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Some Experiments in Partially Coherent Imaging and Modulation Transfer Function Evaluation

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Abstract. A relatively simple and inexpensive scanning optical system is described as a versatile tool for the demonstration and analysis of a variety of optical imaging properties. Some examples of partially coherent imaging and modulation transfer function (MTF) evaluation are presented.

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The evaluation of modulation transfer functions (MTF) and the study of the effect of partial coherence in optical systems are important topics in both instrumental optics, where these factors may drastically influence the quality of an image and in physical and Fourier optics where optical systems are used as analog computers to process images or other pictorial data. The experimental demonstration of these phenomena usually requires specialized and expensive equipment rarely available in a student optics laboratory. The purpose of this paper is to describe a relatively simple and inexpensive optical system which can be used to illustrate and experiment with many of the topics mentioned above.

The optical layout and underlying theory are briefly described in the next section which focuses on an interesting analogy between scanning optics and parallel optics. This analogy is well known but is rarely mentioned in light-optics texts. The following section suggest a few experiments which can be performed with this system either for the purpose of demonstration or as a student laboratory assignment.

1. Scanning Optics

It is well known that for any partially coherent transmission optical system, one can find an equivalent scanning optical layout. This equivalence has been extensively studied and exploited in the context of electron microscopy [1], and has found applications in scanning light microscopy [2], in microdensitometry [3], and in scanning optical image processing systems [4].

This equivalence is rooted in the Helmholtz principle of reversibility. In the context of this paper, it states that the two optical systems of Fig. 1 are equivalent. The first (Fig. 1b) uses a quasimonochromatic spatially incoherent source of intensity distribution $M(\mathbf{X}_s)$ to illuminate an object transparency of amplitude transmittance $f(\mathbf{X}_0)$. The equivalent scanning system (Fig. 1a) uses a low-power laser source to project the coherent point spread function $h(\mathbf{X})$ onto the object transparency. The projected PSF is scanned over the input with a galvanometer mirror scanner. A two-

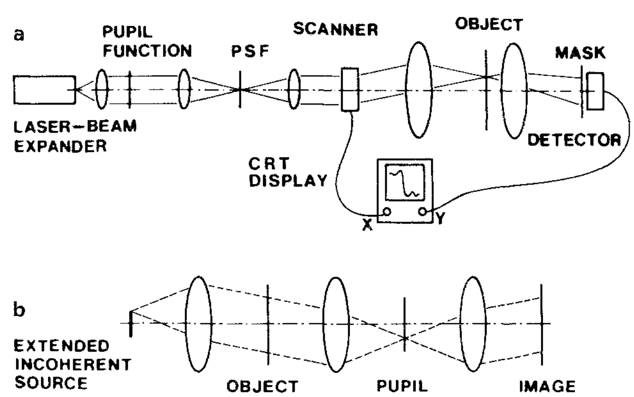

Fig. 1. (a) Laser scanning optical system. (b) Equivalent parallel optics using an extended quasimonochromatic source

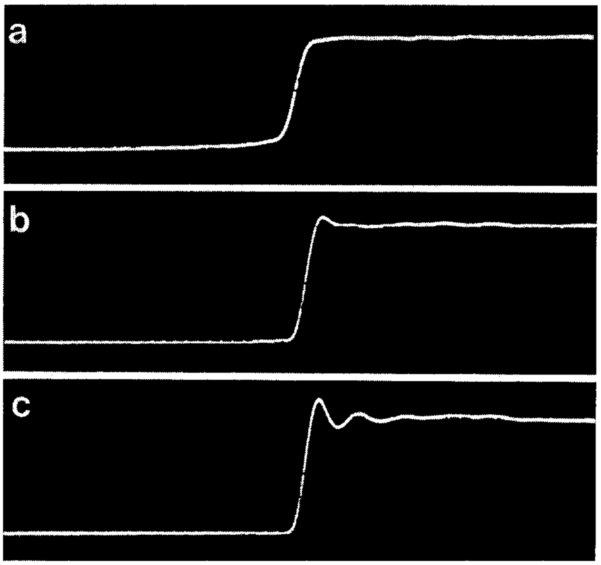

Fig. 2a–c. Images of a step function (edge) with a cutoff frequency of $10\,\mathrm{mm}^{-1}$ in (a) incoherent mode (detector mask larger than the pupil); (b) partially coherent mode (mask size about half the pupil size); (c) coherent mode (mask size about 1/10 the pupil size)

dimensional scanner is required with two-dimensional systems but the most informative experiments can be done with a one-dimensional system. The transmitted-light distribution is Fourier transformed by the last lens of Fig. 1b and falls on a quadratic detector through a mask of intensity transmittance $M(\mathbf{X}_d)$. A photo-multiplier was used in the experiment described below, but in most cases the output irradicance is sufficiently large that a less expensive and more compact solid-state detector could be used.

If both systems of Fig. 1 have the same point spread function $h(\mathbf{X})$, the image can, in both cases, be written as

$$I(\mathbf{X}_i)=\iiint f(\mathbf{X}_0)f^*(\mathbf{X}'_0)h(\mathbf{X}_0-\mathbf{X}_i)h^*(X'_0-X_i)\cdot\exp\left[-\frac{2i\pi}{\lambda f}\mathbf{X}_d\cdot(\mathbf{X}_0-\mathbf{X}'_0)\right]M(\mathbf{X}_d)d\mathbf{X}_0 d\mathbf{X}'_0 d\mathbf{X}_d\,.\quad(1)$$

This assumes all the focal lengths to be equal. Proportionality factors are omitted and * stands for complex conjugate.

In the scanning system, the image is formed point by point according to the scanner's motion $\mathbf{X}_i=\mathbf{X}_i(t)$. The temporal signal can, for example, be displayed on a cathode ray tube (CRT) oscilloscope with the time base (X-axis) scanned in synchronism with the object scanner for 1-D display. For 2-D images, the $x-y$ mode of the CRT can be used with intensity (z-axis) modulation.

2. Experiments

In addition to providing for the pedagogical demonstration of an interesting equivalence principle, the scanning system can be used to demonstrate and analyze a number of properties of partially coherent imaging systems. A few examples are shown in this section. The influence of first-order partial coherence in imaging systems can be studied with the scanning layout by simply changing the detector mask $M(\mathbf{X}_d)$. The two extremes of quasicoherence and quasiincoherence are obtained with, respectively, a pinhole detector, and a detector area larger than the image of the pupil projected onto the detector. In these two extreme cases, (1) reduces to $S(t)=|\int f(\mathbf{X}_0)h(\mathbf{X}_0-\mathbf{X}_i)d\mathbf{X}_0|^2$ in the coherent case, and $S(t)=\int|f(\mathbf{X}_0|^2\,|h(\mathbf{X}_0-\mathbf{X}_i)|^2d\mathbf{X}_0$ in the incoherent case.

To illustrate the technique, Fig. 2 shows the image of a sharp edge with various degrees of coherence. The pupil was a clear aperture of size a. In one dimension, the corresponsing PSF is [5] $h(X_i)=(\sin\pi X_i a/\lambda f)/(\pi X_i/\lambda f)$. Partially coherent images of a wedge are well known [6]. Analytic solutions for the 1-D coherent and incoherent cases are, respectively

$$I_{\mathrm{coh}}(\xi)=\left[\frac{1}{2}+\frac{1}{\pi}\mathrm{Si}(2\pi\xi)\right]^2,\quad(2)$$

$$I_{\mathrm{inc}}(\xi)=\frac{1}{2}+\frac{1}{\pi}\left[\mathrm{Si}(4\pi\xi)-\frac{1-\cos 4\pi\xi}{4\pi\xi}\right],\quad(3)$$

where $\mathrm{Si}(\alpha)=\int_0^\alpha\frac{\sin t}{t}dt$, $\xi=X_iU_c$ is a dimensionless coordinate and $U_c=a/\lambda f$ is the coherent cutoff frequency of the system. The most noticeable differences between the coherent and incoherent images are the fringes in the illuminated region, with a 19% overshoot at $\xi=0.5$ and a shift of the half-intensity point toward the illuminated region. The fringing is clearly visible in Fig. 2 which shows good qualitative agreement with (2) and (3), but the jitter of the scanner was too large to confirm the shift of the edge position. The pupil had a cutoff frequency of about $10\,\mathrm{mm}^{-1}$. The incoherent image of Fig. 2a was obtained with a detector mask slightly larger than the image of the pupil. For Fig. 2b, the mask was about half the size of the pupil and for Fig. 2c, about 1/10.

The effect of coherence on the output of a spatial filtering system is shown in the example of Fig. 3. Here the pupil was a ring with an inner and outer cutoff frequency of $U_1=5\,\mathrm{mm}^{-1}$ and $U_2=10\,\mathrm{mm}^{-1}$, respectively. Next to each profile of the step images, the relative size of the pupil (dotted line) and the detector (shaded) is shown. This clearly illustrates the difference between the incoherent transfer function (the autocorrelation of the pupil) and the coherent transfer function (the pupil itself) [6]. With a mask larger than the pupil, the system is practically incoherent while a mask smaller than the inner ring of the pupil gives a coherent image. In this last case the image is approximately

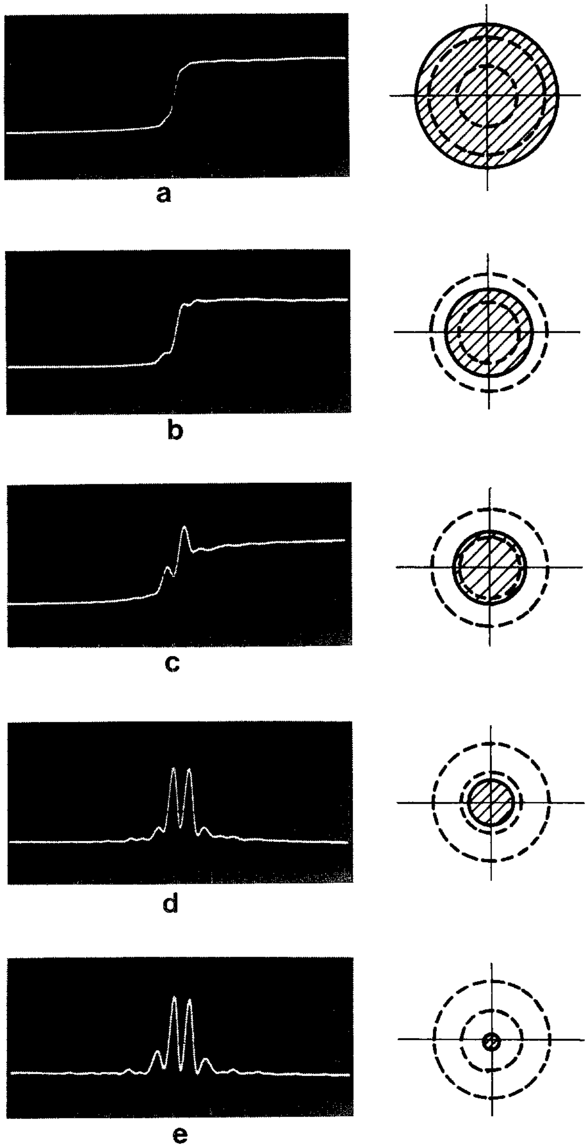

Fig. 3a–e. Images of a step function with an annular pupil (bandpass from 5 to 10 mm^{-1}) and varying mask size (shaded)

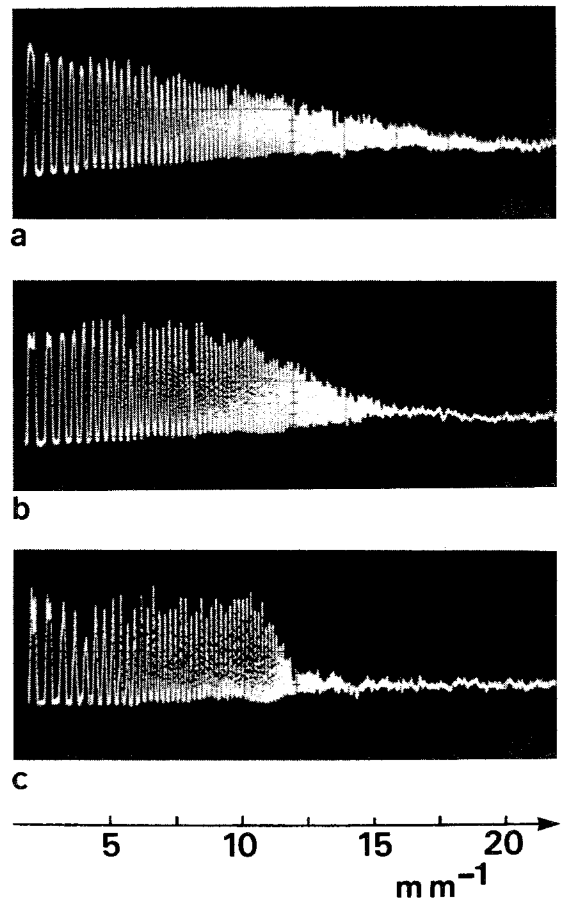

Fig. 4a–c. Transfer function of an optical system with pupil cut-off frequency 10 mm^{-1}. (a) Incoherent; (b) partially coherent; (c) coherent

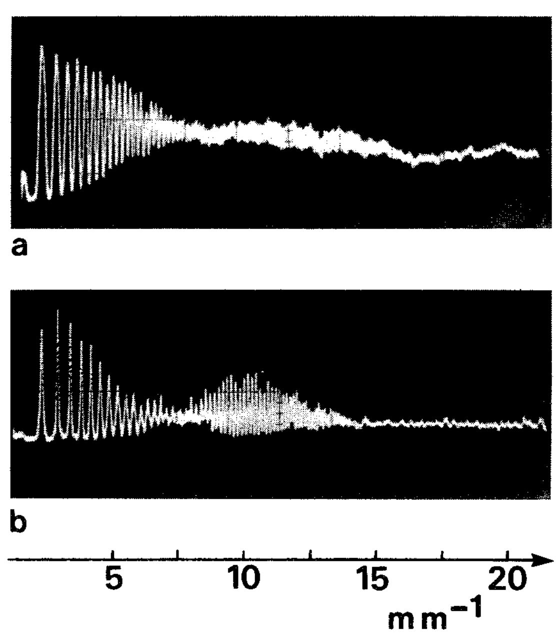

Fig. 5a and b. Transfer function of an optical system with 5% defocusing: (a) incoherent; (b) coherent

proportional to the square of the second derivative of the step.

The scanning system can also be used to display the modulation transfer function (MTF) of optical systems with arbitrary pupils or PSF. If a grating of variable spatial frequency is used as the object transparency, the CRT trace is a direct plot of the MTF. The results of Fig. 4 were obtained with a clear pupil, 10 mm^{-1} cut-off. The three transfer functions shown correspond to (a) the incoherent case where MTF has approximately a triangular shape from zero frequency to $2\,U_c(\approx 20\,mm^{-1})$, (c) the coherent case with a charp cutoff at $U_c(\approx 10\,mm^{-1})$ and (b) an intermediate case of partial coherence. These results were obtained by simply closing a diaphragm in front of the detector.

To simulate an optical system with strong aberrations, an amount of defocusing of about 5% of the system focal length (300 mm) was introduced. The corresponding incoherent and coherent transfer functions shown in Fig. 5 are in good agreement with the expected behavior of defocused optical systems [5].

Conclusions

A few experiments which can be performed with a relatively simple scanning optical system have been described. The output of the system, displayed on a CRT was found to be particularly convenient for demonstration or for student experiments. It allows quantitative measurements to be made in quasi real time, avoiding the time consuming and tedious step of taking pictures and analyzing them with a microdensitometer.

Many other experiments can be done with this scanning system. For example, its use in pattern recognition [7] and in reconstructing coded aperture images [8] have been reported in the literature. It could also serve to model and study a wide variety of instruments from microdensitometers to scanning electron microscopes.

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