on two different classical trajectories. Do we have, here again, to assume that our macroscopic body choses, at random, one of the two possible trajectories (on each of which the center of mass position and velocity are defined at any time), only when a measurement is performed on it? Should we not rather assume, in view of the fact that the body is macroscopic, that the right description is given by CSM, and that the double delta function simply reflects our ignorance?

In this paper we will discuss this puzzling question showing that if we choose the second answer, consistently with the commonly accepted physical content of CSM, we are led also to admit that the widely spread belief in the current interpretation of QM needs revisiting. If we do not wish to do so we have to adopt a strictly subjective conception of the classical world outside us, implying that also macroscopic objects are not localized anywhere before we look at them.

The work is organized in three steps. In the first one, we recall that QM tends, when a proper classical limit is performed, to classical statistical mechanics. This is a result which is independent of any particular "interpretation" of QM. In the second step we show that the uncertainties in position and momentum predicted by QM lead always to an uncertainty product $\Delta x \, \Delta p$ which is the sum of the minimum quantum value $h/2$ and a term which has the same form of the corresponding classical term of statistical origin. We interpret this fact as an indication that the former reflects an intrinsic indeterminacy of the quantum variables, while the latter reflects our ignorance of the state of the individual particles. This is also confirmed by the fact that the first one disappears in the classical limit, while the second does not. One would therefore expect that, in the classical limit, individual systems, while being collectively described by statistical mechanics, should tend to follow classical trajectories. In the third step, we show that this expectation is contradicted by the standard interpretation of QM. Finally, we shall try to find out the consequences of this contradiction on the microscopic world.

Section 2 is devoted to the first step for the case of large values of the action-variable of a single quantum system, while Appendix 1 deals with the same problem for a macroscopic object. Section 3 is devoted to the discussion of the second step. Section 4 is dedicated to the third step and Section 5 to the consequences of the preceding chain of reasoning. In Section 6 we discuss the relevance of our proposal for some open questions in the foundations of quantum mechanics.

## 2. THE CLASSICAL LIMIT OF THE WIGNER FUNCTION

We shall now briefly recall some more or less known facts about the statistical properties of a quantum state in the classical limit. As is well known, Wigner[2] has defined a quantum mechanical function W(x,p) by means of the relation (for simplicity we put $h = 1$, but we shall later introduce, when necessary, $h$ explicitly):

$$W(x,p) = \pi^{-1} \int \psi^*(x+y)\, \psi(x-y)\, \exp(2ipy)\, dy . \quad (1)$$

This function may be used to compute the expectation value of any quantum variable $\mathbf{A(x,p)}$ function of the operators $\mathbf{x}$ and $\mathbf{p}$, by means of an expression:

$$\langle A \rangle = \int \psi^*(x)\, A(x,-i\, \partial/\partial x)\, \psi(x)\, dx = \iint W(x,p)\, \mathcal{A}(x,p)\, dx\, dp, \quad (2)$$

provided one takes

$$\mathcal{A}(x,p) = 2 \int \langle x+z|A|x-z \rangle \exp(-2ipz)\, dz. \quad (3)$$

W(x,p) corresponds to the classical distribution function f(x,p) in phase space, because Eq. (2) is formally identical with the classical expression

$$\langle A \rangle = \iint dx\, dp\, A(x,p)\, f(x,p), \quad (4)$$

where A(x,p) is the classical variable corresponding to $\mathbf{A(x,p)}$.

Of course, W(x,p) is not everywhere positive as the classical f(x,p) and therefore can not be interpreted as a distribution function. Nevertheless, Eq.(2) reduces, generally, to Eq.(4) in the classical limit. In the second part of this section we will show with a few examples that this limit consists essentialy in considering large quantum numbers.

On a formal level, one can simply suppose that $h$ is negligibly small ($h \to 0$)[3] In this case $\mathcal{A}(x,p) \to A(x,p)$, when the quantum variable $\mathbf{A(x,p)}$ has the same form as the classical variable A(x,p), because its matrix elements $\langle x|A|p \rangle$ differ from it only by terms of order $h$ arising from the reordering of the operators $\mathbf{x}$ and $\mathbf{p}$ . On the other hand, it is well known that the time evolution of W(x,p;t) reduces to the Liouville equation

$$\partial W(x,p;t)/\partial t = -(p/m)\, \partial W(x,p;t)/\partial x + (\partial V(x)/\partial x)(\partial W(x,p;t)/\partial p) \quad (5)$$

when V(x) is slowly variable compared to the oscillations of the exponential.

This reduction is a necessary step in order to obtain the classical distribution in the limit $h \to 0$. One needs in fact:

$$\int_A dx\, dp\, W(x,p) \;\to\; \int_A dx\, dp\, f(x,p) \tag{6}$$

for any finite region A of the phase space > h. This limit obviously holds when W(x,p) tends to the corresponding f(x,p). One can easily see that this last limit can be obtained by means of a smoothing procedure when W(x,p) is substantially a large bump in phase space . Introducing the representation

$$\psi(x) = \exp[R(x) + iS(x)/h\,] \tag{7}$$

into Eq.(1) and expanding $R(x \pm y)$, $S(x \pm y)$ up the second order in y, we get, in the limit $h \to 0$,

$$W(x,p) \to f(x,p) = \delta[p-S'(x)]\, \exp[2R(x)]\ . \tag{8}$$

Thus W(x,p) tends to the distribution function in phase space of a Hamiltonian fluid of space density exp[2R(x)] and action function S(x). In practice, Eq.(6) holds in more general cases even when W(x,p) does not tend to the corresponding f(x,p). We will see a typical example when considering in this section a particle between two infinite potential barriers. An interesting particular case of (8) is that of a free particle wave packet initially concentrated in a Gaussian region:

$$\psi(x,0) = (\pi\alpha)^{-1/4} \exp\{-(x-x_o)^2/2\alpha\} \tag{9}$$

Since the energy expectation value e is given by

$$\varepsilon = h^2/4\alpha m, \tag{10}$$

the proper classical limit is provided by the limit e t >> $h$ . One now has:

$$W(x,p;t) \to (4\varepsilon t^2/m)^{-1/2} \exp[-m(x-x_o)^2/4\varepsilon t^2]\ \delta[p-(x-x_o)m/t]\ . \tag{11}$$

This is the classical distribution function f(x,p;t) in phase space of a free Hamiltonian fluid concentrated initially (t=0) at $x=x_0$.

A second example is the stationary state of energy $k^2/2$ of a particle between two infinite potential barriers of distance L. An easy calculation gives

$$W(x,p) = (1/2L)\{\delta(p-k)+ \delta(p+k) - \delta(p)\ [\exp(2ikx)+ \exp(-2ikx)]\} \quad (12)$$

This function does not become positive in the limit of large energies. However, for a given region A of phase space, when the energy $E >> (h\,\pi/2L)^2/2$ (ground state energy), one has

$$\int_A dx\, dp\, W(x,p) \rightarrow \int_A dx\, dp\, f(x,p)=(1/2L)\int_A dx\, dp\, (\delta[p-k]+ \delta[p+k]) \quad (13)$$

For a general potential V(x), one can show that

$$W(x,p) \rightarrow N\, p_E^{-1}[\delta(p-p_E) + \delta(p+p_E)] = N\, \delta(H-E), \quad (14)$$

where $H = (p^2/2m) + V(x)$ and N is the phase space volume on the energy shell. In other words, the statistical properties of the quantum mechanical density matrix for a given energy E tend to those of the corresponding microcanonical ensemble of classical statistical mechanics.

Let us now see what happens when the state is a superposition

$$\psi = c_1\psi_1 + c_2\psi_2\,. \quad (15)$$

The total Wigner function will be the sum of the two Wigner functions of $\psi_1$ and $\psi_2$, weighted with the respective probabilities, plus an interference term, whose general features will be better understood by considering two complementary cases.

The first one occurs when the two wave functions are localized in two separate space regions. Then the contribution to W(x,p;t) of the interference term contains a factor

$$\cos[p(x_1-x_2)/h\,],$$

where $x_1$ and $x_2$ are the mean values of x in $\psi_1$ and $\psi_2$. The contribution of this term to the expectation value of any variable A(x,p) vanishes unless its p-dependence shows the same rapidly oscillating behaviour of the cosine factor. This is certainly not the case for the quantum variables considered here that have a classical limit.

The complementary case obtains when the two wave functions are not localized in separate space regions, but are labeled with two widely different values of the energy $E_1 >> E_2 >> E_0$ (ground state energy). Here again the interference term gives a vanishing contribution to expectation values of variables which have a classical limit. This can be easily seen in the simple example of a particle constrained between two fixed boundaries, for in which the interference term to the Wigner function oscillates with a factor

$$\cos\{[\sqrt{(2m E_1)} - \sqrt{(2m E_2)}]x/h\}.$$

Here the contribution of this term to the expectation value of a variable A(x,p) is negligibly small unless its x dependence shows the same rapidly oscillating behaviour of the cosine factor, a property which we do not expect a variable with a classical limit to possess.

It is therefore clear that the limiting classical statistical ensemble corresponding to the quantum state (15) is always simply the union of the two classical statistical ensembles corresponding to the individual states, weighted with probabilities $|c_1|^2$ and $|c_2|^2$.

The case of a macroscipic body made of N particles is discussed in Appendix 1 by means of a simple model. The result is that there is always a one-to-one correspondence between a quantum mechanical state and a statistical distribution in phase space of classical statistical mechanics, both in the case of a microscopic system and in the case of a macroscopic body. The difference is that in the former the classical limit is attained when a suitable action variable is >> $h$, while in the latter the limit $N \rightarrow \infty$ is sufficient to ensure that the macroscopic variables are correctly described by means of CSM. We shall now discuss the consequences of this correspondence.

## 3. THE STATISTICAL PROPERTIES OF A QUANTUM MECHANICAL STATE

It is now easy to discuss the statistical properties of a quantum mechanical state. It is particularly interesting to see what happens to the uncertainty product $\Delta x \Delta p$. For a free wave packet of average energy e one has always:

$$(\Delta p)^2 = 2\varepsilon m - \langle p\rangle^2 \qquad (16)$$

$$(\Delta x)^2 = (\Delta p)^2 t^2/m^2 + \hbar^2/4(\Delta p)^2 \quad (17)$$

$$(\Delta x\, \Delta p)^2 = (\Delta p)^4 t^2/m^2 + \hbar^2/4 \rightarrow (\Delta x\, \Delta p)_{cl}{}^2, \text{ for } \varepsilon\, t >> \hbar \quad (18)$$

When a potential is present eq.(18) still holds in the approximation when the Wigner function W satisfies the Liouville equation.

Equation (17) resembles closely an old relation which marked a turning point in the history of physics: Einstein's formula for the energy fluctuation of radiation at thermal equilibrium expressed as the sum of two terms of different origin.[4] In the case of radiation, the quantum term arises from its particlelike properties and the classical term from the wavelike ones. In quantum mechanics the reverse happens. In our case the first (particle) term has a classical origin and the second (wave) term a quantum one. This separation, however, has been forgotten since the adoption of the standard interpretation of QM, which considers the fluctuations of the quantum variables as wholly due to their intrinsically undetermined nature. What we propose, on the contrary, is to take seriously this separation as physically meaningful. From this point of view, Eq.(17) means that the spread of a quantum wave packet for large values of $\varepsilon t$ does not arise from an ontologically intrinsic delocalization of the particle, but, as it happens for classical particles, is a trivial consequence of the fact that the region where a particle may be found increases with time if its momentum is not precisely determined.

Stated differently, Eq.(18) indicates that the really intrinsic quantum indeterminacy, reflecting the impossibility of simultaneous existence of position and momentum, is always the minimum one implied by the Heisenberg principle. Higher indeterminacies are instead of statistical nature, reflecting the actual displacement in space of particles with different momenta, and they survive in the classical limit. On the contrary the classical phase space volume available shrinks to a $\delta$ function when the uncertainty is of the order of its minimum, as it happens in the case of coherent states. This picture is particularly relevant for the interpretation of the superposition (15) when the two states are macroscopically different. One can no longer say that only when a variable is measured it assumes a value corresponding to either $\psi_1$ or $\psi_2$.

Since the statistical ensemble described by the density matrix of the state (15) is, in the limit of large values of the action, the weighted union of two disconnected classical phase space distributions, we are almost forced to say that the particle belonged to one or the other distribution even before the measure has taken place. Of course this statement would be completely correct only if the classical limit could be attained. However, it starts being almost correct in the intermediate region between the pure quantum case ($\Delta x\Delta p \approx h/2$) and the classical region ($\Delta x\Delta p >> h/2$). We will explain and justify this statement in the following sections, showing that it is indeed possible to give a more precise meaning to the separation between quantum and classical indeterminacy.

## 4. INCONSISTENCY BETWEEN INTERPRETATIONS OF CSM AND QM

Assume now that a classical distribution function in phase space is at t=0 of the form

$$f(q,p;0) = P_1 f_1(q,p;0) + P_2\, f_2(q,p;0) \tag{19}$$

with $f_1=0$, when $q,p \notin \Gamma_1{}^o$, and $f_2=0$ when $q,p \notin \Gamma_2{}^o$ in phase space, with $\Gamma_1{}^o \cap \Gamma_2{}^o = 0$. Call $q_i{}^o$, $p_i{}^o$ the mean values of q, p in the distribution $f_i$ and $\Delta q_o$, $\Delta p_o$ their mean square values, which we assume for simplicity to be the same for $f_1$ and $f_2$. Suppose furthermore that the space distance $d_o$ between $\Gamma_1{}^o$ and $\Gamma_2{}^o$ ($d_o = \min|q_1-q_2|$; $q_1 \in \Gamma_1{}^o, q_2 \in \Gamma_2{}^o$) is $>> \Delta q_o$. Now we measure q with a resolution $\Delta q_o$ and find the particle in $S_1{}^o$, the space width of $\Gamma_1{}^o$. We might as well have measured p with a resolution $\Delta p_o$, with the result that we would have found the particle in $M_1{}^o$, the momentum width of $\Gamma_1{}^o$. In classical mechanics, of course, both measurements are compatible, but one is sufficient, in this case, to deduce from (19) that at t=0 the point in phase space representing the particle's state is in $\Gamma_1{}^o$. Then we have two possible interpretations of this fact:

(a) We can say that even before our measurement at an earlier time t the phase space point of the system was in $\Gamma_1{}^t$ (the region which subsequently evolved according to Liouville into $\Gamma_1{}^o$ at t=0) because it has followed a trajectory which, starting from a point located within $\Gamma_1{}^t$ goes through a point in $\Gamma_1{}^o$. The position of the particle at the earlier time was therefore within a distance $\Delta q_t << d_t$ from the mean value $q_1{}^t$ given by $f_1(q,p;t)$, with $\Delta q_t$ given by (suppose for simplicity that the particle has propagated freely such that $\Delta p_o$ does not vary with time)

$$\Delta q_t = [(\Delta q_o)^2 + (\Delta p_o t/m)^2]^{1/2} . \qquad (20)$$

It should be stressed at this point that, while the volume of $\Gamma_1{}^t$ is equal to the volume of $\Gamma_1{}^o$, the uncertainty product $\Delta q_t \, \Delta p_t$ always increases with time (both in the backward and in the forward direction) because of (20).

The probabilities $P_1$ $P_2$ in (19) represent therefore our ignorance about the previous localization of the particle and not an actual indetermination of its position in space.

(b) We can say that before the measurement there was no phase space point representing the particle's state in $\Gamma_1{}^t$ or in $\Gamma_2{}^t$ and that therefore the state has been localized in $\Gamma_1{}^o$ by the measurement. In this case $P_1$ and $P_2$ are intrinsic probabilities of localizing the particle either in $\Gamma_1{}^o$ or in $\Gamma_2{}^o$. There is no trajectory followed by the particle from one point of $\Gamma_1{}^t$ to a given point of $\Gamma_1{}^o$.

In both cases, after the measurement the state is no longer represented by the distribution function f(q,p) but is reduced to $f_1(q,p)$, the new state created by the measurement, which evolves successively according to the Liouville equation. However, in the first case the state $f_1(q,p)$ is the state of a new ensemble in which the states of the individual particles are known only within the corresponding uncertainties; but in the second case there is no difference between the state of the particles and the state of the ensemble. Therefore one immediately recognizes that (a) is the usual interpretation of statistical mechanics in terms of classical dynamics, and (b) is an interpretation which closely resembles the conventional interpretation of quantum mechanics in which the observer has an essential role. In spite of the fact that they both lead to the same observable consequences, our choice is biased in favour of the first one by our belief in the existence of an objective world outside our mind.

Let us now consider the corresponding situation in quantum mechanics. Take a state defined by the wave function

$$\psi = (P_1)^{1/2} \psi_1 + (P_2)^{1/2} \exp(if) \, \psi_2 \qquad (21)$$

whose Wigner function tends in the classical limit to (19)

$$W(q,p;0) \rightarrow P_1 f_1(q,p;0) + P_2 f_2(q,p;0) = f(q,p;0) . \qquad (22)$$

The wave functions $\psi_1$and $\psi_2$ have therefore the same mean values and mean square values of q and p as before. Let us assume $\Delta q_o$ to be related to $\Delta p_o$ by the minimum uncertainty

$$\Delta q_o \Delta p_o \approx h/2. \qquad (23)$$

Suppose we measure q with the resolution $\Delta q_o$ and find the particle within the space region $S_1{}^o$, which is the space support of $\psi_1$. We might as well have measured p with resolution $\Delta p_o$, with the result that we would have found the momentum of the particle in $M_1{}^o$, the momentum support of $\psi_1$. In both cases we deduce that the state of the particle at t=0 is represented by $\psi_1$,and evolves subsequently according to the Schrödinger equation. It should be stressed that also in the quantum case the two measurements are compatible, because the two resolutions satisfy the uncertainty relation (23). Both these measurements, therefore, reduce the state (21) but do not change the form of $\psi_1$. However, according to the conventional interpretation of quantum mechanics, we cannot infer, from this fact, that the particle was in $S_1{}^t$ (or $M_1{}^t$) at an earlier time t, because we have to accept that the particle has been located in that region by the act of measurement, and that any statement about its position (or momentum) before the measurement is actually meaningless. Eq.(22), however, forces us to extend this interpretation also to classical statistical mechanics and therefore to adopt interpretation (b), because $S_1{}^t$ ($M_1{}^t$) is the space (momentum) extension of $\Gamma_1{}^t$. *We find therefore an inconsistency if we insist on accepting the conventional interpretation (a) for classical statistical mechanics while retaining the standard interpretation of quantum mechanics.*

## 5. LOCALIZATION OF PARTICLES IN PHASE SPACE AND QUANTUM MECHANICS.

The standard interpretation of quantum mechanics is therefore incompatible with the usual assumption that Newtonian dynamics for individual particles underlies the description of classical statistical ensembles. This suggests that the introduction of the notion of a sort of localization of particles in phase space should implement the conventional formulation of quantum mechanics. This localization, of course, should always be consistent with the minimum uncertainty allowed by the Heisenberg principle. In other words we believe that one may describe the time evolution of a particle's state in terms of a sort of fuzzy trajectory which is undefined within the region of minimum uncertainty, but is sufficiently localized in phase space to exclude that it may instantaneously jump from one small region to another one very far away.

We are not going to construct explicitly a new theory of this sort. We wish however to examine in more detail whether the possibility exists of modifying the standard interpretation of quantum mechanics in order to save our traditional picture of classical mechanics.

We have dealt up to here with the problem of giving a meaning to the statement that a particle was localized into one or the other of two widely separated regions in space even before an actual measurement of its position has been made. In this case, the resolution $\Delta q_0$ is given by the width of each wave packet at the time of measurement. Suppose now one localizes a particle in a space region of extension $\Delta q_0$ around a value $q_0$ within a wave packet of larger extension. Does it still make sense to ask the question: Where was the particle at an earlier time t?

The answer requires a brief discussion of the analogous classical case. Given a distribution function f(q,p,0) in phase space with mean square values $\Delta \underline{q}$ , $\Delta \underline{p}$ of q and p, we can reduce our ignorance by measuring both q and p with resolutions $\Delta q_0$, $\Delta p_0$ such that their product is much smaller than the product $\Delta \underline{q}\, \Delta \underline{p}$.. Eq.(20) will therefore again give us the uncertainty of the position of the particle (supposed to propagate freely) at an earlier time t, in terms of the values chosen for these resolutions. Of course, in classical mechanics we may chose these resolutions as small as we like (or at least as small as our instruments allow us to do). Therefore an ideal measurement with infinite resolutions determines completely the classical Newtonian trajectory.

We may now go back to the quantum case described by a wave function $\psi$ whose Wigner function tends to f(q,p;0) in the classical limit. The uncertainties in q and p given by $\psi$ are now such that $\Delta \underline{q}\, \Delta \underline{p} >> h$ . Again we may reduce our ignorance by measuring q and p with resolutions $\Delta q_0$ $\Delta p_0$ such that their product is $<< \Delta \underline{q}\, \Delta \underline{p}$, but, of course, we cannot make them as small as we like because of the minimum uncertainty relation (23). These ideal measurements however can be performed in such a way as to minimize the uncertainty in the position of the particle (again supposed to propagate freely) at an earlier time t. We obtain from (20), on making the replacement $\Delta p_0 = h/2\, \Delta q_0$,

$$\Delta q_t = \min\, [(\Delta q_o)^2 + (h\, t/2m\Delta q_o)^2]^{1/2} \qquad h/2\,\Delta \underline{p} < \Delta q_o < \Delta \underline{q} \qquad (24)$$

where the minimum is taken with respect to $\Delta q_o$ and depends on t.

For small times one has $\Delta q_t \approx h/2\,\Delta \underline{p}$ ; for intermediate times $\Delta q_t \approx (ht/2m)^{1/2}$, and for large times $\Delta q_t \approx h\,t/2m\,\Delta \underline{q}$.

This result shows that, even if we cannot precisely localize the particle on a trajectory as in classical mechanics, it is still possible to give an upper limit for the extension of the region where the particle was localized before the measurements. This statement, does of course, not conflict in any way with the physical predictions of quantum mechanics, but leads to the correct Newtonian trajectories when the classical limit is performed. An explicit example is discussed in Appendix 2 to better illustrate our point.

## 6. FINAL REMARKS.

We now wish to make a few comments on the relevance of our point of view to some of the controversial issues about the counterintuitive aspects of quantum mechanics.

The first one concerns the various attempts to construct theories which reconcile quantum mechanics with a description of the motion of particles in space by means of random processes. The fundamental physical content of this theories consists in the fact that they reproduce the one-time expectations of quantum mecanics. However, since the existence of trajectories is assumed explicitely, the positions of the particle at two different times are supposed to be correlated. This correlation is not experimentally observed and can be in principle of any kind. However our analisis gives a criterion to decide if such a theory can be accepted or not. The correlation must be in fact strong enough in order that the treajectories become the Newtonian trajectories in the classical limit. More precisely, according to (24), one should have $E[(\xi(t)-\xi(t-dt))^2] \leq ht/m$, where $\xi(t)$ is the process.

One of these theories is Nelson's stochastic mechanics.[5] In this theory, the motion is the result of the joint action of classical and stochastic forces, leading to a continous but non differentiable trajectory, typical of a Markov diffusion process. In fact the stochastic variable $\xi(t)$ describing the particle's position on the trajectory satisfies the stochastic differential equation

$$d\xi = b_+(\xi(t),t)\, dt + dw(t), \qquad (25)$$

where the drift is given by

$$b_+(x,t) = h\, \partial R(x,t)/\partial x + \partial S(x,t)/\partial x, \qquad (26)$$

with R,S as defined in Equation (7) and dw(t) denoting a Brownian process with the same diffusion coefficient $h/2m$ found above.

If we consider now the Stern Gerlach device discussed in Appendix 2, it is clear that, when the conditions are met which ensure that the lateral width of the beams are much smaller than their separation ($p_0 >> \Delta p_0$, and $t >> mh/p_0\Delta p_0$), only the first term survives in the wave function (A21), when $q \approx q_+$, and only the second term survives when $q \approx q_-$ because the Gaussians of width $\Delta q$ are vanishingly small when $q >> q_\pm$. This means that the stochastic trajectories of Nelson's theory are confined within the regions swept by each beam, but do not jump from one beam to the other. The drift contains in fact a term $p_0 dt$ ($-p_0 dt$) in the upper (lower) beam which leads to their further separation. The initial choice for the particle to go into any one of the two beams is intrinsically casual, and occurs with probabilities $P_\pm$ in the region where the two beams overlap, but as time goes on the separation between the two beams increases, and the probability of the particle being kicked out of one beam into the other one becomes smaller and smaller.

The additional content of the formalism of stochastic mechanics, which describes not only the statistical properties of an ensemble, identical of course with those given by quantum mechanics, but also the behaviour of individual particles in terms of stochastic trajectories, is therefore not meaningless, as usually one may think. It is this content which fits in our interpretation of quantum mechanics and does not fit in the conventional one. This does not mean, of course, that this theory should be accepted in its present form as a more satisfactory description of nature than conventional quantum mechanics. It means however, in our opinion, that the search for a reformulation of quantum mechanics, in which its purely statistical content due to our incomplete knowledge of the state of an individual system should be separated from the intrinsic quantum indeterminacy of the system's physical properties, is still an open question worth of deep effort and clever thinking.

The second comment has some relevance for the old problem of the nature of the wave packet reduction as a consequence of a measurement.[6] In order to understand fully the meaning of our point of view, we stress again that the result of a measurement which reduces a wave packet whith uncertainty product $\Delta \underline{q} \Delta \underline{p} >> h$ into a wave packet with uncertainty $\Delta q_0 \Delta p_0 \approx h$, is substantially different from a change in the form of a wave packet which maintains the uncertainty equal to its minimum value. It is very important to avoid confusions between the two.

The first one is irreversible, because our knowledge changes irreversibly. It implies, exactly as it does in classical mechanics, the measurement of both q and p, the only difference with classical mechanics being that now the resolutions $\Delta q_0$ and $\Delta p_0$ must satisfy the minimum uncertainty relation. The state of the individual particle is not reduced: it is only the ensemble's state which is reduced. This measurement eliminates the "empty waves" of a superposition because they are not physical; they only represent our ignorance before the measurement.

The second change is reversible, because it corresponds to an actual change of the individual particle's physical state from a wave packet with $\Delta q'_0 \Delta p'_0 \approx h$ to a wave packet with $\Delta q_0 \Delta p_0 \approx h$ due to its Schrödinger evolution in the presence of a physical interaction. Clearly, there is no reduction in this case, because there is no change in the information we have on the properties of the individual system: What we gain in the definition of q (if $\Delta q_0 < \Delta q'_0$), we loose in the definition of p ($\Delta p_0 > \Delta p'_0$) and vice versa. Obviously, this absence of reduction will not lead to a nightmarish multiplication of worlds,[7] because the first type of reduction (reduction of ignorance) is sufficient to eliminate the proliferation of branches of a composite system's wave function, a proliferation which, by definition, implies a tremendous increase with time of the uncertainty product. At the same time this absence of reduction is sufficient to eliminate the extremely "counterintuitive mutual involvement of physical and mental phenomena",[8] invoked explicitly by von Neumann and implicitly accepted by all theories of measurement which adopt the wave function collapse postulate as a physical irreversible phenomenon that cannot be reduced to the Schrödinger time evolution.

The third comment has to do with the controversy about the objective nature of the properties of a macroscopic body. It is often stated that the possibility of defining variables which maintain an essential quantum nature even in the classical limit makes a superposition of macroscopically different states substantially different from the corresponding statistical mixture (Schrodinger's cat), and forces us to conclude that one can never attribute the character of absolute reality (independent of the observer) to the objects of the world around us. This statement is, according to our viewpoint, totally unjustified.

It is of course true that one can define for a macroscopic body variables strongly depending on the details of its microscopic structure, such as the relative position of some of its atoms, which show, by definition, a typical quantum behaviour. Variables of this kind, however, are completely decoupled from those which have a classical limit, which describe all the collective properties of the body. This means not only that its physical state, in the classical limit, may be completely defined independently of the details of its microscopic structure, but also that the body's classical properties are objective inasmuch as their values, as we have argued at length in the preceeding sections, do not depend on whether they are measured or not.

One concludes therefore that the inconsistency between the standard interpretation of QM and the commonly accepted realistic interpretation of CSM discussed above should be removed if we wish to save our cherished belief that the world is there even if we do not look at it.

We are grateful to our friends G Jona-Lasinio, G F Dell'Antonio, G F De Angelis and N Zanghi for stimulating discussions. We thank L Peliti for his poetical advice.

## APPENDIX 1

We are going now to discuss a model of a macroscopic body made of N particles individually and coupled elastically to the origin, coupled one to each other by an elastic force

$$V= (a/4)\Sigma_{ij}(q_i-q_j)^2 + (b/2)\Sigma_i q_i^2, \quad i,j = 1,...,N. \qquad (A1)$$

By introducing the center of mass coordinate $q= \Sigma_i q_i/N$ and rescaling the coupling constant $a \rightarrow a/N$, we get

$$V=(a/2)\ \Sigma_i(q_i-q)^2+(b/2)\ \Sigma_i\ q_i^2 = ((a+b)/2)\ \Sigma_i(q_i-q)^2+(b/2)Nq^2. \quad (A2)$$

In this form we see that b is the coupling constant of the center of mass to the origin and (a+b) the coupling constant of the individual particles to the center of mass. The 2N independent classical equations derived from the Hamiltonian (m=1)

$$H = (1/2)\Sigma_i p_i^2 + V \qquad (A3)$$

are:

$$dq_i/dt = p_i \qquad (A4a)$$

$$dp_i/dt = -\omega^2(q_i-q) -\Omega^2 q, \quad a+b = \omega^2 \quad b = \Omega^2. \qquad (A4b)$$

Let us now define

$$q'_i = q_i - q, \quad p'_i = p_i - v, \quad v = \Sigma_i p_i / N, \quad i = 1,\dots,N$$

$$q''_\lambda = q'_\lambda - \varepsilon\, q'_N, \quad p''_\lambda = p'_\lambda - \varepsilon\, p'_N, \quad \varepsilon = (-1+\sqrt{N})/(N-1) \quad \lambda = 1,..,N-1,$$

Then it is easy to see that we have 2(N-1) independent equations

$$dq''_\lambda/dt = p''_\lambda \tag{A5a}$$

$$dp''_\lambda/dt = -\omega^2\, q''_\lambda, \quad \lambda = 1,\dots,.N-1, \tag{A5b}$$

and two equations

$$dq/dt = v \tag{A6a}$$

$$dv/dt = -\Omega^2\, q. \tag{A6b}$$

The corresponding quantum variables have the commutators

$$[\mathbf{q}_i, \mathbf{p}_j] = i\hbar\, \delta_{ij} \tag{A7}$$

$$[\mathbf{q}, \mathbf{v}] = i\hbar / N, \quad [\mathbf{q}''_\lambda, \mathbf{p}''_\mu] = i\hbar\ \delta_{\lambda\mu} \quad \lambda,\mu = 1,\dots,N-1 \tag{A8}$$

One should notice that as N (the total mass) $\to\infty$ the commutator of the average (intensive) variables **q** and **v** vanishes, i.e., these variables become classical, while the "internal" variables maintain their quantum nature.

The quantum Hamiltonian corresponding to (A3) becomes, in terms of the new variables,

$$\mathbf{H} = (1/2)\Sigma_\lambda \mathbf{p}''^2_\lambda + (\omega^2/2)\Sigma_\lambda \mathbf{q}''^2_\lambda + (1/2)\, N\, \mathbf{v}^2 + (\Omega^2/2)\, N\, \mathbf{q}^2\,, \tag{A9}$$

and the corresponding Schrödinger equation

$$i\hbar\, \partial\psi/\partial t = \mathbf{H}\, \psi \tag{A10}$$

is obtained by taking

$$v = (-i\hbar /N)\, \partial/\partial q, \quad q=q; \quad p''_\lambda = (-i\hbar)\, \partial/\partial q''_\lambda, \quad q''_\lambda = q''_\lambda \quad \text{(A11)}$$

We take a factorizable wave function $\Psi$ of the form

$$\Psi = \phi(q)\, \chi\, (q''_1, ..., q''_{N-1}), \quad \text{(A12)}$$

with the center of mass wave function describing a coherent state

$$\phi(q) = (N\Omega/\hbar\, \pi)^{-1/4} \exp\{[-N\Omega(q-q_c)^2 + 2iNqv_c - iNq_cv_c - i\hbar\, \Omega t]/2\hbar\} \quad \text{(A13)}$$

Consider for the moment the q-dependent part of $\Psi$. The classical limit of the Wigner function constructed with the wave function (A13), namely

$$f(q,v) = \delta(q-q_c(t))\, \delta(v-v_c(t)) \quad \text{(A14)}$$

is now attained as $N \to \infty$. This means that the center of mass coordinate of a macroscopic body undergoes a classical oscillator motion with coordinate $q_c(t)$ and velocity $v_c(t)$. We might instead consider, however, a wave function of the center of mass which does not shrink to a $\delta$ function in phase space, as (A13) does. A stationary state of energy $E >> \hbar\, \Omega$ would, for example, lead to a Wigner function which tends to (14) as $N \to \infty$. In other words the statistical properties of $\phi(q)$ are the same as those discussed in Section 2 in the corresponding classical limit. The presence of the remaining variables does however make a difference in the case of a macroscopic body.

To see how this comes out, let us consider a state described by a superposition of two wave functions of the form (A12),

$$\Psi = c_1\phi_1\chi_1 + c_2\phi_2\chi_2 \quad \text{(A15)}$$

In the classical limit $N \to \infty$, (A15) leads to a classical Liouville distribution of the form (19) with $f_{1,2}(q,v)$ of the form (A14). This result follows by considering the proper Wigner function of the center of mass variables q,v constructed by integrating over all the microscopic variables. One gets, in fact, the following expressions:

$$W(q,v) = \Sigma_i\, |c_i|^2 W_i(q,v) + \text{Cross Term}, \quad i=1,2, \quad \text{(A16)}$$

$$W_i(q,v) = (N/\pi h)\int dy\ \phi_i^*(q+y)\ \phi_i(q-y)\ \exp[(2iN/h)vy], \quad (A17)$$

$$C.T.=(N/\pi h)\mathcal{R}\{c_1^*c_2\ J_{12}\int dy\ \phi_1^*(q+y)\ \phi_2(q-y)\ \exp[(2iN/h)vy]\} \quad (A18)$$

$$J_{12} = \int...\int \chi_1^*(q''_1,..q''_{N-1})\ \chi_2(q''_1...q''_{N-1})\ dq''_1,...,dq''_{N-1}. \quad (A19)$$

It is now clear that the C. T. is small not only for the reasons that made small the interference contribution to the Wigner function derived from the wave function (15) (which have the same effect in the integral on y of eq.(A18), but also because it is extremely unlikely that $\chi_1$ and $\chi_2$ are exactly the same. In general $\chi_1$and $\chi_2$ are very different microscopically even if they are macroscopically equivalent. Therefore $J_{12}$ becomes vanishingly small very quickly as N becomes large.

## APPENDIX 2

Let us consider a Stern Gerlach device in which a beam of particles initially located in a given space region is split, by means of an inhomogeneous magnetic field interacting with the magnetic moment of each particle, into two beams whose separation increases with time. We study the time evolution of their lateral widths, which at t=0 coincide. If the initial wave packet is

$$\Psi(q,0) = \psi(q,0)\ [c_+\exp(ip_oq/h) + c_-\exp(-ip_oq/h)], \quad (A20)$$

with $\psi(q,0)$ given by (9), the expression of the wave function at a time t turns out to be:

$$\Psi(q,t) = c_+\Psi_+(q,t) + c_-\Psi_-(q,t) \quad (A21)$$

with

$$\Psi_\pm=\exp\{-i(\pm p_oq/h)-ip_o^2t/2mh\}(\alpha/\pi)^{1/4}(\underline{\alpha})^{-1/2}\exp\{-(q-q_\pm)^2/2\underline{\alpha}, \quad (A22)$$

$$\underline{\alpha} = a + ih\ t/m \qquad q_\pm = q_o\pm p_ot/m \quad (A23)$$

Eq.(A22) gives rise to a Wigner function, which is the sum of two widely separated parts:

$$W(q,p;t) = P_+ W_+(q,p;t) + P_- W_-(q,p;t), \quad P_\pm = |c_\pm|^2, \tag{A24}$$

where $W_\pm(q,p;t)$ are given by Eq.(11), with $x_0$ replaced by $q_\pm = q_0 \pm p_0 t/m$. The interference terms are in fact easily found to be vanishingly small, because the overlap of the two Gaussians centered at $q_\pm$ is practically zero. It is easy to work out the mean square values of q,p by means of (A24). We have:

$$(\Delta q)^2 = (\Delta q_0)^2 + (\Delta p_0)^2 t^2/m^2 + 4\, P_+ P_-\, p_0^2 t^2/m^2, \tag{A25}$$

$$(\Delta p)^2 = (\Delta p_0)^2 + 4 P_+ P_-\, p_0^2. \tag{A26}$$

Now one has, in addition to the terms which describe the spread of the individual beams, also the terms which take into account the spread in the position which arises from the possible presence of the particle in either of the two beams. If the lateral width of the individual beams is much smaller than their distance (namely if $p_0 >> \Delta p_0$, and $t >> \hbar m/p_0 \Delta p_0$), the uncertainty product reduces to the classical expression

$$(\Delta q\, \Delta p)_{cl} = 4\, P_+ P_-\, p_0^2 t/m, \tag{A27}$$

which represents the effect of the uncertainty $\pm p_0$ in the momentum of the particle on the uncertainty of its position. Here again, if we measure the position and we find the particle in the region occupied by the beam with momentum $+p_0$, we have to conclude that it was in that beam even before we made the measurement. The probabilities $P_\pm$ represent therefore our ignorance and not an intrinsic delocalization of the particle.

The example of the Stern Gerlach device shows therefore that our interpretation and the conventional one have very different implications. For us the particle is already in one of the two beams before its detection by a counter which may have been placed on its path. The counter is discharged *because* the particle was already in the beam which impinges on it. We stress that this does not imply that coherence has been destroyed once and for all. In fact if the two beams are superimposed again, the occupied phase space is not anymore the union of two classically separated regions, and therefore the typical quantum interference occurs again. In our picture the reduction of the wave function is simply a consequence of the additional information acquired on the state of the particle which allows us to change our description of it, and no problem arises.

For the conventional interpretation it is the other way round: The counter's discharge localizes the particle in the region where it occurs, inducing an abrupt change in the physical entity represented by the wave function. In this case, as already discussed at length, one has to explain many puzzling features of this sudden and irreversible change of the particle's properties.

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## NOTE

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