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Energy-Momentum and Angular Momentum of Isotropic Homogeneous Cosmological Models in a Gauge Theory of Gravity

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It is shown that in isotropic, homogeneous cosmological models in the framework of a gauge theory of gravity, the material and gravitational parts of the energy-momentum, orbital angular momentum, and spin cancel each other out locally, resulting in the global quantities being trivially zero.

1. INTRODUCTION

In (1) Garecki (n.d.) (Garecki, 1995) we investigated the energy-momentum and angular momentum problem in isotropic, homogeneous cosmological models in the framework of general relativity (GR), i.e., in Friedman cosmological models. We used the *geometrically correct* expressions for the energy and for the linear and angular momentum components.

The conclusion from very simple calculations was that in every Friedman cosmological model, independent of the curvature index $k = 0 \pm 1$, the gravitational energy and matter energy, the gravitational linear and angular momentum, and the linear and angular momentum of matter cancel each other out locally and (trivially) globally.

The present paper is devoted to investigation of the analogous problem in isotropic and homogeneous cosmological models in the framework of a gauge theory of gravity (MicGGT) which has been studied by this author for several years (Garecki, 1985a,b, 1989, 1990, 1993).

We also use in the paper only *covariant expressions* and *geometrically correct* integrals (all the integrals are scalars).

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The results of the very simple calculations are the same as the results obtained in the (Garecki, 1995) Garecki (n.d.) (1): in every isotropic, homogeneous cosmological model in MicGGT, independent of the curvature index $k = 0, \pm 1$, the material and gravitational parts of all the quantities considered cancel each other out locally, so that all the global quantities are (trivially) zero.

In the following the Latin indices run over 0, 1, 2, 3; the Greek indices run over 1, 2, 3 and the metric signature is $(+, -, -, -)$. The symbol $:=$ means "by definition" and a comma $_{,i}$ or ∂_i denotes partial differentiation.

2. THEORY OF GRAVITY AND COSMOLOGICAL MODELS

By MicGGT (Garecki, 1985a,b, 1989, 1990, 1993) we mean an example of the so-called "Poincaré gauge gravity theory" (PGT) based on the specific gravitational Lagrangian

$$L_g = \alpha(\Omega^i{}_k \wedge \eta_i{}^k + \Theta^i \wedge \star\Theta_i) + \beta\Omega^i{}_k \wedge \star\Omega^k{}_i \tag{2.1}$$

where $\Omega^i{}_k$ and Θ^i are the curvature and torsion 2-forms of the Riemann–Cartan connection $\omega^i{}_k$, respectively, $\eta_i{}^k = g^{kj}\eta_{ij} = \frac{1}{2}g^{kj}\eta_{ijlm}\vartheta^l \wedge \vartheta^m$ denotes the pseudotensorial 2-form introduced by Trautman (1971), and $\star$ is the Hodge star operator. ϑ^i is the field of coframes (co-tetrads).

The constants α and β are defined by $\alpha = c^4/16\pi G$, $\beta = K^2hc/16\pi$, $K \in \mathbb{R}^+$ (most probably $K = 1$), where h is Planck's constant, c is the speed of the light in vacuum, and G denotes the Newtonian gravitational constant.

The constant K should be determined from experiments. Its theoretical value $K = 1$ comes from probable properties of so-called "quantum gravity."

MicGGT generalizes general relativity (GR) geometrically (it uses the more general Riemann–Cartan geometry instead of the Riemannian) and physically (it uses a more general Lagrangian than GR uses, and a more sophisticated description of matter in terms of the two canonical 3-forms, the energy-momentum t_l and spin $S_i{}^j$).

The field equations of MicGGT have the form

$$\begin{aligned} D \star \Omega^j{}_i = (-)\frac{\alpha}{2\beta}(\vartheta^j \wedge \star\Theta_i - \vartheta_i \wedge \star\Theta^j) \\ - \frac{\alpha}{2\beta}\Theta_k \wedge \eta_i{}^{jk} - \frac{S_i{}^j}{4\beta} \end{aligned} \tag{2.2}$$

$$\begin{aligned} D \star \Theta_l = (-)\frac{1}{2}\Omega^{jk} \wedge \eta_{ljk} + \Bigg[Q^b{}_{lr}Q_b{}^{pr} - \frac{1}{4}\delta^p_l Q^{btr}Q_{btr} \\ + \frac{\beta}{\alpha}\left(\frac{1}{4}\delta^p_l R^{ijrm}R_{ijrm} - R^{ij}{}_{lt}R_{ij}{}^{pt}\right)\Bigg]\eta_p - \frac{t_l}{2\alpha} \end{aligned} \tag{2.3}$$

In the field equations (2.2)–(2.3) D is the exterior covariant derivative, $R^i{}_{klm} = (-)R^i{}_{kml}$ and $Q^i{}_{kl} = (-)Q^i{}_{lk}$ are the curvature and torsion components, respectively; $\eta_{ljk} := \frac{1}{2}|g|^{1/2}\epsilon_{ljkm}\vartheta^m$ and $\eta_p := \frac{1}{6}|g|^{1/2}\epsilon_{pikl}\vartheta^i \wedge \vartheta^k \wedge \vartheta^l$ denote the pseudotensorial forms introduced by Trautman (1971) and ϑ^i denotes the field of the coframes. t_l and S_i^j denote the canonical energy-momentum 3-form and the spin 3-form of the microscopic matter (mixture of quarks and leptons), respectively.

With the constants $\alpha = c^4/16\pi G$, $\beta = K^2hc/16\pi$ the formal (macroscopic) limit $h \to 0$ performed *in vacuum* on the field equations of the MicGGT leads to the vacuum GR equations.

One can obtain a similar situation inside matter with randomly oriented spins by the following two steps: (1) Averaging over randomly oriented spins (Garecki, 1985b) and then (2) taking the formal limit $h \to 0$.

MicGGT is a very good model of the gauge theory of gravitation. It is causal and deterministic (in the same sense as GR is), it admits a Hamiltonian formulation, satisfies Birkhoff's theorem, and attributes energy-momentum tensors to the gravitational field. Moreover, in the framework of the theory there exist very useful energy-momentum and spin complexes and interesting cosmological solutions without singularities, and the global quantities of an isolated, nonradiating system (especially the global energy) vanish (Garecki, 1989, 1993).

By an isotropic, homogeneous cosmological model in the framework of MicGGT we mean any solution to the cosmological equations of the theory with the Friedmann–Lemaitre–Robertson–Walker line element (FLRW line element) that in the comoving coordinates (we put $c = 1$) $x^0 = t$ (t = *cosmic time*), $x^1 = r$, $x^2 = \vartheta$, $x^3 = \varphi$ is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right] \tag{2.4}$$

and with the following torsion components different from zero [in natural frames determined by (2.4)] (Kudin *et al.*, 1979)

$$\begin{aligned}
Q^1{}_{01} &= (-)Q^1{}_{10} = Q^2{}_{02} = (-)Q^2{}_{20} = Q^3{}_{03} = (-)Q^3{}_{30} = \frac{(-)C(t)}{R^2(t)} \\
Q^1{}_{23} &= (-)Q^1{}_{32} = (-)(1 - kr^2)^{1/2}r^2 \sin\vartheta\, D(t)R(t) \\
Q^2{}_{31} &= (-)Q^2{}_{13} = (-)\frac{D(t)R(t)\sin\vartheta}{(1 - kr^2)^{1/2}} \\
Q^3{}_{12} &= (-)Q^3{}_{21} = (-)\frac{D(t)R(t)}{\sin\vartheta\,(1 - kr^2)^{1/2}}
\end{aligned} \tag{2.5}$$

The *curvature index* $k = 0, \pm 1$.

The *scale factor* $R = R(t)$ and the two torsion functions $C(t)$ and $D(t)$ are solutions of the cosmological equations of the theory. We do not cite these equations here because they are very complicated and their solutions will not be needed. We only remark that the symmetries imposed on the cosmological models considered exclude spin and admit as sources a perfect fluid only. Thus, concerning sources, we have in isotropic, homogeneous MicGGT cosmology the same situation as in GR.

Isotropic, homogeneous cosmological models in MicGGT generalize the standard Friedman cosmological models in the framework of GR.

It is very interesting that in order to calculate the energy and other quantities of the isotropic, homogeneous cosmological models in MicGGT *no solution* of the cosmological equations of the theory *is needed*. We only need the general form of the line element given by (2.4) and torsion given by (2.5), the components of the Killing vector fields admitted by (2.4) and (2.5), and the connection and curvature components corresponding to (2.4) and (2.5).

We give here only the components $\xi^i_A(x)$ ($A = 1, 2, 3, 4, 5, 6$) of the six spatial, linearly independent Killing vector fields admitted by (2.4) and (2.5). The connection and curvature components corresponding to (2.4) and (2.5) are given in the Appendix. All the components are given with respect to natural frames and coframes.

The components of the six Killing vector fields admitted by (2.4) and (2.5) are (see, e.g., Kudin *et al.*, 1979)

$$\xi^i_1 = \left(0, (1 - kr^2)^{1/2} \sin\vartheta \cos\varphi, \frac{(1 - kr^2)^{1/2}}{r} \cos\vartheta \cos\varphi, (-)\frac{(1 - kr^2)^{1/2}}{r} \frac{\sin\varphi}{\sin\vartheta}\right) \tag{2.6}$$

$$\xi^i_2 = \left(0, (1 - kr^2)^{1/2} \sin\vartheta \sin\varphi, \frac{(1 - kr^2)^{1/2}}{r} \cos\vartheta \sin\varphi, \frac{(1 - kr^2}{r} \frac{\cos\varphi}{\sin\vartheta}\right) \tag{2.7}$$

$$\xi^i_3 = \left(0, (1 - kr^2)^{1/2} \cos\vartheta, (-)\frac{(1 - kr^2)^{1/2}}{r} \sin\vartheta, 0\right) \tag{2.8}$$

$$\xi^i_4 = (0, 0, \sin\varphi, \cot\vartheta \cos\varphi) \tag{2.9}$$

$$\xi^i_5 = (0, 0, (-)\cos\varphi, \cot\vartheta \sin\varphi) \tag{2.10}$$

$$\xi^i_6 = (0, 0, 0, (-)1) \tag{2.11}$$

The Killing fields ξ^i_A (A = 1, 2, 3) are the generators of *infinitesimal spatial translations* and the Killing fields ξ^i_A (A = 4, 5, 6) are the generators of *infinitesimal spatial rotations*, all performed inside the spaces $x^0 = t = \text{const}$.

3. SINGLE-INDEX COMPLEXES IN MicGGT. ENERGY AND OTHER QUANTITIES OF ISOTROPIC, HOMOGENEOUS COSMOLOGICAL MODELS IN THE THEORY

One can easily transform the field equations (2.2)–(2.3) of MicGGT to the so-called "superpotential form"

$$d \star \Omega^j{}_i = \omega^l{}_i \wedge \star\Omega^j{}_l - \omega^j{}_l \wedge \star\Omega^l{}_i - \frac{\alpha}{2\beta}(\vartheta^j \wedge \star\Theta_i - \vartheta_i \wedge \star\Theta^j) - \frac{\alpha}{2\beta}\Theta_k \wedge \eta_i{}^{jk} - \frac{S_i^j}{4\beta} \tag{3.1}$$

$$d \star \Theta_l = \omega^p{}_l \wedge \star\Theta_p - \frac{1}{2}\Omega^{jk} \wedge \eta_{ljk} + \left[\left(Q^b{}_{lr}Q_d{}^{pr} - \frac{1}{4}\delta^p_l Q^{btr}Q_{btr}\right) + \frac{\beta}{\alpha}\left(\frac{1}{4}\delta^p_l R^{ijrm}R_{ijrm} - R^{ij}{}_{lt}R_{ij}{}^{pt}\right)\right]\eta_p \; (-)\frac{t_l}{2\alpha} \tag{3.2}$$

The left-hand side (or right-hand side) of (3.1) represents the 3-form of the *total spin density* ${}_tS_i^j$ of matter and gravitation and the 2-form $\star\Omega^j{}_i$ gives the *superpotential 2-form* of the total spin.

From (3.1) the *local conservation laws* immediately follow (= continuity equations)

$$d\left[\omega^l{}_i \wedge \star\Omega^j{}_l - \omega^j{}_l \wedge \star\Omega^l{}_i - \frac{\alpha}{2\beta}(\vartheta^j \wedge \star\Theta_i - \vartheta_i \wedge \star\Theta^j) - \frac{\alpha}{2\beta}\Theta_k \wedge \eta_i{}^{jk} - \frac{S_i^j}{4\beta}\right] = d_t S_i^j = 0 \tag{3.3}$$

Equations (3.1) admit the following interesting physical interpretation: the density of the total spin ${}_tS_i^j$ of matter and gravitation is the source of the curvature $\Omega^j{}_i$ of space-time.

If the curvature vanishes, then the matter spin and gravitational spin *cancel each other out* locally, i.e., ${}_tS_i^j = 0$ and (trivially) globally.

On the other hand, the left-hand side (or right-hand side) of (3.2) represents the 3-form of the *canonical energy-momentum complex* of matter and gravitation ${}_tK_l$ and the 2-form $\star\Theta_l$ gives the *superpotential 2-form* for the total energy-momentum density.

One can give the following physical interpretation of equality (3.2): the energy and linear momentum of matter and gravitation form the source of torsion of space-time. If torsion vanishes, then the energy and momentum of matter and gravitation *cancel each other out* locally and (trivially) globally.

The energy-momentum and spin complexes determined by (3.1) and (3.2) *localize* the energy-momentum and spin of the gravitational field. The energy-momentum and spin densities of the gravitational field determined by them vanish $\Leftrightarrow \Theta^i = \Omega^i{}_k = 0$, i.e., $\Leftrightarrow$ if space-time is flat and there is no gravitational field.

Thus, in the framework of MicGGT, we have a very much better situation with the energy-momentum problem than in GR. From the beginning we have covariant, canonical expressions representing energy-momentum and spin densities and we *do not have to add* any differentials to both sides of the (3.1) and (3.2) to obtain such covariant expressions.

In the following we will use the complexes determined by the left-hand sides of (3.1) and (3.2) to calculate energy and other quantities for the isotropic, homogeneous cosmological models in the framework of MicGGT. In order to get geometrically correct integrals, we have to use, as in GR, *single index complex language*. With this aim we transvect (3.1) on an *arbitrary* bivector field $b^{ik} = (-)b^{ki}$ (spin *descriptor*) and (3.2) on an *arbitrary* vector field $\xi^i(x)$ (energy-momentum and orbital angular momentum *descriptor*). Then we get

$$b^{ji} d \star \Omega_{ji} = b^{ji}\, {}_tS_{ij} \tag{3.4}$$

where

$$\begin{aligned} {}_tS_{ij} = (-)\frac{\alpha}{2\beta}(\vartheta_j \wedge \star\Theta_i - \vartheta_i \wedge \star\Theta_j) \\ - \frac{\alpha}{2\beta}\Theta_k \wedge \eta_{ij}{}^k - \frac{S_{ij}}{4\beta} + \omega^l{}_j \wedge \star\Omega_{li} \\ + \omega^l{}_i \wedge \star\Omega_{jl} \end{aligned} \tag{3.5}$$

and

$$\xi^l d \star \Theta_l = \xi^l\, {}_tK_l \tag{3.6}$$

where

$$\begin{aligned} {}_tK_l = (-)\frac{1}{2}\,\Omega^{jk} \wedge \eta_{ljk} &+ \left[\left(Q^b{}_{lr}Q_b{}^{pr} - \frac{1}{4}\,\delta^p_l Q^{btr}Q_{btr}\right)\right. \\ &\left.+ \frac{\beta}{\alpha}\left(\frac{1}{4}\,\delta^p_l R^{ijrm}R_{ijrm} - R^{ij}{}_{lt}R_{ij}{}^{pt}\right)\right]\eta_p \\ &+ \omega^p{}_l \wedge \star\Theta_p - \frac{t_l}{2\alpha} \end{aligned} \tag{3.7}$$

One can easily obtain from (3.4) that

$$d(b^{ji} \star \Omega_{ji}) = b^{ji}\, {}_tS_{ij} + db^{ji} \wedge \star\Omega_{ji} \tag{3.8}$$

and from (3.6)

$$d(\xi^l \star \Theta_l) = \xi^l\, {}_tK_l + d\xi^l \wedge \star\Theta_l \tag{3.9}$$

After choosing suitable *descriptors*, $d(b^{ji} \star \Omega_{ji})$ will be the 3-form of the *total spin density*, matter and gravitation, and the $d(\xi^l \star \Theta_l)$ will be the 3-form of the *total energy-momentum and orbital angular momentum density*, matter and gravitation. These ordinary Cartan 3-forms leads to the *geometrically correct integrals* (the integrals are scalars) representing the global energy, the components of the global linear and orbital angular momentum, and the components of the global spin. In terms of the components with respect to the local coordinates these 3-forms give us the covariant, *single index* energy-momentum-orbital angular momentum and spin complexes as vector densities.

Using the single index complexes, we have, as in GR, a serious problem: how do we choose a suitable descriptor?

In the case of the energy of the isotropic, homogeneous cosmological models the problem is not very hard. Namely, from simple geometric and physical considerations, we immediately obtain that the *energy descriptor* is here given by unit timelike vector field $\xi^i = u^i$, $u^iu_i = 1$ ($u^i = \delta^i_0$ in the coordinates used). This vector field is distinguished physically (u^i is the 4-velocity of the so-called "fundamental or isotropic observers") and geometrically (it determines a hypersurface-orthogonal congruence) in the framework of the isotropic, homogeneous cosmology.

Using the unit timelike vector field $\xi^i = u^i$ ($= \delta^i_0$ in the coordinates used, $x^0 = t$, $x^1 = r$, $x^2 = \vartheta$, $x^3 = \varphi$) as the *energy descriptor*, we get from (3.9) the following integral expression for the total energy, matter and gravitation, contained inside of the hypersurface $x^0 = t = \text{const}$, i.e., on the total energy of the isotropic, homogeneous universe at the moment $x^0 = t = \text{const}$ of cosmic time t:

$$E = \int_{x^0=t=\text{const}} \partial_\beta(2\alpha|g|^{1/2}Q_i{}^{\beta 0}\xi^i)\, dr\, d\vartheta\, d\varphi$$
$$= \oint_{\partial x^0} 2\alpha|g|^{1/2}Q_i{}^{\beta 0}\xi^i\, d\sigma_\beta \tag{3.10}$$

Finally, putting $\xi^i = u^i = \delta^i_0$, we have

$$E = \int_{x^0=t=\text{const}} \partial_\beta(2\alpha|g|^{1/2}Q_0{}^{\beta 0})\, dr\, d\vartheta\, d\varphi$$
$$= \oint_{\partial x^0} 2\alpha|g|^{1/2}Q_0{}^{\beta 0}\, d\sigma_\beta \tag{3.11}$$

∂x^0 is an oriented boundary of the space $x^0 = \text{const}$ and $d\sigma_\beta$ is the integration element over this boundary.

It is easily seen from the formulas given in Section 2 that the torsion components $Q_0{}^{\beta 0}$ which determine the *total energy density of matter and gravitation* in the framework of the isotropic, homogeneous cosmology *identically vanish*. Therefore, as in GR (Garecki, 1995), *in every isotropic and homogeneous cosmological model* in the framework of MicGGT the gravitational energy and matter energy *cancel each other locally and (trivially) globally and we have* $E = 0$.

In order to calculate correctly the other global quantities for isotropic, homogeneous cosmological models in the framework of MicGGT one should use the 3-form $d(\xi^l \star\Theta_l)$ (to calculate the components of the linear and orbital angular momentum) and the 3-form $d(b^{ji} \star \Omega_{ji})$ (to calculate spin components).

As the *descriptors* of the components of the linear and orbital angular momentum one should take the Killing vector fields (2.6)–(2.11).

The *descriptors* of the linear momentum components will be given by the Killing fields $\xi^i_A(x)$ $(A = 1, 2, 3)$ and the *descriptors* of the orbital angular momentum components will be given by the Killing fields $\xi^i_A(x)$ $(A = 4, 5, 6)$.

We obtain from (3.9) the following *integral invariants* representing the components of the global linear and (orbital) angular momentum of the isotropic, homogeneous universes

$$I_A = 2\alpha \int_{x^0=t=\text{const}} (|g|^{1/2}Q_i{}^{\alpha 0}\xi^i_A)_{,\alpha}\, dr\, d\vartheta\, d\varphi \tag{3.12}$$
$$A = 1, 2, 3, 4, 5, 6; \qquad k = 0, \pm 1$$

After very simple calculations we get from (3.12):

1. The components $P_1 = I_1$, $P_2 = I_2$, $P_3 = I_3$ of the global linear momentum of the considered universes *vanish locally*, i.e., the densities (integrands) of these components vanish and (trivially) *globally*, i.e., the integrals (3.12) (trivially) vanish, *independent of* the curvature index $k = 0, \pm 1$.

The components P_1, P_2, P_3 are connected with the Killing vector fields ξ^i_1, ξ^i_2, ξ^i_3 given by (2.6)–(2.8) respectively.

2. The spatial components $M_1 = I_4$, $M_2 = I_5$, $M_3 = I_6$ of the *orbital angular momentum* connected with the Killing vector fields ξ^i_4, ξ^i_5, ξ^i_6 given by (2.9)–(2.11), respectively, also *vanish locally*, i.e., their densities (integrands) vanish and (trivially) *globally*, i.e., the suitable integrals (3.12) (trivially) vanish, *independent* of the curvature index $k = 0, \pm 1$.

We have obtained all the above results by direct calculations of the integrals (3.12), without using Stokes' integral theorem.

Direct calculation of the integrals (3.12) for the Killing fields ξ^i_A ($A = 1, 2, 3, 4, 5, 6$) gives *not only global quantities*, but also the *densities* of these quantities (integrands).

Concerning of the spin, we have from (3.8) in terms of components with respect to natural frames

$$\partial_a(4\beta |g|^{1/2} R^{liaq} b_{li}) = |g|^{1/2}[({}_g S^{qil} + {}_m S^{qil}) b_{li} + 4\beta R^{liaq} \partial_a b_{li}] \quad (3.13)$$

where R^{liaq} are the curvature components and ${}_g S^{qil}$ and ${}_m S^{qil}$ denote of the *spin-tensor* components of gravitation and matter, respectively. Here $b_{li}(x) = (-)b_{il}$ are the components of an arbitrary bivector.

From (3.13) we get the following *integral invariants* representing the components of the *global spin* of matter and gravitation in the isotropic, homogeneous universes:

$$S_A = 4\beta \int_{x^0 = t = \text{const}} (|g|^{1/2} R^{li\alpha 0} b_{Ali})_{,\alpha} dr\, d\vartheta\, d\varphi \quad (3.14)$$

As the *descriptors* of the components of the global spin we propose to take the bivectors

$$b_{Ali} := \nabla_{[l}\xi_{Ai]} = \partial_{[l}\xi_{Ai]} + \tfrac{1}{2} Q^p{}_{il}\xi_{Ap}$$

where $\xi_{Ai}(x)$ are the components of the Killing vector fields which generate infinitesimal rotations and the lower index A enumerates the Killing fields (and descriptors). The maximal number of such vector fields is at most six, so the index A satisfies $0 \le A \le 6$.

The above choice of the *spin descriptors* is the *most natural* and it is connected with the *symmetry properties of space-time.*

In the framework of the isotropic, homogeneous MicGGT cosmology there exist the three Killing vector fields $\xi^i_A(x)$ ($A = 4, 5, 6$) given by (2.9)–(2.11), which are *generators of infinitesimal spatial rotations.* So, in the framework of the isotropic, homogeneous MicGGT cosmology there exist only three integral invariants, which may be interpreted as the spatial components of the global spin of matter and gravitation. We will denote these invariants by $S_1(\xi^i_4)$, $S_2(\xi^i_5)$, $S_3(\xi^i_6)$.

If we calculate these integral invariants by using (3.14), the components of the Killing fields $\xi^i_4(x)$, $\xi^i_5(x)$, $\xi^i_6(x)$ given by (2.9)–(2.11), and the curvature and connection components given in the Appendix, then we obtain that their densities (integrands) *vanish* and, in consequence, the integral invariants (trivially) *vanish.*

4. CONCLUSIONS

We have calculated the nine integral invariants for an isotropic, homogeneous cosmological model in the framework of MicGGT: one invariant E is given by (3.10), the six invariants I_A ($A = 1, 2, 3, 4, 5, 6$) are given by (3.12), and the three invariants S_A ($A = 1, 2, 3$) are given by (3.14).

All nine invariants *vanish locally,* i.e., their densities (integrands) vanish and, in consequence, they (trivially) *vanish globally,* i.e., the all integrals (trivially) vanish.

We have interpreted physically the invariant E as representing the global energy of matter and gravitation, the invariants I_A ($A = 1, 2, 3$) as describing the components P_A ($A = 1, 2, 3$) of the global linear momentum of matter and gravitation, the invariants I_A ($A = 4, 5, 6$) as describing the spatial components M_A ($A = 1, 2, 3$) of the global *orbital* angular momentum of matter and gravitation, and the invariants S_A ($A = 1, 2, 3$) as giving the spatial components of the *global spin* of matter and gravitation.

The sum $M_A + S_A$ ($A = 1, 2, 3$) can be interpreted physically as representing the spatial components of the *total angular momentum* (orbital and spin) of matter and gravitation.

Thus, by using *covariant expressions,* we have proved the surprising fact that the isotropic, homogeneous cosmological models in MicGGT evolve in such a way that, for all t, the densities (integrands) of the all global quantities *invariantly* connected with them *vanish.*

In consequence, all the global quantities which may be *invariantly* defined for such cosmological models also (trivially) *vanish.*

One can physically interpret the above fact as a new *characteristic property* of the isotropic, homogeneous cosmological models in MicGGT.

Any deviation from isotropy and homogeneity *disturbs* this property.

APPENDIX

We give here the nonzero connection and curvature components in the isotropic, homogeneous cosmological models in the framework of MicGGT. The components are given in natural frames determined by the comoving coordinates $x^0 = t$, $x^1 = r$, $x^2 = \vartheta$, $x^3 = \varphi$.

The Nonzero Connection Components

$$\Gamma^1_{11} = \frac{kr}{1 - kr^2}$$

$$\Gamma^0_{11} = \frac{R\dot{R} + C(t)}{1 - kr^2}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}$$

$$\Gamma^1_{22} = (-)r(1 - kr^2)$$

$$\Gamma^0_{22} = r^2(R\dot{R} + C)$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\vartheta$$

$$\Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = \frac{\dot{R}}{R}$$

$$\Gamma^2_{33} = (-)\sin\vartheta\cos\vartheta$$

$$\Gamma^2_{13} = (-)\Gamma^2_{31} = (-)\frac{D(t)R\sin\vartheta}{2(1 - kr^2)^{1/2}}$$

$$\Gamma^1_{33} = \Gamma^1_{22}\sin^2\vartheta$$

$$\Gamma^0_{33} = \Gamma^0_{22}\sin^2\vartheta$$

$$\Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = \frac{R\dot{R} + C}{R^2}$$

$$\Gamma^1_{23} = (-)\Gamma^1_{32} = \frac{(1 - kr^2)^{1/2}r^2\,DR\sin\vartheta}{2}$$

$$\Gamma^3_{12} = (-)\Gamma^3_{21} = \frac{DR}{2\sin\vartheta(1 - kr^2)^{1/2}} \tag{A.1}$$

The Nonzero Curvature Components

$$R^0{}_{101} = (-)R^0{}_{110} = \frac{R(\ddot{R}\dot{R} + \dot{C}) - \dot{R}C}{R(1 + kr^2)}$$

$$R^1{}_{001} = (-)R^1{}_{010} = \frac{R(\ddot{R}\dot{R} + \dot{C}) - \dot{R}C}{R^3}$$

$$R^0{}_{123} = (-)R^0{}_{132} = \frac{r^2DR(R\dot{R} + C)\sin\vartheta}{(1 - kr^2)^{1/2}}$$

$$R^1{}_{023} = (-)R^1{}_{032}$$

$$= (-)\,\frac{(1 - kr^2)^{1/2}r^2\,D(R\dot{R} + C)}{2R}$$

$$R^0{}_{202} = (-)R^0{}_{220} = r^2(R\dot{R} + C)^{\cdot} - \frac{\dot{R}r^2(R\dot{R} + C)}{R}$$

$$R^2{}_{002} = (-)R^2{}_{020} = \left(\frac{R\dot{R} + C}{R^2}\right)^{\cdot} + \frac{\dot{R}(R\dot{R} + C)}{R^3}$$

$$R^0{}_{303} = (-)R^0{}_{330} = R^0{}_{202}\sin^2\vartheta$$

$$R^3{}_{003} = (-)R^3{}_{030} = R^2{}_{002}$$

$$R^2{}_{323} = (-)R^2{}_{332} = R^3{}_{232}\sin^2\vartheta = \left[\frac{(R\dot{R} + C)^2}{R^2} + k + \frac{D^2R^2}{4}\right]r^2\sin^2\vartheta$$

$$R^3{}_{232} = (-)R^3{}_{223} = \left[\frac{(R\dot{R} + C)^2}{R^2} + k + \frac{D^2R^2}{4}\right]r^2$$

$$R^0{}_{213} = (-)R^0{}_{231} = \frac{(R\dot{R} + C)DRr^2\sin\vartheta}{(1 - kr^2)^{1/2}} \tag{A.2}$$

$$R^2{}_{013} = (-)R^2{}_{031} = \frac{(R\dot{R} + C)D\sin\vartheta}{R(1 - kr^2)^{1/2}}$$

$$R^0{}_{312} = (-)R^0{}_{321} = (-)\,\frac{(R\dot{R} + C)DRr^2\sin\vartheta}{(1 - kr^2)^{1/2}}$$

$$R^3{}_{012} = (-)R^3{}_{021} = (-)\,\frac{D(R\dot{R} + C)}{R(1 - kr^2)^{1/2}\sin\vartheta}$$

$$R^1{}_{203} = (-)R^1{}_{230} = \frac{(1 - kr^2)^{1/2}r^2(DR)^{\cdot}\sin\vartheta}{2}$$

$$R^2{}_{103} = (-)R^2{}_{130} = (-)\frac{(DR)^{\cdot} \sin \vartheta}{2(1 - kr^2)^{1/2}}$$

$$R^1{}_{302} = (-)R^1{}_{320} = (-)\frac{(1 - kr^2)^{1/2}(DR)^{\cdot} r^2 \sin \vartheta}{2}$$

$$R^3{}_{102} = (-)R^3{}_{120} = \frac{(DR)^{\cdot}}{2 \sin \vartheta \, (1 - kr^2)^{1/2}}$$

$$R^1{}_{212} = (-)R^1{}_{221} = \left[k + \frac{(R\dot{R} + C)^2}{R^2} - \frac{D^2R^2}{4}\right] r^2$$

$$R^2{}_{112} = R^3{}_{113} = (-)R^2{}_{121} = (-)R^3{}_{131} = (1 - kr^2)^{-1}\left[\frac{D^2R^2}{4} - \frac{(R\dot{R} + C)^2}{R^2} - k\right]$$

$$R^1{}_{313} = (-)R^1{}_{331} = R^1{}_{212} \sin^2 \vartheta$$

$$R^2{}_{301} = (-)R^2{}_{310} = \frac{(DR)^{\cdot} \sin \vartheta}{2(1 - kr^2)^{1/2}}$$

$$R^3{}_{201} = (-)R^3{}_{210} = (-)\frac{(DR)^{\cdot}}{2 \sin \vartheta \, (1 - kr^2)^{1/2}}$$

In the above formulas $k = 0, \pm 1$ is the *curvature index*, $R = R(t)$ is the so-called *scale factor*, and $C = C(t)$ and $D = D(t)$ denote the two intrinsic "torsion functions." Here $\dot{R} := dR/dt$, $\ddot{R} := d^2R/dt^2$; $(DR)^{\cdot} := d(DR)/dt$, and so on.

The three functions $R(t)$, $C(t)$, and $D(t)$ should be determined from the cosmological equations of the theory.

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