Appl. Phys. B 32, 15–20 (1983)

Applied Physics B Photophysics and Laser Chemistry

Diffraction Characteristics of Planar Absorption Gratings

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Received 22 April 1983/Accepted 26 May 1983

Abstract. Planar (co)sinusoidal conductivity (absorption) transmission gratings are analyzed using rigorous coupled-wave theory. The first-order and higher-order diffraction efficiencies are determined over the entire range of possible conductivities and Bragg angles of incidence (or equivalently, grating periods) for H-mode polarization incident plane waves. The maximum possible first diffracted order efficiency is found to be 5.26%. Rigorous results are compared to approximate results from the Raman-Nath theory and the two-wave first-order coupled-wave (Kogelnik) theory. A regime parameter, ϱ_σ, is defined which delineates the regions of Raman-Nath diffraction behavior ($\varrho_\sigma < 1$) and the region of two-wave first-order diffraction theory behavior ($\varrho_\sigma > 1$). Likewise, the angular selectivity characteristics of conductivity gratings are determined from rigorous theory and are compared with corresponding results from approximate theory.

PACS: 42.20, 42.40

Optical diffraction by planar transmission gratings is a subject of fundamental importance in optics. Fields of application include acousto-optics, integrated optics, quantum electronics, holography, and spectroscopy. Grating device functions include laser-beam deflection, modulation, coupling filtering, distributed feedback, distributed Bragg reflection, holographic beam combining, wavelength multiplexing, and wavelength demultiplexing.

A rigorous coupled-wave theory (without approximations) has recently been formulated for lossless dielectric gratings with relative permittivity (dielectric constant) modulation [1]. This analysis has been shown to be general and the approximations used in previous theories have been explicitly quantified [2]. It is the purpose of this paper: 1) to extend the rigorous coupled-wave analysis to (co)sinusoidal conductivity (absorption) gratings, 2) to show that the maximum diffraction efficiency is 5.26% (rather than 3.70% from Kogelnik theory [3] or 4.80% from Raman-Nath theory [4] for these gratings), 3) to define the diffraction regimes and their boundaries for transmission absorption gratings, and 4) to determine rigorously the angular selectivity characteristics of these gratings and compare them to those from approximate theory. To assist in isolating the basic diffraction characteristics from other physical effects, the fundamental case of the same permittivity inside and outside the grating, an unslanted grating (fringes perpendicular to surface), and H-mode polarization (electric field perpendicular to the plane of incidence and perpendicular to the grating vector) is treated.

1. Theory

1.1. Conductivity Grating

The gratings analyzed in this work have a conductivity of the form

$$\sigma(x) = \sigma_0 + \sigma_1 \cos Kx\,. \tag{1}$$

The grating is unslanted with grating vector **K** (of magnitude $K = 2\pi/\Lambda$, Λ being the grating period) along the x-axis. The planar surfaces of the grating medium are at $z=0$ and $z=d$. The plane of incidence is the $x-z$ plane and thus all quantities are invariant in the

y-direction. The permittivity ϵ of the grating is constant and equal to the permittivity of the surrounding medium. The permeability μ is that of free space. In terms of these parameters the attenuation factor $\alpha(x)$ is

$$\alpha(x)=\omega(\mu\epsilon)^{1/2}\{\tfrac{1}{2}[[1+(\sigma/\omega\epsilon)^2]^{1/2}-1]\}^{1/2}, \tag{2}$$

where ω is the angular frequency of the incident light wave. The primary quantities of interest here are the diffraction efficiencies of the first-order and higher-order transmitted diffracted waves. In particular, the maximum value of the first-order diffraction efficiency is obtained for the total range of conductivities and grating periods at Bragg incidence.

1.2. Rigorous Coupled-Wave Theory

The rigorous coupled-wave equations for an unslanted (co)sinusoidal conductivity grating for H-mode polarization are

$$\frac{d^2S_i(z)}{dz^2}-\frac{4\pi}{\lambda}\left(\mathrm{j}\frac{\sigma_0}{\omega\epsilon_0}-\varepsilon_0\cos^2\theta'\right)^{1/2}\frac{dS_i(z)}{dz}+\left(\frac{2\pi}{\Lambda}\right)^2 i(m-i)S_i(z)-\mathrm{j}\frac{\pi}{\lambda}\sigma_1\eta_0[S_{i+1}(z)+S_{i-1}(z)]=0, \tag{3}$$

where $S_i(z)$ is the normalized amplitude of the i^{th} space-harmonic field at any point within the modulated region, λ is the free space wavelength of the incident plane wave, ϵ_0 is the permittivity of free space, ε_0 is the relative permittivity (dielectric constant) inside and outside of the grating, θ' is the angle of incidence in the input region,

$$m=2\Lambda\epsilon_0^{1/2}\sin\theta'/\lambda \tag{4}$$

is the Bragg condition for an unslanted absorption grating ($m=1$ for incidence at the first Bragg angle, θ_B), $\eta_0=(\mu_0/\epsilon_0)^{1/2}$ is the characteristic impedance of free space, and μ_0 is the permeability of free space. These rigorous coupled-wave equations can be solved by state-variable methods [5]. Then with the application of electromagnetic boundary conditions (continuity of tangential E and tangential H at $z=0$ and $z=d$), the diffracted fields and thus the diffraction efficiencies can be calculated for any order, reflected or transmitted [1].

1.3. Two-Wave First-Order Theory

In this approximation to the rigorous theory (Kogelnik theory [3]), the only orders retained in the analysis are $i=0$ and $+1$; the second derivatives of field amplitudes are assumed negligible; and the boundary conditions on the two space-harmonic field amplitudes are assumed to be $S_0(0)=1$ and $S_1(0)=0$. The diffraction efficiency for the first-order transmitted wave according to this theory is given by

$$\mathrm{DE}_1=\exp\left(\frac{-\eta_0\sigma_0 d}{\varepsilon_0^{1/2}\cos\theta'}\right)\sinh^2\left(\frac{\eta_0\sigma_1 d}{4\varepsilon_0^{1/2}\cos\theta'}\right) \tag{5}$$

and the zero-order (undiffracted) transmitted efficiency is predicted to be

$$\mathrm{DE}_0=\exp\left(\frac{-\eta_0\sigma_0 d}{\varepsilon_0^{1/2}\cos\theta'}\right)\cosh^2\left(\frac{\eta_0\sigma_1 d}{4\varepsilon_0^{1/2}\cos\theta'}\right). \tag{6}$$

The maximum first-order diffraction efficiency occurs when $\sigma_1=\sigma_0$ and $\eta_0\sigma_0 d/2\varepsilon_0^{1/2}\cos\theta'=\ln 3$. This maximum efficiency has a value of $\mathrm{DE}_{1,\max}=1/27\simeq 3.70\%$. The results of this well-known two-wave, first-order approximation are used as a comparison for the results obtained from rigorous theory.

1.4. Multiwave First-Order Theory Without Dephasing

In this approximation to the rigorous theory (an extension of the Raman-Nath theory of phase gratings [6–8] to absorption gratings [4]), the second derivatives of the space-harmonic field amplitudes are assumed negligible, dephasing from the Bragg condition is ignored, and the boundary conditions on the space-harmonic field amplitudes are assumed to be $S_0(0)=1$ and $S_i(0)=0$ for $i\neq 0$. The diffraction efficiency predicted for any transmitted diffracted order i is given by

$$\mathrm{DE}_i=\exp\left(\frac{-\eta_0\sigma_0 d}{\varepsilon_0^{1/2}\cos\theta'}\right)\mathrm{I}_i^2\left(\frac{\eta_0\sigma_1 d}{2\varepsilon_0^{1/2}\cos\theta'}\right), \tag{7}$$

where I_i is a modified Bessel function of the first kind of integer order i. The quantity i is equal to the diffracted order. The maximum first-order diffraction efficiency occurs when $\sigma_1=\sigma_0$ and $\eta_0\sigma_0 d/2\varepsilon_0^{1/2}\cos\theta'=1.545$ and has a value of $\mathrm{DE}_{1,\max}\simeq 4.80\%$. The results of this multiwave, first-order theory without dephasing are used as a comparison for the results obtained from rigorous theory.

2. Diffraction Characteristics

To determine the diffraction characteristics of planar (co)sinusoidal conductivity gratings, the first-order and higher-order diffraction efficiencies were calculated using the rigorous coupled-wave theory. The maximum first-order transmitted diffraction efficiency was determined for each value of conductivity modulation and Bragg angle of incidence (or equivalently, the grating period). The rigorously-determined diffraction efficiencies were then compared with results from the two-wave first-order coupled-wave (Kogelnik) theory

Table 1. Maximum diffraction efficiencies for sinusoidal conductive gratings. The maximum diffraction efficiencies (given in percent) are shown for each combination of conductivity and Bragg angle. The amplitude of the conductivity modulation in each case is equal to the average conductivity of the grating. The indices of refraction inside and outside of the grating are equal

σ [mho/m]	Angle of incidence (at first Bragg angle)													
			$s^{-1}(1/9)$	$s^{-1}(1/7)$		$s^{-1}(1/5)$		$s^{-1}(1/3)$						
	1°	5°	6.38°	8.21°	10°	11.54°	15°	19.47°	20°	25°	30°	35°	40°	45°
1	3.704	3.700	3.699	3.698	3.698	3.704	3.702	3.704	3.704	3.698	3.703	3.696	3.703	3.704
10	3.760	3.700	3.700	3.691	3.698	3.704	3.702	3.704	3.704	3.698	3.703	3.696	3.703	3.704
10^2	4.687	3.710	3.703	3.700	3.698	3.704	3.702	3.704	3.704	3.698	3.703	3.696	3.703	3.704
10^3	4.800	4.390	4.118	3.836	3.759	3.737	3.714	3.713	3.705	3.698	3.702	3.695	3.701	3.700
5×10^3	4.794	4.775	4.751	4.659	4.498	4.385	4.000	3.857	3.745	3.683	3.675	3.657	3.609	3.575
9,375	4.777	4.797	4.802	4.788	4.743	4.682	4.459	4.251	4.015	3.674	3.607	3.562	3.529	3.454
10^4	4.773	4.791	4.799	4.794	4.754	4.704	4.499	4.319	4.046	3.673	3.596	3.557	3.499	3.432
14,375	4.747	4.775	4.789	4.808	4.807	4.787	4.659	4.627	4.332	3.693	3.515	3.420	3.330	3.230
28,125	4.641	4.666	4.688	4.748	4.787	4.881	4.793	5.126	4.851	3.590	3.275	3.129	3.020	2.906
5×10^4	4.481	4.520	4.552	4.635	4.700	4.837	4.688	5.256	4.906	3.646	3.137	2.927	2.777	2.639
55,937	4.450	4.488	4.512	4.604	4.672	4.812	4.673	5.260	4.911	3.648	3.139	2.900	2.730	2.576
10^5	4.272	4.320	4.362	4.474	4.526	4.717	4.518	4.197	4.849	3.622	3.062	2.721	2.493	2.295
5×10^5	3.441	3.505	3.569	3.744	3.793	4.050	3.736	4.473	4.151	3.141	2.642	2.318	2.114	1.981
10^6	3.053	3.117	3.180	3.353	3.417	3.687	3.408	4.253	3.928	3.001	2.558	2.279	2.102	1.993

and the Raman-Nath theory. The regions of validity of these approximate theories were then delineated. Similarly, the angular selectivity characteristics were calculated using rigorous theory and compared with results from approximate theory.

2.1. Maximum Diffraction Efficiency

The maximum first-order transmitted diffraction efficiencies in percent for a range of Bragg angles of incidence and grating conductivity amplitudes are presented numerically and graphically in Table 1 and Fig. 1, respectively. The conductivity modulation amplitude is always equal to the average conductivity value, as this is necessary for maximum diffraction efficiency. The wavelength of the incident light is 500 nm, and the grating period is varied to keep the angle of incidence always at the first Bragg angle ($m=1$). The relative permittivity (dielectric constant) both inside and outside the grating is the same in order to eliminate the effects due to discontinuities in the average index of refraction. For near-normal incidence and lower values of conductivity, the maximum diffraction efficiency tends to the value of 4.80% predicted by the Raman-Nath multiwave theory, which neglects dephasing. For conditions of near-normal incidence, there are many closely angularly-spaced propagating diffracted orders and dephasing is indeed expected to be of minor importance. For larger Bragg angles of incidence and lower values of conductivity, the maximum diffraction efficiency tends to the value of 3.70% predicted by the Kogelnik two-wave first-order theory. For these larger angles of incidence, the higher-order

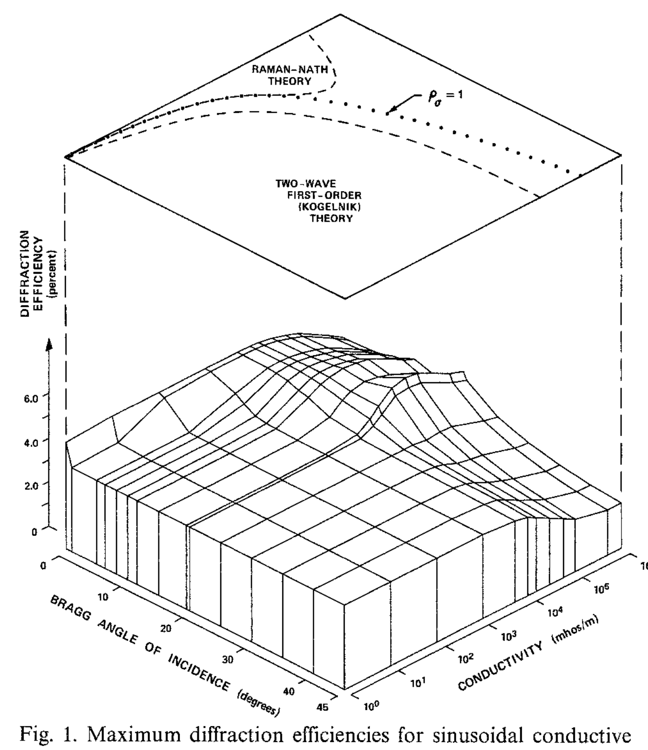

Fig. 1. Maximum diffraction efficiencies for sinusoidal conductive gratings

waves are evanescent and rigorous multiwave theory may be approximated in practice by a two-wave calculation.

A significant structural feature in the resulting maximum diffraction efficiency surface (Fig. 1) occurs for those Bragg angles of incidence at which higher-order diffracted waves are at the transition from propagating to evanescent (cut-off). For example, the angles $\sin^{-1}(1/9)\simeq 6.38°$, $\sin^{-1}(1/7)\simeq 8.21°$, and

Table 2. Example fundamental and higher-order diffraction efficiencies for a Bragg angle incidence of 1.00° for sinusoidal conductivity gratings according to the Raman-Nath, Kogelnik, and rigorous coupled-wave theories. The first case ($\sigma = 1$ mho/m) is in the Bragg regime (Kogelnik theory) and the second case ($\sigma_1 = 10^3$ mho/m) is in the Raman-Nath regime. Other parameters are $\lambda = 0.5$ μm, $\Lambda = 14.325$ μm, $\sigma_1 = \sigma_0$, and thickness chosen to maximize DE_1

Theory	σ_1 [mho/m]	ϱ_σ	Diffraction efficiency [%]					
			DE_0	DE_1	DE_2	DE_3	DE_4	DE_5
Raman-Nath	1	8.12×10^1	19.6	4.49	3.05×10^{-1}	9.66×10^{-3}	1.75×10^{-4}	2.05×10^{-6}
Kogelnik	1	8.12×10^1	14.9	3.70	–	–	–	–
Rigorous coupled-wave	1	8.12×10^1	14.9	3.70	1.38×10^{-4}	5.77×10^{-10}	6.04×10^{-16}	2.28×10^{-22}
Raman-Nath	10^3	8.12×10^{-2}	13.0	4.80	5.98×10^{-1}	3.61×10^{-2}	1.27×10^{-3}	2.93×10^{-5}
Kogelnik	10^3	8.12×10^{-2}	7.84	3.30	–	–	–	–
Rigorous coupled-wave	10^3	8.12×10^{-2}	13.0	4.80	5.98×10^{-1}	3.61×10^{-2}	1.27×10^{-3}	2.85×10^{-5}

$\sin^{-1}(1/5) \simeq 11.54°$ exhibit local diffraction efficiency maxima in the surface and correspond to transitions from 10 to 8 transmitted propagating waves, 8 to 6 waves, and 6 to 4 waves, respectively. For the angle $\sin^{-1}(1/3) \simeq 19.47°$ and a conductivity of 55,937 mho-s/m, the global maximum of 5.260% occurs in the first-order transmitted wave diffraction efficiency. This angle marks the transition from 4 forward-diffracted waves to 2 waves ($i = -1$ and $+2$ become cut-off). Other transitions, of course, occur for specific angles less than $\sin^{-1}(1/9)$. However, the resulting local maxima are masked by the overall Raman-Nath behavior of the surface in that region.

2.2. *Diffraction Regimes*

The regime where the two-wave first-order theory accurately predicts the diffraction characteristics is often referred to as the "Bragg regime". The region where Raman-Nath theory is accurate is called the "Raman-Nath regime". These regions may be distinguished by a regime parameter. The conductivity grating regime parameter ϱ_σ is defined as

$$\varrho_\sigma = \frac{4\pi\lambda}{\eta_0 \sigma_1 \Lambda^2} \tag{8}$$

by analogy to the regime parameter ϱ for phase gratings [9–11] which is

$$\varrho = 2\lambda^2/\varepsilon_1 \Lambda^2, \tag{9}$$

where ε_1 is the amplitude of the relative permittivity modulation of the phase grating. The condition $\varrho_\sigma = 1$ separates the $\sigma - \theta_B$ plane into two regions as shown in Fig. 1. For the region of $\varrho_\sigma > 1$, which includes large Bragg angles of incidence (small grating periods), the two-wave first-order (Kogelnik) result as given by (5) produces accurate results for the fundamental diffracted order ($i = +1$) for conductivities up to about 10^3 mho/m. In the $\varrho_\sigma > 1$ regime, the transmitted power is concentrated primarily in the $i = 0$ and $i = +1$ orders. In fact, the higher-order diffraction efficiencies calculated by rigorous theory were found to obey the condition

$$\sum_{i \neq 0,1} DE_i < 1/\varrho_\sigma^2. \tag{10}$$

That is, the sum of all of the higher-order diffraction efficiencies is less than $1/\varrho_\sigma^2$. This is exactly analogous to the Bragg regime two-wave criterion for phase gratings [11]. Also for $\varrho_\sigma > 1$, the values of the transmitted wave ($i = 0$) efficiency calculated by rigorous theory were compared with the values predicted by (6) from Kogelnik's theory. Good agreement was again found except at high conductivities. An example $\varrho_\sigma > 1$ case showing this agreement is given in Table 2. Since two-wave first-order theory neglects all diffracted orders except the $i = 0$ and $i = +1$ orders, there are no predictions for the higher-order waves using this theory and these are indicated by dashes in Table 2.

For the region of $\varrho_\sigma < 1$, the diffraction efficiencies of all diffracted orders (in addition to the $i = +1$ order) were calculated by rigorous theory and then compared with the values predicted by the Raman-Nath theory, (7). The $\varrho_\sigma < 1$ regime includes near-normal Bragg incidence (large grating periods). In this region the Raman-Nath formula as given by (7) was found to produce accurate results for conductivities up to about 5×10^4 mho/m. This close agreement for the first-order diffracted wave is apparent in Table 1 and Fig. 1. For the zero-order and higher-order diffraction efficiencies, similar good agreement was found. A single typical $\varrho_\sigma < 1$ case showing the agreement with Raman-Nath theory is included in Table 2.

2.3. *Angular Selectivity*

A Bragg condition occurs whenever m in (4) is an integer. Dephasing from the Bragg condition may be produced for a fixed grating by changing the angle of incidence and/or the wavelength. For $m = 1$, it is the first or fundamental Bragg incidence. In this case, there is efficient power transfer from the incident wave to the $i=+1$ diffracted order. Mathematically, this is due to the factor $(m-i)$ being zero in the rigorous coupled-wave equations, (3). This $S_i(z)$ term in the rigorous coupled-wave equations represents dephasing from the Bragg condition. When it is zero, there is no dephasing and Bragg incidence occurs. The two-wave first-order coupled-wave analysis of Kogelnik retains the effects of dephasing from the Bragg condition. The Raman-Nath theory neglects this term entirely, and any angle of incidence and wavelength is treated as Bragg incidence.

The angular selectivity of a grating is a measure of the sensitivity of the diffraction to changes in the angle of incidence. The angular selectivity, $\Delta\theta$, may be defined as the full angular deviation about the first Bragg angle $(m=1)$ which causes a reduction in the diffraction efficiency to one half the value at the Bragg angle. This angular selectivity may be calculated from rigorous coupled-wave theory or from approximate two-wave first-order coupled-wave theory since these theories include dephasing effects. The angular selectivity is given by

$$\Delta\theta=\theta^{+}-\theta^{-}, \tag{11}$$

where θ^{+} and θ^{-} are the angles of incidence, greater than and less than the Bragg angle, respectively, at which the diffraction efficiency has dropped to one half of the value at the Bragg angle. From two-wave first-order (Kogelnik) theory, these quantities are given by

$$\theta^{\pm}=\sin^{-1}\left\{\frac{\sin\theta_{\mathrm{B}}\pm(\xi\Lambda/\pi d)[\cos^{2}\theta_{\mathrm{B}}-(\xi\Lambda/\pi d)^{2}]^{1/2}}{1+(\xi\Lambda/\pi d)^{2}}\right\}. \tag{12}$$

The quantity ξ is a dephasing parameter. If $\xi=0$, there is no dephasing and $\theta^{\pm}=\theta_{\mathrm{B}}$ indicating $\Delta\theta=0$ (incidence at Bragg angle). For the maximum efficiency $(\mathrm{DE}_{1,\max}=1/27)$ in this theory, it is $\xi=0.8952$. The angular selectivity may not be calculated from Raman-Nath theory since that theory does not include any dephasing effects.

A comparison of some angular selectivity results from rigorous theory and from Kogelnik theory are shown in Table 3. In each case the first Bragg angle $\theta_{\mathrm{B}}=30°$, the wavelength $\lambda=500\,\mathrm{nm}$, and the grating is fully modulated $\sigma_1=\sigma_0$. For each conductivity, the thickness that maximizes the first diffracted order power is used. For relatively thick gratings, the rigorous theory and the Kogelnik theory predict the same angular sensitivities. For high conductivity thin gratings, the angular selectivity, $\Delta\theta$, approaches approximately 80°. However, approximate two-wave first-order (Kogelnik) theory, (12), predicts that the angular selectivity approaches 180° in the limit of increasing conductivity.

Table 3. Angular selectivity for sinusoidal conductive gratings. The full angular deviation about the first Bragg angle, $\Delta\theta$, that causes a reduction in the diffraction efficiency to one half of the value at the Bragg angle is given. Both the approximate value of $\Delta\theta$ from Kogelnik's two-wave first-order coupled-wave theory and the value from the present rigorous theory are shown. In each case $\theta_{\mathrm{B}}=30°$, $\lambda=500$ nm, and the grating is fully modulated. The indices of refraction inside and outside of the grating are equal

σ	d	$\Delta\theta$ [degrees]	
[mho/m]	[mm]	Kogelnik's theory	Rigorous theory
10^{-1}	5.05×10^{1}	5.08×10^{-4}	5.08×10^{-4}
1	5.05	5.08×10^{-3}	5.08×10^{-3}
10	5.05×10^{-1}	5.08×10^{-2}	5.08×10^{-2}
10^{2}	5.05×10^{-2}	5.08×10^{-1}	5.08×10^{-1}
10^{3}	5.05×10^{-3}	5.07	5.08
10^{4}	5.16×10^{-4}	4.69×10^{1}	5.06×10^{1}
10^{5}	8.85×10^{-5}	1.37×10^{2}	7.83×10^{1}
10^{6}	1.07×10^{-5}	1.75×10^{2}	7.79×10^{1}
10^{7}	1.02×10^{-6}	1.79×10^{2}	7.84×10^{1}

3. Summary and Discussion

The rigorous coupled-wave equations for (co)-sinusoidal conductivity (absorption) gratings have been presented. These were then solved subject to the appropriate electromagnetic boundary conditions for the first-order and higher-order transmitted diffraction efficiencies for the entire range of possible conductivities and first Bragg angles of incidence (equivalent to the range of possible grating periods). These results were then compared to results from the Raman-Nath and two-wave first-order (Kogelnik) approximate theories. Example results are shown in Table 2. The global maximum diffraction efficiency for the first-order transmitted diffracted wave was found to be 5.26% rather than 4.80% or 3.70% predicted respectively by the Raman-Nath and Kogelnik approximate theories.

A conductivity grating regime parameter was defined as $\varrho_\sigma=4\pi\lambda/\eta_0\sigma_1\Lambda^2$ by analogy to the phase grating regime parameter [9–11]. The condition $\varrho_\sigma=1$ was shown to delineate Raman-Nath diffraction behavior $(\varrho_\sigma<1)$ and two-wave first-order (Kogelnik) diffraction behavior $(\varrho_\sigma>1)$. For sufficiently high conductivities (about 5×10^4 mho/m for Raman-Nath theory and about 10^3 mho/m for Kogelnik theory), it was shown that these approximate theories no longer give accurate results even though the regime parameter condition is met (Fig. 1).

The angular selectivity characteristics of these planar conductivity gratings were analyzed using rigorous coupled-wave theory. Two-wave first-order approximate theory was found to give accurate predictions for conductivities up to about 10^4 mho/m, but overestimated the angular selectivity for higher conductivities.

H-mode polarization (electric field perpendicular to the plane of incidence and perpendicular to the grating vector) has been analyzed. However, E-mode polarization may be treated in the same manner by starting with the E-mode coupled-wave equations, as shown in [12].

Acknowledgement. This work was sponsored by the National Science Foundation and the Joint Services Electronics Program.

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