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Topology of Gap Nodes in Superfluid ^{3}He: π_4 Homotopy Group for ^{3}He-B Disclination

P. G. Grinevich and G. E. Volovik

L. D. Landau Institute for Theoretical Physics, Moscow, USSR, and NORDITA, DK-2100 Copenhagen, Denmark

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The topologically stable zeros in the energy spectrum of Fermi excitations in superfluid ^{3}He both in uniform phases and in textures are classified. This generalizes the classification of the defects of the order parameter in real coordinate space to the classification of zeros in the gap, which are the more general defects in coherent superfluid or superconducting states both in real space and momentum **k** *space. The zeros are described by classes of mappings of the spherical surfaces* S^n, *embracing the* $(6-n-1)$*-dimensional manifold of zeros in six-dimensional* $(\mathbf{k}, \mathbf{r})$ *space, into the space of the Bogolyubov–Nambu matrices, which describe the Fermi excitations. The examples of topologically nontrivial manifolds of zeros are discussed, including the closed line of zeros in five-dimensional space, which is described by the* π_4 *homotopy groups and exists in the core of the* ^{3}*He-B disclination. This object demonstrates the coupling between the real space topology of disclination and the extended space topology of zeros in the disclination core.*

1. INTRODUCTION

The nodes in the gap of the fermionic quasiparticle spectrum in superfluids and superconductors play an important part in the low-temperature physics of these coherent systems, since in the nodes the coherence is broken and normal excitation dynamics is essential even at $T=0$. The nodes can exist in real **r** space (an order parameter may become zero, e.g., on the axes of quantized vortices or in the middle of domain walls separating differently oriented vacuums of superfluid or superconducting states) as well as in momentum **k** space (the order parameter becomes zero at two points of the Fermi surface in superfluid ^{3}He-A; points and lines of zeros on the Fermi surface are possible in heavy-fermion superconductors (see, e.g., ref. 1) or in high-T_c superconductivity[2]; some symmetry classes of superfluidity and superconductivity necessarily have zeros in the momentum space.[3]

The real space and momentum space zeros in the gap have many common consequences for low-temperature properties of superfluids and superconductors. This is not surprising, since these zeros are just different projections of the more general manifold of zeros in extended six-dimensional (**k**, **r**) space. This gives the possibility of deforming the real space zeros into the momentum space zeros simply by rotating the manifold of zeros in (**k**, **r**) space. This results in the existence of the coreless vortices in ^{3}He-A (for review of vortices see ref. 4); in the ^{3}He-B quantized vortices due to such transformations, the singularity in real space flows out into extra dimensions and the vortex core becomes superfluid (refs. 4 and 5).

The properties of coherent systems essentially depend on the type of manifold of zeros. The most important manifolds are those that are characterized by nontrivial topology, such as quantized vortices in real space of ^{3}He-B and point zeros in the **k** space of the ^{3}He-A. Due to their topology, they are extremely stable toward external perturbations, including impurities. The impurities make the superconducting gap more isotropic; as a result, the topologically trivial zeros may disappear. On the other hand, the topologically nontrivial zero manifold not only survives in the presence of impurities due to topological stability, but its projection onto **k** space increases and this may produce a nonzero density of states. (In ref. 6 a nonzero density of states due to impurities was obtained for the superconducting system with lines of zeros; this is, however, in some contradiction with the result of the present paper, where we in particular show that such lines of zeros are topologically unstable.)

One may state that systems with the same topological class of zero manifold have similar properties. For example, ^{3}He-A and quantum electrodynamics (QED) with massless chiral fermions have the same topology of zero manifold in extended space, which results in the same chiral anomaly and vacuum polarization effects.[7] Different topology results in different low-temperature anomalies and in different types of the Wess-Zumino terms in action (see, e.g., ref. 8, where the anomalous behavior is considered for Cooper pairing with orbital momentum $l > 1$ and with the topological invariant for the manifold of zeros being different from that in ^{3}He-A). Therefore the classification of possible zero manifolds is of importance.

The zeros in superfluids and superconductors correspond to the so-called diabolical points in the Hamiltonian eigenvalue problem,[9,10] where the contact of different branches of the energy spectrum occurs: in our case in the zero manifold the contact of quasiparticle and quasihole spectra takes place, since the gap becomes zero on the manifold.[11] The topological classification of diabolical points that corresponds to the classification of nondegenerate unitary or orthogonal matrices of infinite dimension was given in refs. 12 and 13. However, this cannot be directly applied for superfluids and superconductors, where the Bogolyubov–Nambu 4×4 matrices have specific structure, resulting in additional topologically nontrivial manifolds of zeros.

Here we consider the topological classification of the zero manifolds in superfluid ^{3}He (Section 2) and give (in Section 3) an example of a new type of topologically nontrivial zero manifold, which is described by both π_3 and π_4 homotopy groups and appears to be due to the specific form of the Bogolyubov–Nambu matrix in superfluid ^{3}He. The manifold has a form of closed loop in five-dimensional space and appears in the core of topologically stable disclination in ^{3}He-B, corresponding to the nontrivial element of the B-phase homotopy group $\pi_1(SO_3)=Z_2$. This construction reflects the correspondence between π_4 and π_1 groups[14] and thus between the real space topology of disclination and extended space topology of zeros in the core of disclination.

2. HOMOTOPY GROUPS OF MANIFOLD OF ZEROS

The Hamiltonian that describes the fermionic quasiparticles in superfluid ^{3}He is the 4×4 Bogolyubov–Nambu matrix in spin and particle–hole spaces:

$$H=\begin{pmatrix}\varepsilon\delta_{ab}; & \Delta_{ab}\\ \Delta^{\dagger}_{ab}; & -\varepsilon\delta_{ab}\end{pmatrix} \tag{1}$$

where $\varepsilon(\mathbf{k},\mathbf{r})$ is the normal-state quasiparticle energy counted from the Fermi surface, e.g., $v_F(k-k_F)$; $\Delta_{ab}(\mathbf{k},\mathbf{r})$ is the spinor gap function, which depends both on the momentum $\mathbf{k}$ of particles forming the Cooper pair and on the coordinate $\mathbf{r}$ of the center of mass of the Cooper pair. Such a semiclassical approach is valid if the $\mathbf{r}$ dependence of Δ is slow on the inverse Fermi momentum (k_F^{-1}) scale. Since the Cooper pairing in superfluid ^{3}He occurs in a state with the spin $S=1$, the gap function is symmetric over the spin indices and may be expressed in terms of complex vector $\mathbf{d}(\mathbf{k},\mathbf{r})$:

$$\Delta_{ab}(\mathbf{k},\mathbf{r})=i\sigma_2\sigma_\alpha d_\alpha(\mathbf{k},\mathbf{r}) \tag{2}$$

where σ_α are the Pauli 2×2 matrices and $\mathbf{d}$ is an odd function of $\mathbf{k}$ due to Fermi statistics.

We define the manifold of zeros as a set of points in six-dimensional $(\mathbf{k},\mathbf{r})$ space in which at least one branch of the quasiparticle energy spectrum becomes zero, i.e., at least one eigenvalue of Eq. (1) is zero; the latter takes place when $\det H=0$. Since for this determinant one has $\det H=\varepsilon^4+\varepsilon^2\,\mathrm{Tr}(\Delta\Delta\dagger)+|\det\Delta|^2$, the manifold of zeros is a set of points at which simultaneously

$$\varepsilon=0 \tag{3a}$$

and

$$\det \Delta = \mathbf{d}^2 = 0 \tag{3b}$$

The topological classification of the manifold of zeros in six-dimensional space is just the generalization of the topological classification of real space defects in the order parameter field (on the topology of defects see reviews in refs. 15). In the latter case we consider the slower dependence of $\mathbf{d}$ on the real space coordinate $\mathbf{r}$: it is slow on the scale of the coherence length $\xi \gg k_F^{-1}$. At the distance $r \gg \xi$ from the real space defect one has locally the vacuum state corresponding to the given superfluid phase (^{3}He-A, ^{3}He-A$_1$, ^{3}He-B, polar phase, planar phase, etc.) with its own manifold R of vacuum states. The form of the vector $\mathbf{d}$ for enlisted phases is

$$\mathbf{d}_\alpha(\mathbf{k}, \mathbf{r}) = A_{\alpha i}(\mathbf{r}) k_i \tag{4}$$

with the order parameter $A_{\alpha i}$ proportional to the following Ansatz for each of five phases:

$$\hat{z}_\alpha(\hat{x}_i + i\hat{y}_i); \quad (\hat{x}_\alpha + i\hat{y}_\alpha)(\hat{x}_i + i\hat{y}_i); \quad \delta_{\alpha i}; \quad \hat{z}_\alpha \hat{z}_i; \quad (\delta_{\alpha i} - \hat{z}_\alpha \hat{z}_i) \tag{5}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are unit vectors. The vacuum manifold R for each phase is obtained by spin, orbital, and phase rotations

$$A_{\alpha i} \to R^S_{\alpha\beta}(\mathbf{r}) R^L_{ij}(\mathbf{r}) \{\exp[i\phi(\mathbf{r})]\} A_{\beta j} \tag{6}$$

To find the classes of defects in a given superfluid phase one has to enclose the defect, which corresponds to a real space set of zeros in the order parameter field, by an n-dimensional spherical surface S^n and map S^n into the space R. The nonequivalent classes of mapping form the homotopy groups $\pi_n(R)$, which describe classes of $(3-n-1)$-dimensional defects in three-dimensional real space, i.e., the point defects for $n=2$, linear defects (vortices, dislocations, disclinations, etc.) for $n=1$, and domain walls for $n=0$.

In our case we consider more rapid variation of the vector $\mathbf{d}$ and approach the scales deep inside the core of defects and with the order parameter far from the vacuum manifold of the given phase. Instead of the vacuum manifold R we must consider the whole space of vectors $\mathbf{d}$ or the whole space of matrices H in Eq. (1) except for matrices with zero eigenvalue. To find the classes of manifolds of zeros, which are the six-dimensional analogues of real space defects in the coherent state, one must embrace the manifold of zeros by S^n in six-dimensional space and map these spherical surfaces into the space R^* of all regular matrices H in Eq. (1), i.e., excluding those that have at least one zero eigenvalue. The corresponding homotopy groups $\pi_n(R^*)$ will give information on the classes of topologically stable zero manifolds of dimension $(6-n-1)$ in six-dimensional space.

It can be shown that the space R^* is the so-called suspension Σ over the space $(S^1\times S^2)/Z_2$:

$$R^*=\Sigma[(S^1\times S^2)/Z_2] \tag{7}$$

where the suspension $\Sigma(M)$ over the space M means a product of M and the segment $x\in[0,1]$ such that the space M is contracted into a point at $x=0$ and at $x=1$.

The homotopy groups π_n of this space R^* are as follows:

$$\pi_1=0; \qquad \pi_2=Z; \qquad \pi_3=Z\times Z_2; \qquad \pi_4=Z_2\times Z_2\times Z_2 \tag{8}$$

The fundamental group $\pi_1(R^*)$ is trivial, which means that there are no topologically stable lines of zeros on the Fermi surface of superfluid ^{3}He [one-dimensional line of zeros in momentum $\mathbf{k}$ space corresponds to the four-dimensional manifold of zeros in extended $(\mathbf{k},\mathbf{r})$ space, which is described by the π_1 group]. Therefore, if the line of zeros exists due to some symmetry, as in a polar phase of superfluid ^{3}He, this line would disappear under external perturbations, violating the symmetry.

The second homotopy group π_2 describes both the linear defects (quantized vortices) in real space and pointlike zeros (boojums) in momentum space (both have a three-dimensional manifold of zeros in extended space). Therefore the continuous transformation is possible between these two extreme cases of orientation of three-dimensional manifolds of zeros if they have the same topological invariant. This explains the transformation[4,5] of the singular vortex in ^{3}He-B into the vortex with the superfluid core consisting of ^{3}He-A. In first case the zero manifold is concentrated on the vortex axis in real space and on the whole two-dimensional Fermi sphere in momentum space; together these form the three-dimensional manifold. In the second case of ^{3}He-A in the core of the vortex there are pointlike zeros in momentum space for all three-dimensional real space of the vortex core, and this also comprises a three-dimensional manifold of zeros.

The topological invariant, integer N, for these three-dimensional manifolds of zeros can be written in terms of the Green's function $G=(i\omega-H)^{-1}$:

$$N=\frac{1}{24\pi^2}\int \mathrm{Tr}(G\,\partial G^{-1}\wedge G\,\partial G^{-1}\wedge G\,\partial G^{-1}) \tag{9}$$

where the integral is over the three-dimensional surface enclosing the three-dimensional manifold of zeros of G^{-1} in seven-dimensional $(\mathbf{k},\mathbf{r},\omega)$ space [zero in G^{-1} occurs when simultaneously $\omega=0$ and H has zero eigenvalue; therefore the group π_2 for a zero manifold of matrices H corresponds to the group π_3 for matrices G^{-1} producing the π_3-invariant N in Eq. (9)].

The manifolds of zeros with nonzero N are topologically stable and survive at any external perturbations. Nonzero N occurs also for quantum electrodynamics with massless chiral electrons, where the fermionic spectrum is described by the Weyl matrix:

$$H(\mathbf{k}, \mathbf{r}) = [\mathbf{k} - \frac{e}{c}\,\mathbf{A}(\mathbf{r})] \cdot \boldsymbol{\sigma} \tag{10}$$

Therefore ^{3}He-A and QED share many common properties, including the chiral anomaly, which is just the consequence of nonzero N.[7,11]

The third homotopy group π_3 describes the two-dimensional manifold of zeros in extended space. The example of a topologically nontrivial manifold of this type was found inside the domain wall separating two differently oriented vacuum states in ^{3}He-B.[16] This manifold was investigated in ref. 16 assuming the realizations with real vector $\mathbf{d}$ everywhere. For the real vectors $\mathbf{d}$ the space of regular matrices H is topologically equivalent to the three-dimensional sphere S^3, since this space is a set of values of ε and $\mathbf{d}$ that satisfy the condition $\varepsilon^2 + \mathbf{d}^2 > 0$. The third homotopy for this space is $\pi_3(R^*(\mathbf{d} - \text{real})) = \pi_3(S^3) = Z$ and the topological invariant is given in terms of the unit 4-vector n_a, with $a = 0, 1, 2, 3$, $n_0 = \varepsilon/(\varepsilon^2 + \mathbf{d}^2)^{1/2}$, and $n_i = d_i/(\varepsilon^2 + \mathbf{d}^2)^{1/2}$:

$$\tilde{N} = \frac{1}{2\pi^2} \int dx_1\, dx_2\, dx_3\, e_{abcd}\, n_a\, \partial_1 n_b\, \partial_2 n_c\, \partial_3 n_d \tag{11}$$

where the integral is over a closed three-dimensional surface enclosing the two-dimensional zero manifold in six-dimensional space. It can be shown that the zeros with even $\tilde{N}$ disappear if $\mathbf{d}$ becomes complex, while zeros with odd $\tilde{N}$ are stable toward the complex perturbations of vector $\mathbf{d}$ and belong to the nontrivial element of the subgroup Z_2 of $\pi_3(R^*)$. The zeros found in ref. 16 have $\tilde{N} = 1$.

Now we discuss a physical example of the fourth homotopy group in superfluid ^{3}He.

3. π_4 HOMOTOPY IN THE CORE OF ^{3}He-B DISCLINATION

The example of the zero manifold described by the nontrivial element of the homotopy group $\pi_4(R^*)$ was found in the core of disclination in superfluid ^{3}He-B. This disclination belongs to the nontrivial element of the first homotopy group of the ^{3}He-B vacuum manifold. According to Eqs. (5) and (6), the vacuum manifold of ^{3}He-B is a set of the order parameters of the form

$$A_{\alpha i} = R_{\alpha i}\, e^{i\Phi} \tag{12}$$

where $R_{\alpha i}$, the real orthogonal matrices, form the space SO_3, while the phase factors Φ form the space $U(1)$; together this gives the ^{3}He-B vacuum manifold R_B (see, e.g., ref. 15):

$$R_B = SO_3 \times U(1) \tag{13}$$

with the first homotopy group

$$\pi_1(R_B) = Z_2 \times Z \tag{14}$$

The disclination is described by the nontrivial element of its subgroup Z_2 corresponding to rotations $R_{\alpha i}$.

The simplest realization of this disclination for the asymptotic region far from the core is the 2π rotation about the disclination axis z:

$$R_{ij}(\phi) = \hat{z}_i\hat{z}_j + (\delta_{ij} - \hat{z}_i\hat{z}_j) \cos \phi + e_{ijk}\hat{z}_k \sin \phi \tag{15}$$

where ϕ is an azimuthal angle of the cylindrical coordinate frame (z, r, ϕ). Let us continue this structure into the core region $r < \xi$. The naive continuation of the form $A_{\alpha i}(r, \phi) = f(r) R_{\alpha i}(\phi)$ with $f(\infty) = 1, f(0) = 0$ gives a three-dimensional manifold of zeros on the disclination axis, i.e., at $r = 0$. This manifold belongs to the trivial element of the $\pi_2(R^*)$ group, since for real realization the second homotopy group is trivial: $\pi_2(R^*(\mathbf{d}\text{ real})) = \pi_2(S^3) = 0$. Therefore it is unstable toward contraction into the manifold with lower dimension.

This contraction occurs if we choose the continuation in such a manner that the order parameter does not become zero on the disclination axis and corresponds to the polar phase [see Eq. (5)] on the axis

$$A_{ij}(r, \phi) = [1 - f(r)]\hat{z}_i\hat{z}_j + f(r) R_{ij}(\phi) \tag{16}$$

The $\mathbf{d}(\mathbf{k}, \mathbf{r})$ field for this Ansatz is

$$\mathbf{d}(\mathbf{k}, \mathbf{r}) = \hat{\mathbf{z}} k_z + f(r)\mathbf{k}_\perp \cos \phi + f(r)\mathbf{k} \times \hat{\mathbf{z}} \sin \phi \tag{17}$$

According to Eq. (3), the zeros of the matrix H are in the points

$$k_z = 0, \qquad |\mathbf{k}_\perp| = k_F, \qquad r = 0 \tag{18}$$

This is the two-dimensional manifold in six-dimensional $(\mathbf{k}, \mathbf{r})$ space, or, if we discard the z coordinate along the disclination axis, since there is no z dependence of the matrix H, the zeros form the one-dimensional manifold (closed loop on the equator of the Fermi sphere) in reduced five-dimensional $(\mathbf{k}, x, y)$ space. Each point of this line belongs to the nontrivial element of the Z_2 subgroup of the $\pi_3(R^*)$ homotopy group; this can be verified by calculating the topological invariant $\tilde{N}$ over the three-dimensional spherical surface S^3, e.g., $k_z^2 + (k_x - k_F)^2 + x^2 + y^2 = R^2$, which embraces the point ($k_z=0, k_y=0, k_x=k_F, x=0, y=0$) of this zero manifold; the calculations give $\tilde{N}=1$.

It is important that the dimension of the manifold of zeros cannot be reduced further in spite of the first impression that the closed line on the Fermi sphere can be contracted into a point. The latter is forbidden by the special form of the vector **d**: the vector **d** is the odd function of **k** and, if the zero appears at some **k** on the Fermi surface, another zero should simultaneously appear with the opposite momentum −**k** on the Fermi sphere. Therefore, this topologically stable line of zeros couples the diametrically opposite points on the Fermi sphere and as a result cannot be contracted into a point.

Now we show that besides the homotopy group π_3 that describes the elements of this line of zeros, the closed line as a whole has a nontrivial topology related to the $\pi_4(R^*)$ homotopy group. Let us consider the four-dimensional spherical surface S^4:

$$(x/\xi)^2+(y/\xi)^2+(k_x/k_F)^2+(k_y/k_F)^2+(k_z/k_F)^2=R^2 \tag{19}$$

with $R>1$.

This surface embraces the line of zeros in five-dimensional ($\mathbf{k}, x, y$) space and therefore the function $\mathbf{d}(\mathbf{k}, \mathbf{r})$ in Eq. (17) produces the continuous mapping of this S^4 into the space R^*(**d** real), which is topologically equivalent to S^3. The classes of the mapping $S^4\to S^3$ form the homotopy group $\pi_4(S^3)=Z_2$ and we can easily find to which element of the π_4 group our mapping belongs. The case is that this mapping has an important property: in the asymptotic region $r>\xi$ it corresponds to the nontrivial element of the group $\pi_1(SO_3)=Z_2$, which describes the disclination in the $R_{\alpha i}$ field [see Eqs. (13)-(15)]. But this is just the criterion of nontrivial π_4-homotopy: according to a well known theorem,[14] there is one-to-one correspondence between the elements of $\pi_4(S^3)$ and the elements of $\pi_1(SO_3)$. Thus the closed line of zeros has as a whole a nontrivial topological charge related to the fourth homotopy group. It can be shown that this charge is stable toward the complex deformation of the vector **d** and corresponds to the nontrivial element of one of three Z_2 groups comprising the group $\pi_4(R^*)$ in Eq. (8).

Of course the Ansatz (16) for disclination does not necessarily correspond to the minimum energy among the linear defects of the given topological class (according to ref. 17, the real disclination calculated in the Ginzburg-Landau region has quite a different structure). However, all the disclinations of this class necessarily have the same topology of zeros: the manifold of zeros is a closed loop in five-dimensional ($\mathbf{k}, x, y$) space, which may be oriented in this space in a manner different from that for Ansatz (16), but nevertheless has the same properties: it is described by nontrivial elements of both π_3 and π_4 homotopy groups. This is reminiscent of such a disclination loop in nematic liquid crystals,[15] which is described both by π_1 and π_2 groups.

4. CONCLUSION

We have found four homotopy groups describing the manifolds of zeros in the energy spectrum of fermions in superfluid ^{3}He. All the lines of zeros in three-dimensional momentum **k** space are topologically unstable and can be destroyed by external perturbations; this is important for heavy-fermion superconductors: if, due to some crystal symmetry they have a lines of zeros and therefore the T^2 law for the low-temperature specific heat, then after some special deformation of the crystal that violates the symmetry, the zeros would disappear, producing an exponential law for the specific heat. On the other hand, among the manifolds of zeros with lower dimension there exist topologically stable manifolds: the points in the three-dimensional **k** space, such as zeros on the Fermi surface of the ^{3}He-A; the points in four-dimensional (**k**, **r**) space, such as the instantons in the ^{3}He-B domain wall[16]; and a pointlike object in five-dimensional (**k**, x, y) space, such as the closed loop inside the ^{3}He-B disclination.

There are several remaining problems related to the manifold of zeros.

1. Objects with even lower dimension, described by higher homotopy groups, are under investigation.

2. The method of relative homotopy groups should be applied to obtain the general relations between the real space topology of defects outside the core of defects and extended (**k**, **r**) space topology in the core. Such relations have been found in particular cases of the ^{3}He-B disclination and of the ^{3}He-B vortices.[5,4]

3. The calculation of the Wess-Zumino terms in the action for different cases of the topology of the spectrum is necessary: three different types of Wess-Zumino terms have already been introduced for superfluid ^{3}He,[18,19,7] corresponding to different homology groups. These Wess-Zumino terms are important not only for the dynamics of the superfluids and their defects; they also define the quantum statistics of defects.

4. The quantum mechanical calculation of the fermionic spectrum in the vicinity of the zero manifold should be performed. In this paper we used a semiclassical approach, considering the **k** and **r** variables as commuting. It is known, however, that for calculating some physical quantities, such as density of states, the semiclassical approximation is too crude.[20] Therefore, a generalization of the index theorem, applied for the π_2 manifold in ref. 21, to the manifold of zeros of higher homotopy group should also be done.

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