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Waveform of the Amplitude Modulated Laser Light by Means of a Mechanical Chopper

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Abstract. The model which reproduces accurately and conveniently the waveform of the amplitude modulated cw laser light in the TEM_{00} mode by a mechanical chopper is developed. It is found that its predictions are in good agreement with the strict, completely accurate but rather lengthy and inconvenient treatment and with the experiment. The criterion for the efficient amplitude modulation of the laser light by a mechanical chopper is formulated.

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Amplitude modulation of laser light in the TEM_{00} mode by means of a mechanical chopper is frequently employed in diverse fields of optics with photoacoustic spectroscopy being one of the recent additions [1, 2]. The time dependence of the amplitude modulated laser power $P = P(t)$ is conveniently described by the expression of the following form [3]

$$P(t)/P_0 = [1 + mf(t)]/2. \tag{1}$$

Here $P_0, m,$ and $f(t)$ denote the power of the incident laser light, the modulation depth and the modulation function, respectively. The modulation function is usually taken to be of the square wave, harmonic or of the trapezoidal form (Fig. 1) [1–6]. For theoretical considerations the harmonic modulation function is usually preferred for the sake of simplicity in spite of the fact that in most situations the trapezoidal form represents the real situation more accurately [4]. In fact, even this last one is only an approximation. Moreover, it fails badly under certain circumstances as we shall see. It is the purpose of the present note to clarify the situation by giving a model (Sect. 1) which is not mathematically cumbersome and which represents the waveform of the chopper modulated laser light with sufficient accuracy. The predictions of this, first model, in various situations are presented, compared with the second, completely accurate (and rather cumbersome) model (Sect. 1) and with the experiment (Sect. 2). Furthermore, the criterion for the efficient amplitude modulation of the laser light by means of a mechanical chopper is presented.

1. The Model

Consider a laser light in the TEM_{00} mode of radius a propagating along the z-axis and impinging on a system of paralel slots of width b and mark spaces of width β which move with a constant velocity V along the negative y-axis (Fig. 2). Integration of the laser light intensity, corresponding to the TEM_{00} mode

$$I(x, y) = I_0 \exp[-2(x^2 + y^2)/a^2], \tag{2}$$

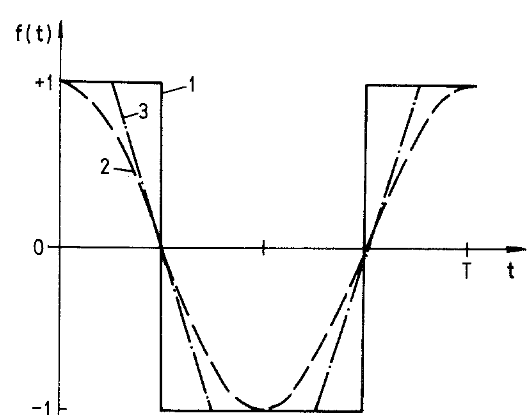

Fig. 1. The modulation function of the square wave "1", harmonic "2", and trapezoidal "3" form

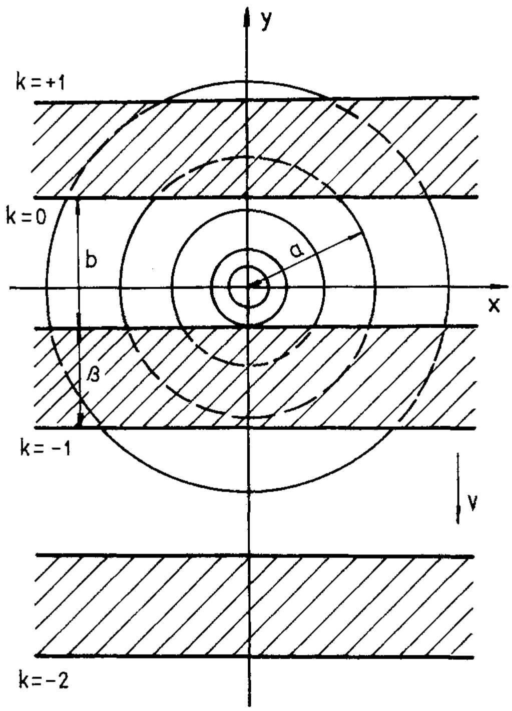

Fig. 2. Laser light in the TEM_{00} mode propagating along the z-axis and impinging on a system of paralel slots and mark spaces moving along the negative y-axis

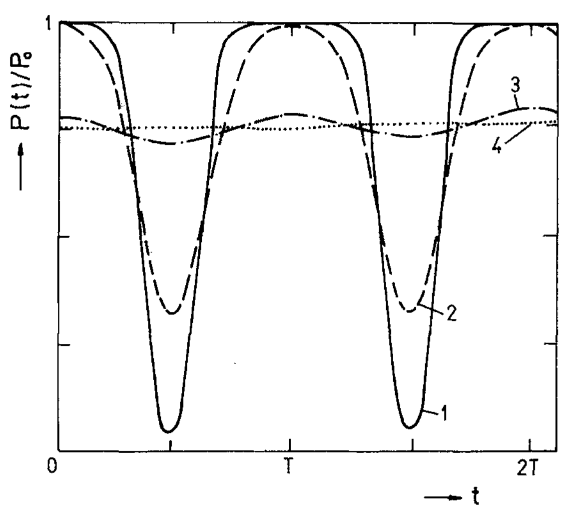

Fig. 3. Waveform of the amplitude modulated laser light calculated from *(3)* for $V=25$ m/s, $b=6\times10^{-3}$ m, $\beta=b/3$ and $a=b/6$ "*1*", $a=b/3$ "*2*", $a=b$ "*3*" and $a=2b$ "*4*"

over the system of moving slots gives for the normalized transmitted laser power, as a function of time, the following expression:

$$P(t)/P_0=\frac{1}{2}\sum_k\left\{\operatorname{erf}\frac{2^{1/2}}{a}\left[y_k(t)+\frac{b}{2}\right]-\operatorname{erf}\frac{2^{1/2}}{a}\left[y_k(t)-\frac{b}{2}\right]\right\}. \tag{3}$$

Here

$$P_0=\pi a^2 I_0/2\,, \tag{4}$$

is the power of the incident laser light, and

$$y_k(t)=k(b+\beta)-Vt\,,\qquad k=0,\pm1,\ldots, \tag{5}$$

is the y-coordinate of the center of the k^{th} slot at time t. Equation (3), together with (5), replaces (1). This function is periodic, with period

$$T=(b+\beta)/V\,, \tag{6}$$

and its values can be easily calculated with the aid of the rational approximation for the error function [7]. Figures 3 and 4 represent several different cases of the amplitude modulated laser light which were calculated using this model.

In the case when $a\ll b,\beta$ the laser beam impinges completely on one slot at most at any instant. In such a situation in the sum over k in (3) only one term contributes significantly (the one corresponding to the slot which is at that particular moment centered on the laser beam). Therefore, one can reproduce the waveform of the modulated laser light by the following simplified expression

$$P(t)/P_0\approx\frac{1}{2}\left[\operatorname{erf}\frac{2^{1/2}}{a}\left(\frac{b}{2}-Vt\right)+\operatorname{erf}\frac{2^{1/2}}{a}\left(\frac{b}{2}+Vt\right)\right], \tag{7}$$

with $|t|<T/2$. On the other hand, when $a\gg b,\beta$, the laser light passes at any instant through many slots simultaneously and (3) gives simply

$$\frac{P(t)}{P_0}\approx\frac{b}{b+\beta}=\frac{\text{transmitting area}}{\text{total area}}=\text{const}\,. \tag{8}$$

In (3) many terms contribute and although the pattern is moving (Fig. 2) the transmitted power is constant (Figs. 3 and 4).

In the intermediary case, $a\simeq b,\beta$, several (usually 3 or 4) slots contribute significantly (the contribution of the others being negligible) and the modulation depth is reduced in comparison to the case when $a\ll b,\beta$.

In all cases considered, the time averaged value $\overline{P(t)}/P_0$ of the transmitted laser power is also given by (8), that is, it is constant independent of the value of the radius of the Gaussian beam.

With $t=nT(n=0,\pm1,\ldots)$ using (3, 5, and 6) one obtaines the expression for the largest value of the normalized transmitted power

$$\frac{P_{\max}}{P_0}=\frac{1}{2}\sum_k\left\{\operatorname{erf}\frac{2^{1/2}}{a}\left[k(b+\beta)+\frac{b}{2}\right]-\operatorname{erf}\frac{2^{1/2}}{a}\left[k(b+\beta)-\frac{b}{2}\right]\right\}. \tag{9}$$

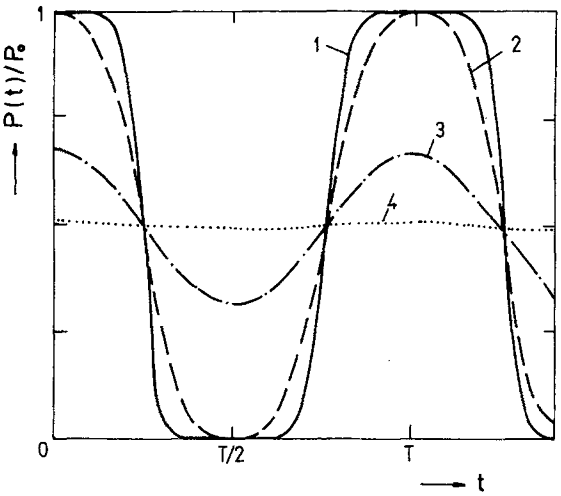

Fig. 4. Waveform of the amplitude modulated laser light calculated from *(3)* for $V=25$ m/s, $b=6\times 10^{-3}$ m, $\beta=b$ and $a=b/6$ "*1*", $a=b/3$ "*2*", $a=b$ "*3*" and $a=2b$ "*4*"

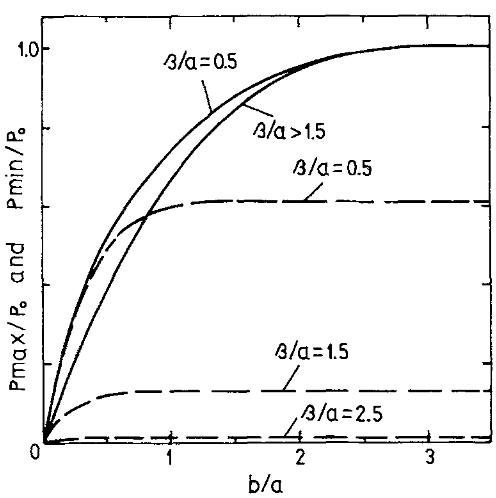

Fig. 5. P_{max}/P_0 (full lines) and P_{min}/P_0 (dotted lines) as a function of b/a for the β/a values indicated

Similarly, with $t=(n+1/2)T$ one obtaines, for the smallest value of the transmitted power, the expression

$$\frac{P_{min}}{P_0}=\frac{1}{2}\sum_k\left\{\operatorname{erf}\frac{2^{1/2}}{a}\left[\left(k-\frac{1}{2}\right)(b+\beta)+\frac{b}{2}\right]-\operatorname{erf}\frac{2^{1/2}}{a}\left[\left(k-\frac{1}{2}\right)(b+\beta)-\frac{b}{2}\right]\right\}. \tag{10}$$

These two quantities measure the modulation depth. They are depicted on Fig. 5 as a function of b/a for several different values of β/a. One can see that for the efficient amplitude modulation of the laser light by a mechanical chopper one must fulfill the following condition:

$$b,\beta \gtrsim 2.7a. \tag{11}$$

The condition (11), which requires that the widths of the slots and mark spaces be at least 2.7 times greater than the radius of the Gaussian beam, is essential for all practical applications requiring efficient amplitude modulation (then P_{max}/P_0 departs less than 1% from unity, and similarly P_{min}/P_0 departs less than 1% from zero). One should note that the change in the velocity V causes change in the modulation period only; the modulation depth being unaffected.

The model presented (Fig. 2) is only an approximation to the real situation which is shown in Fig. 6. Here, one relaxes the assumption that the slot edges are paralel. One can treat this second, completely accurate model in quite a similar manner to obtain for the transmitted laser power as a function of time the following expression

$$\frac{P(t)}{P_0}=\sum_{k=1}^{N}\int_{\phi_-(t)}^{\phi_+(t)}(J_1-J_2)\exp\left(-\frac{2R^2\sin^2\phi}{a^2}\right)d\phi + \int_0^{2\pi}\left[J_3+\frac{R\cos\phi}{(2\pi)^{1/2}a}\right]\exp\left(-\frac{2R^2\sin^2\phi}{a^2}\right)d\phi, \tag{12}$$

where

$$\phi_\pm(t)=k(\phi_1+\phi_2)-\Omega t\pm\phi_1/2, \tag{13}$$

are the upper and the lower polar angle boundaries of the k^{th} slot ($k=1,2,\ldots,N$) at time t, and where

$$J_i=J_i(\phi)\equiv\frac{\exp(-G_i^2)}{2\pi}-\frac{R\cos\phi\operatorname{erf}G_i}{(2\pi)^{1/2}a}, \tag{14}$$

and

$$G_i\equiv 2^{1/2}(r_i-R\cos\phi)/a, \tag{15}$$

with $i=1,2,3$. The meaning of other quantities appearing in these equations is clear from the Fig. 6. The remaining integrations over ϕ in (12) must be performed numerically. In (12) the first term on the right-hand side gives the contribution of the N slots while the second term represents the contribution of the laser light which passes outside the chopper disc. This second term is negligible; it is retained in (12) only for the sake of completeness. The discussion of various cases given after (6) is applicable, mutatis mutandis, to this model also. The numerical comparison of the two models, (3–5 and 12–15), reveals that in all cases of practical interest the difference between the two is less than 0.5%, thus confirming the validity of the much simpler model depicted on Fig. 2 and making the use of the strict treatment of Fig. 6 practically unnecessary.

2. Experimental Verification

A simple experimental setup is arranged to demonstrate the validity of the model expressed by (3 and 5).

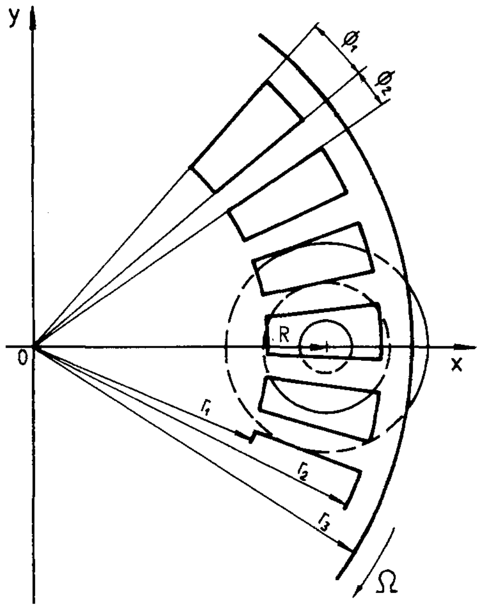

Fig. 6. Laser light in the TEM_{00} mode propagating along the direction normal to the plane of the figure and impinging on a chopping disc rotating with constant angular velocity Ω

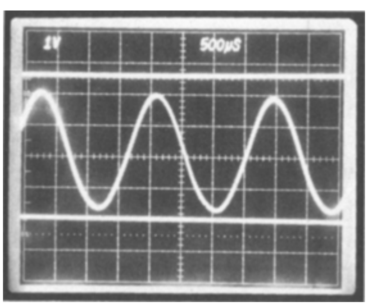

Fig. 7. Waveform of the chopper modulated TEM_{00} laser beam with the following parameters: $a = 1.93$ mm (beam radius), $b = 3.10$ mm (slot width), $\beta = 3.60$ mm (mark space width); chopping frequency $f = 545$ Hz

The beam of a helium-neon laser having 1 mW of output power in the TEM_{00} mode is amplitude modulated by a frequency stabilized chopper and detected by a fast silicon PIN photodiode. A variable beam expander is inserted between the laser and the chopper to modify the radius of the laser beam. A convergent lense of appropriate aperture located behind the chopper is used to focus the expanded and chopped beam onto the photodiode surface. Photodiode output is monitored by a camera-equipped oscilloscope. Oscilloscope traces shown in Figs. 7 and 8 exemplify two characteristic cases of different modulation depths. Relevant parameters are given in figure captions.

In order to determine experimentally the radius of the Gaussian beam, transverse intensity distribution was measured using scanning technique [8]. Least square fit to the experimental data was employed to obtain the appropriate Gaussian function, thus giving the value of the beam radius.

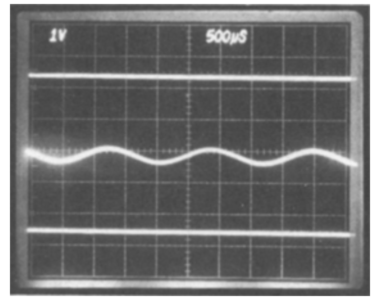

Fig. 8. Waveform of the chopper modulated TEM_{00} laser beam with the following parameters: $a = 5.74$ mm (beam radius), $b = 4.60$ mm (slot width), $\beta = 5.00$ mm (mark space width); chopping frequency $f = 615$ Hz

The lowest oscilloscope traces on Figs. 7 and 8 show the photodiode output level when it is not illuminated, while the uppermost traces show the level of the full illumination. Since $\beta > b$ for both cases, the modulated beam waveform is not centered between these two levels, but is shifted downwards, as expected.

It is found that the experimentally determined waveforms (Figs. 7 and 8) coincide to within experimental error with the computed values of $P(t)/P_0$, (3 and 5), when appropriate values of parameters a, b, β, and $V = f(b + \beta)$ are inserted.

3. Conclusions

The waveform of the amplitude modulated laser light in TEM_{00} mode by means of a mechanical chopper is given with (3 and 5), which, in comparison with the completely accurate model (12–15) and with the experimental observations shows to be of a very high accuracy and at the same time convenient for use. The simple expression (1), previously frequently used in the literature, departs quite significantly from the true waveform in almost all cases of practical interest. The analysis presented reveals also that for the efficient modulation the widths of the slots and mark spaces must be at least 2.7 times greater than the radius of the Gaussian beam, thus giving a criterion important for practical applications.

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