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The Planck Aether Model for a Unified Theory of Elementary Particles

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A dense assembly of an equal number of two kinds of Planck masses, one having positive and the other one negative kinetic energy, described by a nonrelativistic nonlinear Heisenberg equation with pointlike interactions, is proposed as a model for a unified theory of elementary particles. The dense assembly of Planck masses leads to a vortex field below the Planck scale having the form of a vortex lattice, which can propagate two types of waves, one having the property of Maxwell's electromagnetic and the other one the property of Einstein's gravitational waves. The waves have a cutoff at a wavelength equal to the vortex lattice constant about $\sim 10^3$ times larger than the Planck length, reproducing the GUT scale of elementary particle physics. The vortex lattice has a resonance energy leading to two kinds of quasiparticles, both of which have the property of Dirac spinors. Depending on the resonance energy, estimated to be $\sim 10^7$ times smaller than the Planck energy, the mass of one of these quasiparticles is about equal to the electron mass. The mass of the other particle is much smaller, making it a likely candidate for the much smaller neutrino mass. Larger spinor masses occur as internal excitations, with a maximum of four such excitations corresponding to a maximum of four particle families. Other vortex solutions may describe the quark–lepton symmetries of the standard model. All masses, with the exception of the Planck mass particles, are quasiparticles for which Lorentz invariance holds, with the Galilei invariance at the Planck scale dynamically broken into Lorentz invariance below this scale. The assumed equal number of Planck masses with positive and negative kinetic energy makes the cosmological constant exactly equal to zero.

1. INTRODUCTION

Relativistic quantum field theories lead to a divergent zero-point vacuum energy with an ω^3 frequency spectrum, which is the only one invariant under a Lorentz transformation, but general relativity suggests a cutoff of the zero-point energy at the Schwarzschild radius of this

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energy, with the Schwarzschild radius equal to the Planck length $r_P = (G\hbar/c^3)^{1/2} \simeq 10^{-33}$ cm (G is Newton's constant). Each region in space having this length would thereby form a black hole with a mass equal to the Planck mass $m_P = (\hbar c/G)^{1/2} \simeq 10^{-5}$ g, suggesting that space is densely filled with Planck-mass black holes (Wheeler, 1968; Hawking, 1978). Accordingly, the mass density of the vacuum should be of the order $\varrho_v \sim m_P/r_P^3 = c^5/\hbar G^2 \sim 10^{95}$ g/cm^3, large enough to put the mass of the entire universe in a cube with the side length of less than 1 fermi. The cosmological constant corresponding to this mass density is $\Lambda_v \simeq 1/r_P^2 \sim 10^{66}$ cm^{-2}, whereas observational astronomy suggests a value of the order $\Lambda < 10^{-54}$ cm^{-2}. Expressed in Planck length units, one has $\Lambda_v \sim 1$ and $\Lambda < 10^{-120}$, demonstrating the smallness of the cosmological constant. Because of its very small upper bound, it has been suggested that $\Lambda = 0$.

The large discrepancy between the actual value of Λ and the value predicted by quantum gravity demonstrates the existence of a highly perfect compensation mechanism. A mechanism of this kind is not unfamiliar to physics. It is realized in the high degree of electric charge neutrality in condensed matter physics. To separate the positive from the negative charges in a 1 cm^3 piece of condensed matter would require a force of $\sim 10^{15}$ tons. In analogy, we therefore propose that the observed mass neutrality of space is explained by assuming that space is densely filled with an equal number of positive and negative Planck masses. But in order to avoid the decay of the positive into negative Planck masses, the Planck masses must obey an exactly nonrelativistic law of motion. Only then does the Hamilton operator commute with the particle number operator, conserving the number of both the positive and negative Planck masses. We call this hypothesis the Planck aether hypothesis because a zero-point energy cutoff at some high energy results in a distinguished reference system with the assembly of positive and negative Planck masses at rest in this system. A mass neutral vacuum would make the cosmological constant equal to zero, and only in regions comparable to the Planck length would the mass neutrality be violated, in the same way as charge neutrality would be violated in regions comparable to the Debye length.

It is assumed that the Planck masses interact locally through contact-type forces as in Heisenberg's nonlinear spinor theory, but are otherwise not the source of any field or charge. Long-range fields are rather the result of the contact-type interactions, very much as the long-range phonon field in condensed matter physics is the result of short-range interactions between adjacent atoms. In the proposed model, the Planck masses therefore assume the role of a kind of nondestructible Leibnizian monads, with

Leibniz's dictum that the monads shall have no windows reflected in their being no source of long-range fields.

A field theory for the Planck masses has to be based on the two Planck relations

$$Gm_P^2 = \hbar c \tag{1.1}$$

$$m_P r_P c = \hbar \tag{1.2}$$

from which one derives the Planck length $r_P = (\hbar G/c^3)^{1/2} \simeq 1.6 \times 10^{-33}$ cm and the Planck mass $m_P = (\hbar c/G)^{1/2} \simeq 2.2 \times 10^{-5}$ g.

With the exception of the Planck masses, all elementary particles have to be viewed as quasiparticles as quantized modes of the Planck aether. With the wave propagation velocity of these modes equal to the velocity of light, these quasiparticles obey Lorentz invariance as a dynamic symmetry, as in the older pre-Einstein view held by Lorentz, Poincaré, and others, very nicely explained in an article by Shupe (1985) for a "water-wave world".

From this viewpoint, special relativity would be invalid near the Planck length, but because quantum effects become important there, classical gravity would have to be replaced by some kind of quantum gravity. No theory of quantum gravity yet exists, but Einstein's gravitational field equations suggest that it should lead to a system of "wormholes." More recent studies, however, have shown that such a world of wormholes is unstable (Redmount and Sven, 1993). This is in addition to the unphysical feature of such a world which can bring into close contact regions of space and time at a macroscopic scale separated from each other by arbitrarily large distances. None of these problems arises in the Planck aether model, but for the model to have any chance to reflect physical reality, it must lead to Einstein's gravitational field equations, not at the Planck scale, but in the asymptotic limit of energies small compared to the Planck energy. It will be shown that the model not only can do that, but in addition also can reproduce both Maxwell's and Dirac's equations, including a value of the typical spinor mass in terms of the Planck mass.

The Planck aether hypothesis replaces the Lorentz group with the Galilei group as the fundamental symmetry. The system of galaxies forms an almost crystal-like large-scale structure, not known at the time Einstein formulated his special theory of relativity, defining a system at rest with these galaxies suggesting a fundamental field, with the matter in the galaxies plausibly generated by this field. The idea that the Galilei group rather than the Lorentz group is the fundamental kinematic symmetry of nature therefore makes a lot of sense. In contrast to classical aether models, the Planck aether is not an additional substance, but rather the fundamental field from which all particles and their interactions would have to be derived, very much as it was originally envisioned by Einstein and later by Heisenberg.

2. FIELD EQUATION FOR THE PLANCK MASSES

As the fundamental law describing the conjectured dense assembly of the positive and negative Planck masses, a two-component operator field equation is chosen,

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_P} \nabla^2 \psi_\pm \pm 2\hbar c r_P^2 (\psi_\pm^\dagger \psi_\pm - \psi_\mp^\dagger \psi_\mp)\psi_\pm \tag{2.1}$$

with the operators $\psi_\pm$, $\psi_\pm^\dagger$ obeying the canonical commutation relations

$$\begin{aligned} &[\psi_\pm(\mathbf{r})\psi_\pm^\dagger(\mathbf{r}')] = \delta(\mathbf{r}-\mathbf{r}') \\ &[\psi_\pm(\mathbf{r})\psi_\pm(\mathbf{r}')] = [\psi_\pm^\dagger(\mathbf{r})\psi_\pm^\dagger(\mathbf{r}')] = 0 \end{aligned} \tag{2.2}$$

The coupling constant $2\hbar c r_P^2$ of the nonlinear term is explained as follows: The expectation value $\langle \psi_\pm^\dagger \psi_\pm \rangle$ of a vacuum densely filled with an equal number of positive and negative Planck masses, each occupying the volume r_P^3, is $1/2r_P^3$. It thus follows that $2\hbar c r_P^2 \langle \psi_\pm^\dagger \psi_\pm \rangle = \hbar c/r_P = m_P c^2$. The fundamental law (2.1) can be interpreted as a nonrelativistic nonlinear Heisenberg equation, similar to Heisenberg's nonlinear spinor field equation proposed by him as a model of elementary particles (Heisenberg, 1954, 1957). The problem of Heisenberg's relativistic theory, that it had to assume a Hilbert space with indefinite metric and hence negative probabilities, is avoided in a nonrelativistic theory described by (2.1), because it always leads to a Hilbert space with a positive-definite metric.

As a classical field equation, (2.1) can be derived from the Lagrange density

$$\begin{aligned} \mathscr{L}_\pm = {} & i\hbar \varphi_\pm^* \dot{\varphi}_\pm \mp \frac{\hbar^2}{2m_P} (\nabla \varphi_\pm^*) \cdot (\nabla \varphi_\pm) \\ & \mp 2\hbar c r_P^2 \left[\frac{1}{2} \varphi_\pm^* \varphi_\pm - \varphi_\mp^* \varphi_\mp \right] \varphi_\pm^* \varphi_\pm \end{aligned} \tag{2.3}$$

Variation with regard to φ^* leads to

$$i\hbar \frac{\partial \varphi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_P} \nabla^2 \varphi_\pm \pm 2\hbar c r_P^2 [\varphi_\pm^* \varphi_\pm - \varphi_\mp^* \varphi_\mp] \varphi_\pm \tag{2.4}$$

The Hamilton density belonging to (2.3) is

$$\mathscr{H}_\pm = \pm \frac{\hbar^2}{2m_P} (\nabla \varphi_\pm^*) \cdot (\nabla \varphi_\pm) \pm 2\hbar c r_P^2 \left[\frac{1}{2} \varphi_\pm^* \varphi_\pm - \varphi_\mp^* \varphi_\mp \right] \varphi_\pm^* \varphi_\pm \tag{2.5}$$

With $\mathscr{H}_\pm$ given by (2.5), one can derive the quantized Heisenberg equation of motion for the field operators $\psi_\pm \psi_\pm$:

$$i\hbar\dot{\psi}_\pm = [\psi_\pm, H] \tag{2.6}$$

and one finds that (2.6) agrees with (2.1). It therefore follows that the quantized equation (2.1) and classical equation (2.4) are of the same form, for (2.1) even true for a nonlinear classical field equation.

Because

$$i\hbar\dot{N}_\pm = [N_\pm, H] = 0 \tag{2.7}$$

where

$$N_\pm = \int \psi_\pm^\dagger \psi_\pm \, d\mathbf{r} \tag{2.8}$$

is the particle number operator, the number of each Planck mass species is conserved.

The fundamental law is invariant under global $U(1)$ transformations, which, according to Noether's theorem, leads to a conserved current. It is the mass current to be expressed by a continuity equation. Because (2.1) is exactly nonrelativistic, it is, with the exception of a phase, invariant under a Galilei transformation. It is also invariant under the transformations

$$\begin{aligned} \psi_\pm &\to -\psi_\mp \\ \psi_\pm^\dagger \psi_\pm &\to -\psi_\mp^\dagger \psi_\pm \\ m_\mathrm{P} &\to -m_\mathrm{P} \end{aligned} \tag{2.9}$$

belonging to the $SU2$ group. Because the $SU2$ group is isomorph with the $SO3$ rotation group of three-dimensional space, one may see it as the reason for the three-dimensionality of position space (von Weizsäcker, 1971, p. 271).

The fundamental law is not invariant under the Lorentz group and also not under any local gauge group. The invariance under these groups would, for this reason, have to be derived dynamically.

3. HARTREE–FOCK APPROXIMATION

To obtain solutions of the nonlinear quantized field equation (2.1), suitable approximation methods must be used. Perturbation theory contradicts the spirit of the theory, because before perturbation theory can be applied, a spectrum of elementary particles should be derived nonperturbatively. If the temperature of the Planck mass fluid is close to absolute zero, which means that $kT \ll m_\mathrm{P} c^2$, each component is superfluid and is described by a completely symmetric wave function. Under these circumstances, one can use the Hartree–Fock approximation.

In the less accurate Hartree approximation, one sets the expectation value of the product of three field operators equal to the product of their expectation values,

$$\begin{aligned} \langle\psi^\dagger_\pm\psi_\pm\psi_\pm\rangle &\simeq \varphi^*_\pm\varphi^2_\pm \\ \langle\psi^\dagger_\mp\psi_\mp\psi_\pm\rangle &\simeq \varphi^*_\mp\varphi_\mp\varphi_\pm \end{aligned} \tag{3.1}$$

where $\langle\psi_\pm\rangle = \varphi_\pm$, $\langle\psi^\dagger_\pm\rangle = \varphi^*_\pm$. Taking the expectation value of (2.1), one then recovers the classical field equation (2.4). In the more accurate Hartree–Fock approximation, where the exchange interactions are taken into account, one has to consider the symmetric wave function of two identical Planck masses

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\varphi_1(\mathbf{r})\varphi_2(\mathbf{r}') + \varphi_1(\mathbf{r}')\varphi_2(\mathbf{r})] \tag{3.2}$$

The expectation value for a delta-function-type contact interaction between the identical Planck mass particles is

$$\langle\psi(1,2)|\delta(\mathbf{r}-\mathbf{r}')|\psi(1,2)\rangle = 2\varphi^2_1(\mathbf{r})\varphi^2_2(\mathbf{r}) \tag{3.3}$$

with the direct and exchange integrals making an equal contribution. In the Hartree–Fock approximation, one therefore has to put instead of (3.1)

$$\begin{aligned} \langle\psi^\dagger_\pm\psi_\pm\psi_\pm\rangle &\simeq 2\varphi^*_\pm\varphi^2_\pm \\ \langle\psi^\dagger_\mp\psi_\mp\psi_\pm\rangle &\simeq \varphi^*_\mp\varphi_\mp\varphi_\pm \end{aligned} \tag{3.4}$$

In this approximation, one obtains from (2.1) the nonlinear Schrödinger equation

$$i\hbar\frac{\partial\varphi_\pm}{\partial t} = \mp\frac{\hbar^2}{2m_P}\nabla^2\varphi_\pm \pm 2\hbar c r^2_P[2\varphi^*_\pm\varphi_\pm - \varphi^*_\mp\varphi_\mp]\varphi_\pm \tag{3.5}$$

Putting

$$\begin{aligned} n_\pm &= \varphi^*_\pm\varphi_\pm \\ n_\pm v_\pm &= \mp\frac{i\hbar}{2m_P}[\varphi^*_\pm\nabla\varphi_\pm - \varphi_\pm\nabla\varphi^*_\pm] \end{aligned} \tag{3.6}$$

we can transform (3.5) into the so-called hydrodynamic form

$$\frac{\partial\mathbf{v}_\pm}{\partial t} + (\mathbf{v}_\pm\cdot\nabla)\mathbf{v}_\pm = -2c^2r^3_P\nabla(2n_\pm - n_\mp) + \frac{1}{m_P}\nabla Q_\pm \tag{3.7a}$$

$$\frac{\partial n_\pm}{\partial t} + \nabla(n_\pm\mathbf{v}_\pm) = 0 \tag{3.7b}$$

where we have used (1.2). In (3.7a), $Q_\pm$ is the so-called quantum potential

$$Q_\pm = \frac{\hbar^2}{2m_P} \frac{\nabla^2 (n_\pm)^{1/2}}{(n_\pm)^{1/2}} \tag{3.8}$$

In most cases where the distances are large compared to the Planck length, it can be neglected. The connection between (3.5) and (3.7a) (3.7b) is given by

$$\begin{aligned} \varphi_\pm = A_\pm e^{iS_\pm}, \qquad A_\pm > 0, \qquad 0 \le S_\pm \le 2\pi \\ n_\pm = A_\pm^2, \qquad \mathbf{v}_\pm = \pm \frac{\hbar}{m_P} \operatorname{grad} S_\pm \end{aligned} \tag{3.9}$$

showing that curl $\mathbf{v}_\pm = 0$. The uniqueness of $S_\pm$ requires that

$$\oint \mathbf{v}_\pm \cdot d\mathbf{r} = 0 \tag{3.10}$$

but the uniqueness of $\varphi_\pm$ only requires that

$$\oint \mathbf{v}_\pm \cdot d\mathbf{r} = \pm nh/m_P, \qquad n = 0, 1, 2, \ldots \tag{3.11}$$

implying that there can be multiply quantized vortices as solutions of the Hartree–Fock approximation. From the multitude of these solutions, those able to describe physical reality must be chosen. In principle, this could be done by a variational method. This is a problem similar to the corresponding one in condensed matter physics, where from atomic wave functions one has to find those which can lead to a periodic structure as it is realized by nature in the crystal lattice of a solid. In a quite analogous way, we have to guess the most likely wave function made up from the wave functions of the quantized vortices.

4. STEADY-STATE VORTEX SOLUTIONS

For steady-state solutions $\partial/\partial t = 0$ and by neglecting the quantum potential one obtains by adding and subtracting (3.7a) (a, b constants of integration)

$$\frac{v_+^2 + v_-^2}{2} = -2c^2 r_P^3 (n_+ + n_-) + a \tag{4.1a}$$

$$\frac{v_+^2 - v_-^2}{2} = -6c^2 r_P^3 (n_+ - n_-) + b \tag{4.1b}$$

For singly quantized line vortices, one has in cylindrical polar coordinates

$$\begin{aligned} |v_\pm| = v_\varphi &= c(r_P/r), \qquad r > r_P \\ &= 0, \qquad\qquad\;\; r < r_P \end{aligned} \tag{4.2}$$

At $r = r_P$ the centrifugal force is balanced by the force of the quantum potential, leading to a finite radius of the vortex core. The continuity equation (3.7b) requires that $\nabla(n_\pm) \perp v_\pm$. For a dense assembly of positive and negative Planck masses, one has at $r \to \infty$, $n_+ + n_- = 1/r_P^3$. Because for $r \to \infty$, $v_\pm \to 0$, it follows that $a = 2c^2$. Likewise, since for $r \to \infty$, $n_+ - n_- = 0$, it follows that $b = 0$.

To satisfy (4.1b) for singly quantized vortices, one must have $v_+^2 = v_-^2$; hence

$$v_- = \pm v_+ \tag{4.3}$$

and

$$n_+ - n_- = 0 \tag{4.4}$$

The solution $v_- = v_+$ consists of two corotating positive–negative mass vortices, and the solution $v_- = -v_+$ of two counterrotating positive–negative mass vortices. With the value $a = 2c^2$, furthermore, putting $n_+ = n_- = n_\pm$, one obtains (by inserting into (4.1a) $v_\pm^2 = v_\varphi^2$, with v_φ^2 given by (4.2)) for the particle number density distribution inside the vortex,

$$n_\pm = (1/2r_P^3)[1 - (1/2)(r_P/r)^2] \tag{4.5}$$

If space is densely filled with vortices, forming a vortex field of what is sometimes called a vortex sponge, the vortices snap and reconnect by mutual collisions, with the likely ultimate outcome a lattice of vortex rings. In classical hydrodynamics, the lattice of line vortices realized in the Karman vortex street has been analyzed by Schlayer (1928), who found that a stable configuration exists for a distance of separation between the vortices ~300 times larger than the radius of their core. For a three-dimensional lattice of vortex rings, a stable arrangement with a larger ratio of the lattice constant to the vortex core radius is likely because there the vortex rings influence each other from all three directions of space. A stability analysis of such a configuration seems to be very difficult, but it might be possible that the relevant nondimensional number, the ratio of the lattice constant to vortex core radius, can be obtained from experiments in superfluid helium, assuming that this number is universal. It also is plausible that the radius R of the vortex rings should be of the same order of magnitude as the distance of separation between adjacent vortex rings.

A ring vortex of radius R has a resonance frequency under elliptic deformation of the ring given by

$$\omega_v \simeq c r_P / R^2 \tag{4.6}$$

leading to a resonance energy for the positive and negative mass vortex:

$$\hbar\omega_v \simeq \pm m_P c^2 (r_P/R)^2 \tag{4.7}$$

As will be shown below, this resonance energy can explain Dirac spinors as excitonic quasiparticles, and their mass in terms of the Planck mass.

5. LONGITUDINAL WAVES

For small-amplitude disturbances one obtains by adding and subtracting (3.7a) and by neglecting Q

$$\frac{\partial}{\partial t}(v_+ + v_-) = -2c^2 r_P^3 \nabla(n'_+ + n'_-) \tag{5.1a}$$

$$\frac{\partial}{\partial t}(v_+ - v_-) = -6c^2 r_P^3 \nabla(n'_+ - n'_-) \tag{5.1b}$$

and for (3.7b)

$$\frac{\partial n'_\pm}{\partial t} + n_\pm \nabla v_\pm = 0 \tag{5.2}$$

where $n'_\pm$ are small disturbances imposed on $n_\pm$. Eliminating $n'_\pm$ from (5.1a), (5.1b) and (5.2), one obtains two wave equations

$$\frac{\partial^2}{\partial t^2}(v_+ + v_-) = c^2 \nabla^2 (v_+ + v_-) \tag{5.3}$$

$$\frac{\partial^2}{\partial t^2}(v_+ - v_-) = 3c^2 \nabla (v_+ - v_-) \tag{5.4}$$

The first of these two describes waves that propagate with c, the second one with $\sqrt{3}c$. The first wave has the characteristic property of a compression wave and couples the vortex rings. The meaning of the other, "fast" wave is less obvious.

For short wavelengths approaching the Planck length, the wave equations are modified by the quantum potential. In the limit in which the quantum potential dominates, the equation of motion obtained from (3.7a) is

$$\frac{\partial v_\pm}{\partial t} = \frac{1}{m_P} \nabla Q_\pm \tag{5.5}$$

With the help of (5.2), it becomes

$$\frac{\partial^2 v_\pm}{\partial t^2} = -\frac{\hbar^2}{4m_\mathrm{P}^2}\nabla^4 v_\pm \tag{5.6}$$

leading to the nonrelativistic dispersion relation for free Planck masses

$$\omega = \pm\hbar k^2/2m_\mathrm{P} \tag{5.7}$$

6. TRANSVERSE WAVES

The vortex lattice, resp. vortex field, established below the Planck scale can propagate two kinds of waves, one having the property of an electromagnetic and the other the property of a gravitational wave. If described by a lattice of vortex rings, there are two distinct disturbances of the rings possible, each leading to a wave, one representing an elliptic deformation, the second one a tilting rotation. The wave associated with a tilting rotation was analyzed by Thomson (1887), who showed that it leads to a transverse wave which for small amplitudes simulates the waves derived from Maxwell's equations. The other deformation, as we will show, leads to Einstein's gravitational waves.

In deriving these waves, we present a somewhat simplified derivation. We assume that the vortex field can be described by Euler's equations for a frictionless fluid. Then let $\mathbf{v} = \{v_x, v_y, v_z\}$ be the undisturbed velocity of the vortex field and $\mathbf{u} = \{u_x, u_y, u_z\}$ a small superimposed velocity disturbance, and let us take only those solutions for which $\operatorname{div}\mathbf{v} = \operatorname{div}\mathbf{u} = 0$. Going to the continuum limit, the vortex lattice goes from R to r_P, where r_P can be chosen arbitrarily small. The x component of the equation of motion for a disturbance $\mathbf{u}$ is

$$\frac{\partial u_x}{\partial t} = -(v_x + u_x)\frac{\partial(v_x + u_x)}{\partial x} - (v_y + u_y)\frac{\partial(v_x + u_x)}{\partial y} - (v_z + u_z)\frac{\partial(v_x + u_x)}{\partial z} - \frac{1}{\varrho}\frac{\partial p}{\partial x} \tag{6.1}$$

From the continuity equation $\operatorname{div}\mathbf{v} = 0$, one has

$$v_x\frac{\partial v_x}{\partial x} + v_x\frac{\partial v_y}{\partial y} + v_x\frac{\partial v_z}{\partial z} = 0 \tag{6.2}$$

Subtracting (6.2) from (6.1) and taking the y–z average, one finds

$$\frac{\partial u_x}{\partial t} = -\frac{\partial(\overline{v_y v_x})}{\partial y} - \frac{\partial(\overline{v_z v_x})}{\partial z} \tag{6.2a}$$

and similarly, by taking the $x-z$ and $x-y$ averages,

$$\frac{\partial u_y}{\partial t} = -\frac{\partial(\overline{v_x v_y})}{\partial x} - \frac{\partial(\overline{v_z v_y})}{\partial z} \tag{6.2b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial(\overline{v_x v_z})}{\partial x} - \frac{\partial(\overline{v_y v_z})}{\partial y} \tag{6.2c}$$

With the condition $\operatorname{div} \mathbf{u} = 0$, one obtains from (6.2a)–(6.2c) that

$$\overline{v_i v_k} = -\overline{v_k v_i} \tag{6.3}$$

Taking the x component of the equation of motion, multiplying it by v_y and then taking the $y-z$ average; taking the y component multiplied by v_x and then taking the $x-z$ average; and finally subtracting the first from the second equation, one finds

$$\frac{\partial}{\partial t}(\overline{v_x v_y}) = -v^2\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \tag{6.4}$$

where $v^2 = \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$ is the average microvelocity of the vortex field. Putting $\phi_z = -\overline{v_x v_y}/2v^2$, we find that (6.4) is just the z component of

$$\frac{\partial \boldsymbol{\phi}}{\partial t} = \frac{1}{2} \operatorname{curl} \mathbf{u} \tag{6.5}$$

where $\phi_x = -\overline{v_y v_z}/2v^2$, $\phi_y = -\overline{v_z v_x}/2v^2$. Equations (6.2a)–(6.2c) then take the form

$$\frac{\partial \mathbf{u}}{\partial t} = -2v^2 \operatorname{curl} \boldsymbol{\phi} \tag{6.6}$$

Elimination of $\boldsymbol{\phi}$ from (6.5) and (6.6) results in a wave equation for $\mathbf{u}$,

$$-(1/v^2)\, \partial^2\mathbf{u}/\partial t^2 + \nabla^2\mathbf{u} = 0 \tag{6.7}$$

In the continuum limit, making the transition $R \to r_P$, one has for the microvelocity $v^2 = c^2$. In this limit (6.7) describes a transverse wave propagating with the velocity of light c. In reality, though, $R \sim 10^3 r_P$, which means that the equation describing this wave would break down at an energy corresponding to the scale R, that is, at an energy of the order 10^{16} GeV, the energy of the grand unification scale.

With $v = c$ and putting $\mathbf{u} = \mathbf{E}$ and $\boldsymbol{\phi} = -(1/2c)\mathbf{H}$, we find that (6.5) and (6.6) have the same form as the two Maxwell vacuum field equations

$$-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = \operatorname{curl} \mathbf{E} \tag{6.8}$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \operatorname{curl} \mathbf{H} \tag{6.9}$$

To derive the second transverse wave mode, we add (6.2) and (6.1) and take the average over x, y, and z:

$$\frac{\partial u_x}{\partial t} = -\frac{\partial \overline{v_x^2}}{\partial x} - \frac{\partial \overline{v_x v_y}}{\partial y} - \frac{\partial \overline{v_x v_z}}{\partial z} \tag{6.10a}$$

and similarly

$$\frac{\partial u_y}{\partial t} = -\frac{\partial \overline{v_y^2}}{\partial y} - \frac{\partial \overline{v_y v_z}}{\partial z} - \frac{\partial \overline{v_y v_x}}{\partial x} \tag{6.10b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial \overline{v_z^2}}{\partial z} - \frac{\partial \overline{v_z v_x}}{\partial x} - \frac{\partial \overline{v_z v_y}}{\partial y} \tag{6.10c}$$

Combining (6.10a)–(6.10c) with the condition $\operatorname{div} \mathbf{u} = 0$ leads to

$$\frac{\partial^2}{\partial x_i \partial x_k} (\overline{v_i v_k}) = 0 \tag{6.11}$$

and for (6.10a)–(6.10c) one can write

$$\frac{\partial u_k}{\partial t} = -\frac{\partial}{\partial x_i} (\overline{v_i v_k}) \tag{6.12}$$

Multiplying the v_i component of the equation of motion with v_k and vice versa, its v_k component with v_i, taking both, and adding the average, one finds

$$\frac{\partial}{\partial t} (\overline{v_i v_k}) = -v^2 \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \tag{6.13}$$

From (6.12) one has

$$\frac{\partial^2 u_k}{\partial t^2} = -\frac{\partial}{\partial t \partial x_i} (\overline{v_i v_k}) \tag{6.14}$$

and from (6.13)

$$\frac{\partial}{\partial x_i \partial t} (\overline{v_i v_k}) = -v^2 \left(\frac{\partial}{\partial x_k} \frac{\partial u_i}{\partial x_i} + \frac{\partial^2 u_k}{\partial x_i^2} \right) = -v^2 \frac{\partial^2 u_k}{\partial x_i^2} \tag{6.15}$$

the latter because of $\operatorname{div} \mathbf{u} = 0$. Eliminating $\overline{v_i v_k}$ from (6.14) and (6.15) and putting as before $v^2 = c^2$ finally results in

$$\frac{\partial^2 u_k}{\partial t^2} = c^2 \frac{\partial^2 u_k}{\partial x_i^2} \tag{6.16}$$

or

$$\nabla^2 \mathbf{u} - \frac{1}{c^2} \frac{\partial^2 \mathbf{u}}{\partial t^2} = 0 \tag{6.17}$$

To show that (6.17) can describe a gravitational wave propagating in the x direction, one has to compare it with the line element of a linearized gravitational wave (Landau and Lifshitz, 1975)

$$ds^2 = ds_0^2 + h_{22}\,dx_2^2 + 2h_{23}\,dx_2\,dx_3 + h_{33}\,dx_3^2 \tag{6.18}$$

where

$$h_{22} = -h_{33} = f(t - x/c), \qquad h_{23} = g(t - x/c) \tag{6.19}$$

with f and g two arbitrary functions, and ds_0^2 the line element in the absence of a gravitational wave. We compare this result with the deformation of an elastic body described by a distorted line element as follows (Landau and Lifshitz, 1970, p. 2):

$$ds^2 = ds_0^2 + 2\varepsilon_{ik}\,dx_i\,ds_k \tag{6.20}$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial \varepsilon_i}{\partial x_k} + \frac{\partial \varepsilon_k}{\partial x_i}\right) \tag{6.21}$$

In (6.20) and (6.21) $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_y, \varepsilon_z)$ is the displacement vector, which is related to the velocity disturbance vector $\mathbf{u}$ by

$$\mathbf{u} = \frac{\partial \boldsymbol{\varepsilon}}{\partial t} \tag{6.22}$$

In an elastic medium, transverse waves obey the wave equation

$$\nabla^2 \boldsymbol{\varepsilon} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{\varepsilon}}{\partial t^2} = 0 \tag{6.23}$$

Because of (6.22), this is the same as (6.17). From the condition $\operatorname{div} \mathbf{u} = 0$ and (6.22) it also follows that $\operatorname{div} \boldsymbol{\varepsilon} = 0$.

For a transverse wave propagating into the x-direction, $\varepsilon_x = \varepsilon_1 = 0$. The condition $\operatorname{div} \boldsymbol{\varepsilon} = 0$ then leads to

$$\frac{\partial \varepsilon_2}{\partial x_2} + \frac{\partial \varepsilon_3}{\partial x_3} = \varepsilon_{22} + \varepsilon_{33} = 0 \tag{6.24}$$

hence

$$\varepsilon_{33} = -\varepsilon_{22} \tag{6.25}$$

The identity with a gravitational wave follows by putting

$$2\varepsilon_{ik} = h_{ik} \tag{6.26}$$

and by assuming that a vortex sponge behaves under symmetric deformations like an elastic body.

The reason a vortex sponge (resp. lattice of vortex rings) can propagate two types of transverse waves is that the tensor $\overline{v_i v_k}$ describing the microturbulence of the vortex field has a symmetric, $\overline{v_i v_k} = \overline{v_k v_i}$, and an antisymmetric, $\overline{v_i v_k} = -\overline{v_k v_i}$, part. A vortex field of this kind is only possible in a frictionless fluid where small-scale vortices can occur at high wave numbers and, for this reason, plays no role in classical turbulence theory describing a fluid with friction. It is through the quantum mechanical phenomenon of superfluidity, not known at the time William Thomson put forward the hypothesis of a frictionless, fluidlike aether, that a microturbulence with an antisymmetric part is possible at all. Because a vortex sponge can have both a symmetric and antisymmetric part, it has two transverse wave modes. It is for this reason that it can in a unique way attain Einstein's goal of unifying the gravitational with the electromagnetic field, and we see that for this unification to be possible, quantum theory plays an important role.

7. COUPLING TO MATTER

Both Maxwell's and Einstein's equations describe fields coupled to matter, Maxwell's equations through the electric charge (resp. four-current) and Einstein's equations through the mass (resp. energy-momentum tensor).

In the Planck aether model, the phenomenon of charge can be understood to result from the quantum potential. The quantum potential leads to a finite radius of the vortex core, equal for the positive and negative mass component of the vortex, but it can do more than that. In the combination $(1/m_P)\nabla Q_\pm$ it depends on m_P^2, independent of the sign of m_P, and for this reason can break the symmetry between the positive and negative Planck masses. The quantum potential describes the zero-point quantum fluctuations of the Planck masses bound in the vortex, in particular near the core of the vortex, where these fluctuations give rise to an energy density ε, which by order of magnitude is

$$|\varepsilon| \sim \hbar c/r_P^4 \tag{7.1}$$

By order of magnitude, it is also equal to the energy density g^2 that a Newtonian gravitational field g of a Planck mass m_P would have at the distance r_P:

$$g \sim \sqrt{G} m_P/r_P^2 \tag{7.2}$$

(because $Gm_P^2 = \hbar c$). The interpretation of this result is as follows: The zero-point energy fluctuations of the Planck masses bound in the vortex filaments are the source of virtual phonons setting up a Newtonian-type gravitational force field, with the coupling constant $Gm_P^2 = \hbar c$. The gravitational charge is, for this reason, reduced to the zero-point fluctuations of the Planck masses bound in the vortices. The gravitational field produced by the zero-point fluctuations depends on the sign of m_P, as does the kinetic energy $m_P v^2/2$. It therefore explains the principle of equivalence, which says that a positive mass attracts both positive and negative masses, and likewise a negative mass repels both positive and negative masses. Because the field energy resulting from the gravitational interaction of a positive with a negative mass is positive, this positive energy must show up as an excess of the positive over the negative kinetic fluid energy. This is a higher-order effect, resulting from the zero-point collective excitations of the positive and negative mass components. For a densely packed assembly of positive and negative Planck masses, the energy density of one species of Planck masses would by itself be very huge ($\sim 10^{95}$ g/cm^3), explaining why a very small imbalance in the positive over the negative kinetic fluid energy can give the double vortex a net, but in comparison to the Planck mass, small positive mass.

In the absence of an interaction between its positive and negative masses, the mass of each positive-negative mass double vortex vanishes. Space can therefore without the expenditure of energy be filled with a large number of such vortices. If the gravitational interaction is switched on, the double vortices assume a positive mass, but the mutual gravitational interaction energy between all the double vortices is negative. Because the total energy must remain zero, this simply means that the positive energy of the double vortices is compensated by the negative gravitational interaction energy with other double vortices. As a consequence, the cosmological constant remains zero.

According to (6.23)–(6.26), a small-amplitude gravitational wave is derived from the equation

$$\Box h_{ik} = 0 \tag{7.3}$$

In the presence of matter, it can be brought into the form ($\kappa = 8\pi G/c^4$) (Landau and Lifshitz, 1975)

$$\Box h_{ik} = \kappa \Theta_{ik} \tag{7.4}$$

where Θ_{ik} is the energy-momentum tensor.

It was shown by Gupta (1954) that by splitting Θ_{ik} into its matter part T_{ik} and gravitational field part t_{ik}

$$\Theta_{ik} = T_{ik} + t_{ik} \tag{7.5}$$

one can bring (7.4) into Einstein's form,

$$R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik} \tag{7.6}$$

The physical meaning of the splitting into a matter and a gravitational field part can be understood as follows: As explained below, matter composed of Dirac spinors comprises quasiparticles made up from the positive and negative mass components of the Planck aether. The masses of these quasiparticles are representative for the matter part T_{ik}. But because the vortex field also has a kinetic fluid energy, the magnitude of which depends on the interaction between the quasiparticles, this kinetic energy makes an additional contribution to Θ_{ik}. In the Planck aether model, it is representative for the gravitational field part t_{ik} of the energy-momentum tensor.

Because the kinetic energy of the Planck masses is a continuous function, and because the kinetic energy of the positive Planck masses can be compensated by any amount by the negative kinetic energy of the negative Planck masses, the gravitational charge is a continuous parameter, depending on the degree of cancellation between the positive and negative kinetic energies. It is for this reason that the gravitational charge $Gm^2/\hbar c$ of a Dirac spinor quasiparticle can be much smaller than the gravitational charge of a Planck mass, which is $Gm_P^2/\hbar c = 1$.

The situation is quite different for the electromagnetic coupling constant. Vector field equations like Maxwell's equations, can only have a vector four-current as their source, satisfying a continuity equation

$$\frac{\partial \varrho_e}{\partial t} + \operatorname{div} \mathbf{j}_e = 0 \tag{7.7}$$

where ϱ_e and $\mathbf{j}_e$ may be called the electric charge and current density, respectively. A quantity which in the Planck aether model satisfies a continuity equation of the form (7.7) and at the same time can act as a source for a charge are Planck masses bound in the vortex filaments. According to (2.7), their number is conserved and according to (7.1), their zero-point energy gives them a charge of the order $Gm_P^2/\hbar c = 1$. By comparison, the electric charge has the coupling strength $e^2/\hbar c \simeq 1/137$ instead, but this coupling strength becomes larger at higher energies, being at lower energies reduced by vacuum polarization. A value $e^2/\hbar c \sim Gm_P^2/\hbar c = 1$ at high energies is predicted by grand unified theories. It was noticed by Nussinov (1988) that all other coupling constants are within a few orders of the same magnitude, giving substantial support to the hypothesis that the phenomenon of charge has its origin in the zero-point fluctuations of Planck masses bound in the vortex filaments.

With charges as the source of the elctromagnetic field, one would have

$$\operatorname{div} \mathbf{E} = 4\pi\varrho_e \tag{7.8}$$

and in order to satisfy (7.7), a term must then be added to the Maxwell equation. It thereby becomes

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\,\mathbf{j}_e = \text{curl}\,\mathbf{H} \tag{7.9}$$

The other Maxwell equation (6.8) is purely kinematic, as its hydrodynamic form (6.5) demonstrates, and therefore is unchanged. Finally, because of (6.5), div $\boldsymbol{\phi} = 0$ and it follows that

$$\text{div}\,\mathbf{H} = 0 \tag{7.10}$$

We now can say that not only is Einstein's goal to unify the gravitational with the electromagnetic field reached in a very satisfactory way through the hydrodynamics of the vortex lattice, but that this unification reduces the problem of quantum gravity to the quantum mechanics of the superfluid Planck aether. In this regard, it is important to remark that in Gupta's interpretation of Einstein's field equation, space is Euclidean with Einstein's field equations to be supplemented by a gauge condition, fixing the coordinate system. In any case, the arbitrariness of the coordinate systems, which is at the source of the difficulties in quantum gravity, cannot arise in the Planck aether model, which always has a distinguished reference system at rest with the superfluid Planck aether.

8. DIRAC SPINOR QUASIPARTICLES

Because a positive and a negative mass vortex occupy the same position in space, they form a mass dipole, with the components of the dipole interacting gravitationally. As will be shown, at the resonance energy given by (4.7), the mass dipole can form an excitonic quasiparticle having the property of a Dirac spinor.

The interaction is different for corotating as compared to counterrotating vortices. For corotating vortices, the interaction is determined by Newton's law. In either case, the interaction energy is positive. Both configurations can be viewed as a mass dipole with a superimposed mass monopole.

We first analyze the case of two corotating vortices in the framework of a simple nonrelativistic two-body model. According to (4.7), the mass of the positive and negative vortex resonance is

$$m_v^{\pm} = \pm m_{\text{P}}(r_{\text{P}}/R)^2 \tag{8.1}$$

Through the zero-point fluctuations, the corresponding positive and negative mass components in the double vortex interact gravitationally. This interaction energy generates a small positive mass $m \ll m_v^+$. We add this small positive mass to m_v^+ by putting $m^+ = m_v^+ + m$, with m_v^-

unchanged, hence putting $m^- = m_v^-$. Because the gravitational interaction energy between m_v^+ and m_v^- is small, one has $m^+ - |m^-| \ll m^+$, with $m^+ > |m^-|$. As the resonance energy can propagate through the vortex lattice as an excitonic quasiparticle, a bound state of a positive and negative mass forms likewise an excitonic two-body system. Approximating it by two pointlike masses m^+, m^-, with m^+ larger than $|m^-|$, one has what has been called a pole–dipole particle, extensively studied by Hönl and Papapetrou (1939*a*,*b*) as a model for a Dirac spinor. Its center of mass is not located between m^+ and m^-, but rather outside. And because its translation generates angular momentum, the pole–dipole particle executes a circular motion around its center of mass (Fig. 1). It is this motion which simulates the "Zitterbewegung" derived by Schrödinger (1930, 1931) as the trajectory of a particle described by Dirac's wave equation.

Let m^+ be separated by the distance r from m^-, and let the distance of m^+ from the center of mass be r_c. If $m \ll m^+ \simeq |m^-|$, then $r \ll r_c$. Conservation of the center of mass requires that

$$m^+ r_c = |m^-|(r_c + r) \tag{8.2}$$

For the angular momentum of the pole–dipole particle, one obtains

$$J = [m^+ r_c^2 - |m^-|(r_c + r)^2]\omega \tag{8.3}$$

where ω is the angular velocity around S. With $m = m^+ - |m^-|$ and $p = m^+ r \simeq |m^-| r \simeq m r_c$, where m is the mass pole and p the mass dipole, one finds from (8.2) and (8.3) that

$$J = -m^+ r r_c \omega = -pv \tag{8.4}$$

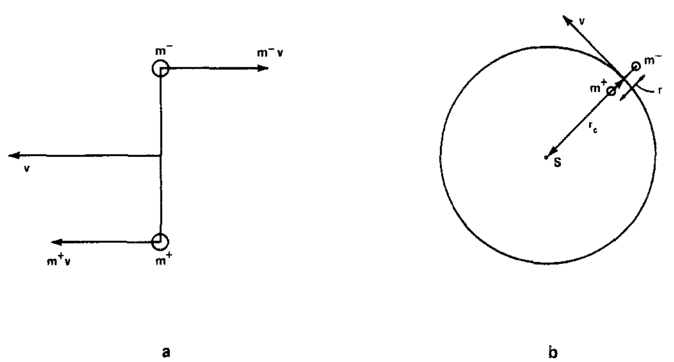


Fig. 1. (a) A translation of a mass dipole generates angular momentum; (b) a pole–dipole executes a circular motion around its center of mass S.

In the limit $v \to c$, one has

$$J = -mcr_c \tag{8.5}$$

Putting $J = -\hbar/2$, it would follow that

$$r_c = \frac{\hbar}{2mc} \tag{8.6}$$

which is the radius of the "Zitterbewegung" derived by Schrödinger from Dirac's equation. The angular momentum is negative because m^- is separated by a larger distance from S than m^+. The factor 1/2 is more difficult to explain, but otherwise this simple model suggests a close relationship between the pole–dipole configuration and a particle described by Dirac's equation.

Applying the solution of the well-known nonrelativistic quantum mechanical two-body problem with Coulomb interaction to the pole–dipole particle with Newtonian interaction, we can obtain an expression for m. For the Coulomb interaction, the ground-state energy is

$$W_0 = -\frac{1}{2}\frac{m^* e^4}{\hbar^2} \tag{8.7}$$

where m^* is the reduced mass of the two-body system, with the potential energy $-e^2/r$ for two charges $\pm e$ of opposite sign. The gravitational potential energy of two masses of opposite sign is $+Gm^+|m^-|/r \simeq + G|m_v^\pm|^2/r$ instead, and one thus has to make the substitution $e^2 \to -G|m_v^\pm|^2$. The reduced mass is

$$\frac{1}{m^*} = \frac{1}{m^+} + \frac{1}{m^-} = \frac{1}{m^+} - \frac{1}{|m^-|} \simeq -\frac{m}{|m_v^\pm|^2} \tag{8.8}$$

Finally, putting $W_0 = mc^2$, one finds from (8.7) that

$$m = (1/\sqrt{2})|m_v^\pm|^3/m_P^2 \tag{8.9}$$

and with (8.1) that

$$m/m_P = (1/\sqrt{2})(r_P/R)^6 \tag{8.10}$$

This expression explains why m is so many orders of magnitude smaller than m_P. For $r_P/R \sim 10^{-3}$, one would have $m/m_P \sim 10^{-18}$.

The Bohr radius for the hydrogen atom is

$$r_B = \hbar^2/m^* e^2 \tag{8.11}$$

Making the substitution for e^2 and m^*, one finds for the corresponding radius of the pole–dipole particle

$$r_v = \hbar/\sqrt{2}|m_v^\pm|c \tag{8.12}$$

For $R/r_P \sim 10^3$, it follows from (8.1) that $r_v \sim 10^{-27}$ cm.

To derive the Dirac equation from the pole–dipole particle configuration alone, requires half-integer angular momentum quantization. If Lorentz invariance is understood as a dynamic symmetry, only those configurations which are Lorentz invariant are in a stable equilibrium. A rigid rotator always leads to an integer angular momentum quantization, but this is, in general, not true for a nonrigid rotator. It was shown by Delcrétaz *et al.* (1986) that half-integer angular momentum quantization can occur in rotating molecules provided the rotation is accompanied by a time-dependent periodic deformation. For the angular momentum quantization to be $(1/2)\hbar$ simply requires that the period for the deformation must be twice as long as the period of rotation. In the pole–dipole particle configuration, the rules of quantum mechanics permit radial s-wave oscillations of m^+ against m^-. For the nonrelativistic pole–dipole particle configuration, this leads just to the correct angular momentum quantization, as can be seen as follows: From Bohr's angular momentum quantization rule $m^* r_v v = \hbar$, where $v = r_v \omega$, one obtains by inserting the values for m^* and r_v that $mc^2 = -(1/2)\hbar\omega$, and because of $\omega = c/r_c$ that $mr_c c = -(1/2)\hbar$. Therefore, even in the nonrelativistic limit, the correct angular momentum quantization rule is obtained, the only one consistent with Dirac's relativistic wave equation. For the mutual oscillating velocity, one finds $v/c = (|m_v^\pm|/m_P)^2 = (r_P/R)^4 \ll 1$, showing that our nonrelativistic approximation appears quite well justified.

For a relativistic treatment of the pole–dipole particle configuration, one has to distinguish the reference system at rest with the two rapidly moving m^+ and m^- particles from the system at rest with their common center of gravity. In the system at rest with the two particles, their gravitational interaction energy is

$$m_0 c^2 = \frac{G|m_v^\pm|^2}{r} \tag{8.13}$$

Putting $m^+ = m_v^+ + m_0$, $m^- = m_v^-$, momentum conservation requires that

$$m^+ \gamma_+ r_c = |m^-|\gamma_-(r_c + r) \tag{8.14}$$

where $\gamma_+ = (1 - v_+^2/c^2)^{-1/2}$, $\gamma_- = (1 - v_-^2/c^2)^{-1/2}$, $v_+ = r_c\omega$, $v_- = (r_c + r)\omega$. For $r \ll r_c$, one can expand (putting $\gamma_+ \equiv \gamma$)

$$\gamma_- = \gamma\left(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + \cdots\right) \tag{8.15}$$

The dipole moment of the pole–dipole particle is

$$p = m^+ r \simeq |m^-| r = \frac{m^+\gamma - |m^-|\gamma_-}{\gamma_-} r_c \tag{8.16}$$

or because of (8.15)

$$p \simeq m_0 r_c / \gamma^2 \tag{8.17}$$

and the energy is

$$E/c^2 = m = m^+\gamma - |m^-|\gamma_- \simeq p\gamma / r_c \tag{8.18}$$

Finally, the angular momentum (putting $r_c \omega \simeq c$) is

$$J = [M^+ \gamma r_c^2 - |m^-|\gamma_-(r_c + r)^2]\omega \simeq -p\gamma c \simeq -mcr_c \tag{8.19}$$

In (8.18) and (8.19), m is the mass of the pole–dipole particle in the center-of-mass rest frame.

From (8.17) and (8.18) it follows that

$$m = m_0/\gamma \tag{8.20}$$

and hence from (8.13)

$$m = \frac{G|m_y^\pm|^2}{c^2 \gamma r} \tag{8.21}$$

From (8.18) and $r_c = \hbar/2mc$, with $p \simeq |m^\pm| r$, one has

$$2\gamma |m_v^\pm| rc = \hbar \tag{8.22}$$

From (8.21) and (8.22), one can eliminate r, which means that one can go to the limit $r \to 0$ of a mass dipole. One obtains

$$m = 2G|m_v^\pm|^3/\hbar c = 2|m_v^\pm|^3/m_{\mathrm{P}}^2 \tag{8.23}$$

Finally, eliminating $|m_v^\pm|$ with the help of (8.1), one finds

$$m/m_{\mathrm{P}} = 2(r_{\mathrm{P}}/R)^6 \tag{8.24}$$

Since $m_{\mathrm{P}} r_{\mathrm{P}} c = \hbar$ and $m_G R c = \hbar$, where m_G is the mass at the GUT scale ($m_G c^2 \simeq 10^{16}$ GeV), one can write instead of (8.24)

$$m/m_{\mathrm{P}} = (2m_G/m_{\mathrm{P}})^6 \tag{8.25}$$

With $m_{\mathrm{P}} = (\hbar c/G)^{1/2}$ one can then write down the remarkable equation

$$G = (2/m)^2 \hbar c (m_G/m_{\mathrm{P}})^{12} \tag{8.26}$$

If the value of m in (8.26) is set equal to the electron mass, one finds that

$$\frac{R}{r_{\mathrm{P}}} = \frac{m_{\mathrm{P}}}{m_G} = \left(\frac{2}{m}\right)^{1/6} \left(\frac{\hbar c}{G}\right)^{1/12} \simeq 6000 \tag{8.27}$$

The mass obtained for the case of two counterrotating vortices is obtained in a similar way, except that the gravitational interaction is reduced by the factor $\gamma^{-2} = (1 - v^2/c^2)$, where v is the rotational velocity (Pfister and Schedel, 1987). The value of γ can be estimated by the uncertainty principle. If applied to the vortex resonance energy, it is

$$\gamma m_v Rc \sim \hbar \tag{8.28}$$

With $m_v c^2 = m_P c^2 (r_P/R)^2$, one thus has

$$\gamma \sim R/r_P \tag{8.29}$$

For the counterrotating vortices, one thus has instead of (8.24)

$$m/m_P = (2r_P/R)^8 \tag{8.30}$$

If one assigns the mass of excitons coming from the corotating vortices to the mass of the electron and the mass of the counterrotating vortices to the mass of the neutrino, one has for the neutrino–electron mass ratio

$$m_\nu/m_e = (m_G/m_P)^2 \simeq 3 \times 10^{-8} \tag{8.31}$$

implying a neutrino mass equal to $m_\nu \sim 10^{-2}$ eV.

The relativistic treatment assumed that the pole–dipole particle can be treated like a rigid rotator, thereby leading to an integer quantization of its angular momentum. The nonrelativistic treatment with internal motion of the pole–dipole particle correctly predicted the half-integer spin angular momentum quantization, but it led to a mass formula differing from the mass formula (8.24) by the factor $2^{-3/2}$.

The expression (8.10) for m was computed in a comoving system of the pole–dipole particle. In a system at rest with its center of mass, the pole–dipole particle executes a rapidly circulating motion with the radius r_c around its center of mass. This motion, however, does not alter the value of m, as can be seen as follows: According to (8.20), in the center-of-mass system the mass pole has to be multiplied by the factor $1/\gamma$, and the reduced mass m^* in the Bohr–Sommerfeld quantization rule has to be multiplied by γ. But because $G|m_v^{\pm}|^2$ plays the role of the gravitational coupling constant, it should be the same in the comoving and rest frames. As a result, the factor $1/\gamma$ and γ compensate each other, leaving (8.10) unchanged.

Therefore, if the internal motion of the pole–dipole particle is taken into account, (8.25) is changed into

$$m/m_P = (1/\sqrt{2})(m_G/m_P)^6 \tag{8.32}$$

(8.26) into

$$G = (\hbar c/2m^2)(m_G/m_P)^{12} \tag{8.33}$$

and, finally, (8.27) into

$$\frac{R}{r_P} = \frac{m_P}{m_G} = \frac{1}{2^{1/12} m^{1/6}} \left(\frac{\hbar c}{G} \right)^{1/12} \simeq 5000 \tag{8.34}$$

9. WAVE MECHANICAL TREATMENT OF SPINOR QUASIPARTICLES

A generalized Lagrange formalism ideally suited to treat the pole–dipole configuration has been developed by Bopp (1946, 1949) in his field mechanics.[2] In the presence of negative masses, the Lagrange function must be of the form $L = L(q_k, \dot{q}_k, \ddot{q}_k)$, because even without external forces present, a mass dipole is self-accelerating.

The Euler–Lagrange equations of the variational principle

$$\delta \int L(q_k, \dot{q}_k, \ddot{q}_k)\, dt = 0 \tag{9.1}$$

lead to a set of two canonical equations, one for the macrovariables describing the system as a whole, and one for microvariables describing the Zitterbewegung-type degrees of freedom.

For the relativistic four-vector of the velocity

$$u_\alpha = dx_\alpha / ds \equiv \dot{x}_\alpha, \qquad ds = (1 - \beta^2)^{1/2}\, dt \tag{9.2}$$

where $\beta = v/c$, $x_\alpha = (x_2, x_2, x_3, ict)$, one has

$$F = u_\alpha^2 = -c^2 \tag{9.3}$$

With units where $c = 1$, one can take instead of (9.1) the variational principle

$$\delta \int \Lambda(x_\alpha, u_\alpha, \dot{u}_\alpha)\, ds = 0 \tag{9.4}$$

With (9.3) as a subsidiary condition, the Euler–Lagrange equations, with λ a Lagrange multiplier, are

$$\frac{d}{ds} \left(\frac{\partial(\Lambda + \lambda F)}{\partial u_\alpha} - \frac{d}{ds} \frac{\partial \Lambda}{\partial \dot{u}_\alpha} \right) - \frac{\partial \Lambda}{\partial x_\alpha} = 0 \tag{9.5}$$

For a field-free configuration without external forces, the Lagrange function depends only on $\dot{u}_\alpha$. With the arbitrary function $f(Q)$ one can write for Λ

$$\Lambda = -f(Q), \qquad Q = \dot{u}_\alpha^2 \tag{9.6}$$

[2]Bopp's theory has been also discussed in considerable detail in a review article by Hönl (1952).

with the equation of motion (9.5) taking the form

$$\frac{d}{ds}\left\{[f(Q)-4Qf'(Q)]u_\alpha+2\frac{d}{ds}[f'(Q)\dot{u}_\alpha]\right\}=0 \tag{9.7}$$

and the dipole moment equal to

$$p_\alpha=-2f'(Q)\frac{du_\alpha}{ds} \tag{9.8}$$

For the transition to wave mechanics one needs the canonical representation of the equation of motion. From $\int \Lambda\, ds=\int L\, dt$, one obtains by separating the space and the time parts

$$\begin{gathered} L=\Lambda(1-v^2)^{1/2}=-f(Q)(1-v^2)^{1/2} \\ Q=\frac{1}{[(1-v^2)^{1/2}]^4}\left[\dot{\mathbf{v}}^2+\left(\frac{\mathbf{v}\cdot\dot{\mathbf{v}}}{(1-v^2)^{1/2}}\right)^2\right] \end{gathered} \tag{9.9}$$

where $L=L(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$.

With

$$\mathbf{P}=\frac{\partial L}{\partial \mathbf{v}}-\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{v}}},\qquad \boldsymbol{\theta}=\frac{\partial L}{\partial \dot{\mathbf{v}}} \tag{9.10}$$

one obtains the canonical equations

$$\begin{gathered} \dot{\mathbf{P}}=-\frac{\partial H}{\partial \mathbf{r}},\qquad \dot{\mathbf{r}}=\frac{\partial H}{\partial \mathbf{P}} \\ \dot{\boldsymbol{\theta}}=-\frac{\partial H}{\partial \mathbf{v}},\qquad \dot{\mathbf{v}}=\frac{\partial H}{\partial \boldsymbol{\theta}} \end{gathered} \tag{9.11}$$

P and **r** are here the macrovariables, **θ** and **v** the microvariables. With these variables, the angular momentum conservation law takes the form

$$\mathbf{r}\times\mathbf{P}+\mathbf{v}\times\boldsymbol{\theta}=\text{const} \tag{9.12}$$

The first term represents the external angular momentum of the macromotion and the second term the internal spin-type angular momentum of the micromotion. With **P** and **θ** given by (9.10) one obtains the Hamilton function ($\mathbf{P}=\{\boldsymbol{\theta}, i\theta_4\}$)

$$H=\mathbf{v}\cdot\mathbf{P}+\dot{\mathbf{v}}\cdot\boldsymbol{\theta}-L=H(\mathbf{r},\mathbf{P};\mathbf{v},\boldsymbol{\theta}) \tag{9.13}$$

From (9.10) one finds that

$$\begin{gathered} \boldsymbol{\theta}=-\frac{2f'(Q)}{(1-v^2)^{3/2}}\left[\dot{\mathbf{v}}+\frac{(\mathbf{v}\cdot\dot{\mathbf{v}})\mathbf{v}}{1-v^2}\right] \\ \dot{\mathbf{v}}=-\frac{1}{2}\frac{(1-v^2)^{3/2}}{f'(Q)}[\boldsymbol{\theta}-(\mathbf{v}\cdot\boldsymbol{\theta})\mathbf{v}] \end{gathered} \tag{9.14}$$

Eliminating $\dot{\mathbf{v}} \cdot \boldsymbol{\theta}$ from these equations leads to

$$4Qf'(Q)^2 = R = (1 - v^2)[\boldsymbol{\theta}^2 - (\mathbf{v} \cdot \boldsymbol{\theta})^2] \tag{9.15}$$

from which the function $Q = Q(R)$ can be obtained, and by which $\dot{\mathbf{v}}$ can be eliminated from H,

$$H = \mathbf{v} \cdot \mathbf{P} + (1 - v^2)^{1/2} F(R) \tag{9.16}$$

where

$$F(R) = f(Q) - 2Qf'(Q) \tag{9.17}$$

For the linear dependence

$$f(Q) = k_0 + (1/2)k_1 Q \tag{9.18}$$

where k_0 and k_1 are constants, one finds

$$H = k_0(1 - v^2)^{1/2} - (1/2k_1)(1 - v^2)^{3/2}[\boldsymbol{\theta}^2 - (\boldsymbol{\theta} \cdot \mathbf{v})^2] \tag{9.19}$$

which has the same form as the Dirac Hamiltonian.

Putting

$$\begin{gathered} \mathbf{P} \to \frac{\hbar}{i} \frac{\partial}{\partial \mathbf{r}} \\ \mathbf{v} \to \boldsymbol{\alpha} \\ (1 - v^2)^{1/2} \to \alpha_4 \end{gathered} \tag{9.20}$$

where $\alpha = \{\boldsymbol{\alpha}, \alpha_4\}$ are the Dirac matrices, one obtains the Dirac equation:

$$\frac{\hbar}{i} \frac{\partial \psi}{\partial t} + H\psi = 0 \tag{9.21}$$

where

$$H = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 m \tag{9.22}$$

with

$$\alpha_\mu^2 = 1; \qquad \alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 0, \qquad \mu \neq \nu \tag{9.23}$$

and the mass given by

$$m = k_0 - (1/2k_1)[\boldsymbol{\theta}^2 - (\boldsymbol{\theta} \cdot \mathbf{v})^2] \tag{9.24}$$

For the linear dependence (9.18) one obtains from (9.7)

$$\frac{d}{ds}(2\lambda u_\alpha + k_1 \ddot{u}_\alpha) = 0 \tag{9.25}$$

or

$$2\dot{\lambda} u_\alpha + 2\lambda \dot{u}_\alpha + k_1 \dddot{u}_\alpha = 0 \tag{9.26}$$

Differentiating (9.3) with regard to s, one has

$$u_\alpha \dot{u}_\alpha = 0, \qquad u_\alpha \ddot{u}_\alpha + \dot{u}_\alpha^2 = 0, \qquad u_\alpha \dddot{u}_\alpha + 3\dot{u}_\alpha \ddot{u}_\alpha = 0 \tag{9.27}$$

and by which (9.26) becomes

$$-2\dot{\lambda} - 3k_1 \dot{u}_\alpha \ddot{u}_\alpha = -2\dot{\lambda} - \frac{3}{2} k_1 \frac{d}{ds}(\dot{u}_\alpha^2) = 0 \tag{9.28}$$

Summation over v gives

$$2\lambda = k_0 - (3/2)k_1 \dot{u}_v^2 \tag{9.29}$$

where k_0 appears here as a constant of integration. Inserting (9.29) into (9.25), we can eliminate the Lagrange multiplier,

$$\frac{d}{ds}\left[\left(k_0 - \frac{3}{2} k_1 \dot{u}_v^2\right)u_\alpha + k_1 \ddot{u}_\alpha\right] = 0 \tag{9.30}$$

To show that (9.30) is the equation of motion for a pole–dipole particle, one writes it as follows:

$$\begin{aligned} \frac{dP_\alpha}{ds} &= 0 \\ P_\alpha &= \left(k_0 - \frac{3}{2} k_1 \dot{u}_v^2\right)u_\alpha + k_1 \ddot{u}_\alpha \end{aligned} \tag{9.31}$$

where P_α are the components of the momentum-energy four-vector. For $k_1 = 0$ one has $P_\alpha = k_0 u_\alpha$, which by putting $k_0 = m$ is the four-momentum of a spinless particle with rest mass m.

With the mass dipole moment computed from (9.8)

$$p_\alpha = -k_1 \dot{u}_\alpha \tag{9.32}$$

conservation of angular momentum is given by

$$\frac{d}{ds} J_{\alpha\beta} = 0 \tag{9.33}$$

where

$$J_{\alpha\beta} = [\mathbf{x}, \mathbf{P}]_{\alpha\beta} + [\mathbf{p}, \mathbf{u}]_{\alpha\beta} \tag{9.34}$$

and where $[\mathbf{x}, \mathbf{P}]_{\alpha\beta} = x_\alpha P_\beta - x_\beta P_\alpha$. For a particle at rest $P_k = 0$, $k = 1, 2, 3$, one has

$$J_{kl} = [\mathbf{p}, \mathbf{u}]_{kl} = p_k u_l - p_l u_k, \qquad k, l = 1, 2, 3 \tag{9.35}$$

which is the spin angular momentum.

The energy of a pole–dipole particle at rest, for which $u_4 = \gamma$, is determined by the fourth component

$$P_4 = im = i\left(k_0 - \frac{3}{2}k_1\dot{u}_v^2\right)\gamma \tag{9.36}$$

but it can also be obtained from

$$P_\alpha u_\alpha = -\gamma m = \left(k_0 - \frac{3}{2}k_1\dot{u}_v^2\right)u_\alpha^2 + k_1\ddot{u}_\alpha u_\alpha = -\left(k_0 - \frac{1}{2}k_1\dot{u}_v^2\right) \tag{9.37}$$

The mass m therefore obeys the double equation

$$m = \left(k_0 - \frac{3}{2}k_1\dot{u}_v^2\right)\gamma = \left(k_0 - \frac{1}{2}k_1\dot{u}_v^2\right)\gamma^{-1} \tag{9.38}$$

To keep m finite in the limit $v \to 1$, resp. $\gamma \to \infty$, one must have

$$\begin{aligned} &\lim_{\gamma\to\infty}\left(k_0 - \frac{3}{2}k_1\dot{u}_v^2\right) \to 0 \\ &\lim_{\gamma\to\infty}\left(k_0 - \frac{1}{2}k_1\dot{u}_v^2\right) \to \infty \end{aligned} \tag{9.39}$$

which means that $k_0 \to (3/2)k_1\dot{u}_v^2$ and $k_0 \to \infty$.

From (9.31) with $P_k = 0, k = 1, 2, 3$, it follows for a circular orbit of radius r_c that

$$p = \left(k_0 - \frac{3}{2}k_1\dot{u}_v^2\right)r_c \tag{9.40}$$

or because of (9.36)

$$p = mr_c/\gamma \tag{9.41}$$

With $\mathbf{u} = \gamma\mathbf{v}$, one obtains for the spin angular momentum

$$J_z = -pu = -mvr_c \simeq -mcr_c \tag{9.42}$$

which is the same as (8.19).

10. HIGHER PARTICLE GENERATIONS FOR SPINOR QUASIPARTICLES

Internal excitations of the positive–negative mass pole–dipole configuration can lead to excited states. Because of the highly nonlinear interaction, there is a finite number of such excited states. In conjunction with the observed finite number of particle families, it is suggested that the excited states are a representation of the higher particle generations.

Excited states are possible with more general nonlinear dependencies $f(Q)$ (Bopp, 1946, 1949). For the wave mechanical treatment of this problem it is convenient to use the four-dimensional representation by making a canonical transformation

$$\boldsymbol{\theta} \cdot d\mathbf{v} + \boldsymbol{\theta}_0 \, d\mathbf{v}_0 + u_\alpha \, dp_\alpha = d\Phi(\mathbf{v}, v_0, p_\alpha) \tag{10.1}$$

with the generating function

$$\Phi = \frac{v_0}{(1 - v^2)^{1/2}} (\mathbf{v} \cdot \boldsymbol{\theta} + i\theta_4) \tag{10.2}$$

and where v_0, θ_0 are superfluous coordinates. Expressed in the new variables, one has

$$R = -\frac{1}{2} M_{\alpha\beta}^2, \qquad M_{\alpha\beta} = u_\alpha \theta_\beta - u_\beta \theta_\alpha \tag{10.3}$$

and with $P_\alpha = \{\mathbf{P}, iH\}$ one finds for (9.16)

$$K = u_\alpha P_\alpha + (-u_\alpha^2)^{1/2} F(R) = 0 \tag{10.4}$$

Because $u_\alpha^2 = -1$ and $u_\alpha p_\alpha = 0$, the superfluous coordinates v_0 and θ_0 can be eliminated. Putting

$$\begin{aligned} P_\alpha &= \frac{\hbar}{i} \frac{\partial}{\partial x_\alpha} \\ p_\alpha &= \frac{\hbar}{i} \frac{\partial}{\partial u_\alpha} \end{aligned} \tag{10.5}$$

one obtains the wave equation

$$K\psi \equiv \left[\left(u_\alpha, \frac{\hbar}{i} \frac{\partial}{\partial x_\alpha} \right) + F(R) \right] \psi(x, u) = 0 \tag{10.6}$$

where

$$R = -\frac{1}{2} M_{\alpha\beta}^2, \qquad M_{\alpha\beta} = \frac{\hbar}{i} \left(u_\alpha \frac{\partial}{\partial u_\beta} - u_\beta \frac{\partial}{\partial u_\alpha} \right) \tag{10.7}$$

For $\mathbf{P} = 0$, the wave function has the form

$$\psi(x, u) = \psi(u) e^{-i\varepsilon t/\hbar} \tag{10.8}$$

with the wave equation for $\psi(u)$:

$$F(R)\psi(u) = \frac{\varepsilon}{(1 - v^2)^{1/2}} \psi(u) \tag{10.9}$$

or if G is the inverse function for F,

$$R\psi(u) = G\left(\frac{\varepsilon}{(1-v^2)^{1/2}}\right)\psi(u) \tag{10.10}$$

From the condition $u_\alpha p_\alpha = 0$ it follows that

$$R = -\hbar^2 \frac{\partial^2}{\partial u_\alpha^2} \tag{10.11}$$

With (θ, ϕ spherical polar coordinates)

$$u_\alpha = [\sinh\alpha \cdot \sin\theta \cdot \cos\phi, \sinh\alpha \cdot \sin\theta \cdot \sin\phi, \sinh\alpha \cdot \cos\theta, i\cosh\alpha]$$
$$\psi = \psi_0/\sinh\alpha \tag{10.12}$$
$$\text{tgh}\,\alpha = v$$

the wave equation becomes

$$-\left[\frac{\partial^2}{\partial\alpha^2} - 1 - \frac{M^2}{\sinh^2\alpha}\right]\psi_0 = G(\varepsilon\cosh\alpha)\psi_0 \tag{10.13}$$

where

$$M^2 = (\mathbf{v}\times\boldsymbol{\theta})^2 = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} \tag{10.14}$$

having the eigenvalues $j(j+1)$, where j is an integer. The wave equation, therefore, finally becomes

$$\frac{d^2\psi_0}{d\alpha^2} = V(\alpha)\psi_0 = \left[1 + \frac{j(j+1)}{\sinh^2\alpha} - G(\varepsilon\cosh\alpha)\right]\psi_0 \tag{10.15}$$

The eigenvalues can be obtained by the WKB method, with the factor $j(j+1)$ be replaced by $(j+1/2)^2$ to account for the singularity at $\alpha = 0$. The eigenvalues are then determined by the equation

$$J = \frac{1}{\pi}\int_{\alpha_1}^{\alpha_2} [-V(\alpha)^{1/2}]\,d\alpha = n + \frac{1}{2}, \qquad n = 0, 1, 2, \ldots \tag{10.16}$$

with

$$V(\alpha) = 1 + \frac{(j+1/2)^2}{\sinh^2\alpha} - G(\varepsilon\cosh\alpha) \tag{10.17}$$

Of special interest are the cases where $j = -1/2$, because they correspond to the correct angular momentum quantization rule for the Zitterbewegung. For $j = -1/2$ one simply has

$$V(\alpha) = 1 - G(\varepsilon\cosh\alpha) \tag{10.18}$$

To obtain an eigenvalue requires a finite value of the phase integral (10.16). The function $G(x)$, $(x = \varepsilon \cosh \alpha)$, must therefore qualitatively have the form of a parabola cutting the line $G = 1$ at two points x_1, x_2 between which $G(x) > 1$. One can then distinguish two limiting cases: first if $\varepsilon \ll 1$ and second if $\varepsilon \gg 1$. In both cases one may approximate (10.16) as follows:

$$J \simeq (1/\pi)\overline{[-V(\alpha)]^{1/2}}(\alpha_2 - \alpha_1) \tag{10.19}$$

In the first case $\alpha \gg 1$, and one has

$$\alpha \simeq \ln\left[\frac{x}{\varepsilon} + \left(\frac{x^2}{\varepsilon^2} - 1\right)^{1/2}\right] \approx \ln\left(\frac{2x}{\varepsilon}\right) - \frac{\varepsilon^2}{4x^2} + \cdots \tag{10.20}$$

hence

$$\alpha_2 - \alpha_1 \simeq \ln\left(\frac{x_2}{x_1}\right) + \frac{x_2^2 - x_1^2}{x_1^2 x_2^2}\frac{\varepsilon^2}{4} + \cdots \tag{10.21}$$

In the second case one has

$$\alpha \simeq \sqrt{2}\left(\frac{x}{\varepsilon} - 1\right)^{1/2} \tag{10.22}$$

or if $x/\varepsilon \gg 1$ simply

$$\alpha \simeq (2x/\varepsilon)^{1/2} \tag{10.23}$$

hence

$$\alpha_2 - \alpha_1 \simeq \left(\frac{2}{\varepsilon}\right)^{1/2}(x_2^{1/2} - x_1^{1/2}) \tag{10.24}$$

One therefore sees that the phase integral has for $\varepsilon \ll 1$ the form $J = a + b\varepsilon^2$, but for $\varepsilon \gg 1$ the form $J = a/\sqrt{\varepsilon}$. The $J(\varepsilon)$ curve can for this reason cut twice the lines $J = 1/2$ $(n = 0)$ and $J = 3/2$ $(n = 1)$.

In Fig. 2 we have adjusted the phase integral to account for the electron, muon, and tau electron, where $\varepsilon = E/mc^2$, with m the electron mass. The fact that the mass ratio of the tau and muon are so much smaller than the mass ratio of the muon and electron suggests that both the muon and tau result from cuts of the line $J = 3/2$. Because of the proximity on the $J = 3/2$-line, it is unlikely that the phase integral would cut the line $J = 5/2$ or higher. Since for large values of ε, $J \propto 1/\sqrt{\varepsilon}$, it follows that there must be one more eigenvalue for which $J = 1/2$, which from the position of the first three families is guessed to be around $80{,}000 mc^2 \simeq 40$ GeV. Our result therefore suggests that there are no more than four particle families.

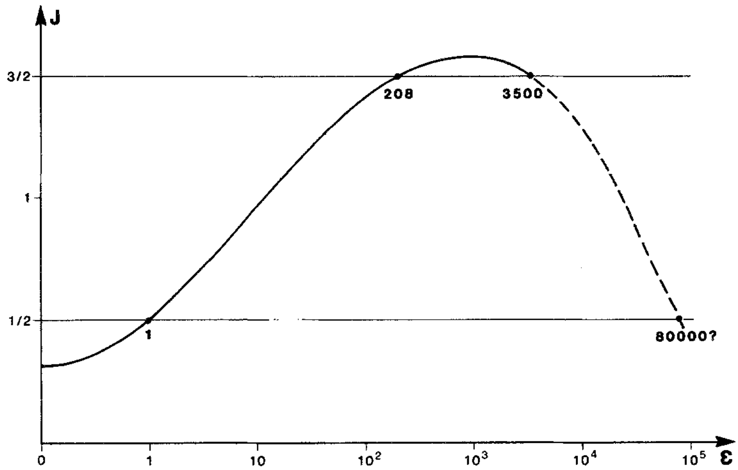


Fig. 2. Qualitative form of the phase integral $J = J(\varepsilon)$ to determine the number of families and their masses.

In the framework of the proposed model a more definite conclusion has to await the determination of the structure function $f(Q)$ from the mass distribution of the exciton made up from the positive–negative mass vortex resonance.

11. QUARK–LEPTON SYMMETRIES

In the model, all particles are quasiparticles like the phonons and excitons in condensed matter physics. For this reason, one may expect that the quark–lepton symmetries should have a condensed matter physics analogy. We claim that it is provided by the fractional quantum Hall effect. It occurs in a very pure thin sheet confining a two-dimensional electron gas. As Laughlin (1983*a*,*b*) has shown, an electron gas can be described by the wave function

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \left[\prod_{j<k} (z_j - z_k)^\mu\right]\left[\prod_j e^{-|z_j|^2/4l^2}\right] \tag{11.1}$$

where $z_j = x_j - iy_j$ is the coordinate of the jth electron in complex notation and $l^2 = \hbar c/eH$ (obtained from $ml^2\omega = \hbar$ and $\omega = eH/mc$ for the lowest Landau level). The magnetic field H is directed perpendicular to the sheet. If μ is an odd integer, the wave function is completely antisymmetric, obeying Fermi statistics, and is made up from states of the first Landau level with the kinetic energy equal to $(1/2)\hbar\omega$ per electron. For the square of the wave function one has

$$|\psi|^2 = e^{-\beta H}$$
$$\beta H = 2\mu \sum_{j<k} \ln|\mathbf{r}_j - \mathbf{r}_k| + (1/2l^2) \sum_j |\mathbf{r}_j|^2 \tag{11.2}$$

which is the probability distribution $|\psi|^2$ of a one-component two-dimensional plasma.

For $\mu = 1$, the wave function is a Slater determinant, but this wave function does not describe the situation actually observed. Numerical calculations for four to six electrons done by Laughlin, rather, show that the wave function (11.1) for $\mu = 3$ gives a much better agreement. It is this wave function which satisfactorily explains the fractional quantized Hall effect, in which plateaus in the conductivity are found to occur in multiple steps of $(1/3)e^2/h$.

The physical meaning of the wave function (11.1) can be understood if one keeps all electrons, except one, fixed in their position and carries out a closed loop motion of the one electron around a point at which the wave function vanishes. This displacement produces the phase shift

$$\Delta\phi = (e/\hbar c) \oint \mathbf{A} \cdot d\mathbf{s} = (e/\hbar c) \int \mathbf{H} \cdot d\mathbf{f} \tag{11.3}$$

where $\mathbf{H} = \text{curl}\,\mathbf{A}$. Accordingly, there should be

$$Z = (e/\hbar c) \int \mathbf{H} \cdot d\mathbf{f} \tag{11.4}$$

vortices within the area $\int d\mathbf{f}$. To satisfy the Pauli principle there must be at least one vortex at the position of each electron. In Laughlin's wave function there are exactly μ vortices for each electron. We therefore have to put $Z = \mu$. The fractional quantized Hall effect then simply means that the charge of one vortex is $e/3$ provided $\mu = 3$, and it follows that in the two-dimensional electron fluid each electron splits into three vortices of charge $e/3$.

The quantization condition for the vortices in the presence of a magnetic field is given by

$$\oint \mathbf{v} \cdot d\mathbf{s} = \frac{h}{m} \nu - \frac{e}{mc} \int \mathbf{H} \cdot d\mathbf{f}, \qquad \nu = 1, 2, 3, \ldots \tag{11.5}$$

which shows that the presence of a magnetic field causes the occurrence of the vortices in the electron fluid. If a magnetic field is adiabatically applied to the electron fluid, the Helmholtz theorem

$$\frac{d}{dt} \oint \mathbf{v} \cdot d\mathbf{s} = 0 \tag{11.6}$$

states that if the circulation $\oint \mathbf{v} \cdot d\mathbf{s}$ is zero before a magnetic field is applied, it remains zero thereafter. This, of course, does not imply that the circulation inside the contour taken in (11.6) cannot differ from zero, because the circulation of different vortices can add up to zero, as would be the case for four vortices with equal and opposite circulation. A vortex configuration with the total angular momentum $(1/2)\hbar$ could be constructed from two vortices with opposite circulation quantum number $\nu = \pm 1$ and one vortex with $\nu = 1$ with the spin quantum numbers adding up to a total angular momentum $(1/2)\hbar$.

To explain the fractionally charged quarks, an analogy to the fractional quantum Hall effect is tempting to explore. We propose the conjecture that it results from the splitting up of the electron and neutrino wave functions into vortices, with the splitting up caused by a strong field acting like a magnetic field. Leaving aside for the moment the question of what that field might be, the occurrence of three-quark configurations suggests that this field acts within a thin sheet, with the vortices perpendicular to and confined within this sheet and with a minimum of three vortices needed to define the orientation of a planar sheet.

If the electron and neutrino wave functions split up in a similar way as happens in the fractionally quantized Hall effect, the quark–lepton symmetries can easily be understood. The angular momentum of a vortex in units of $\hbar$ is equal to the circulation quantum number ν. The vortices for which $\nu = 1$ we call A, those for which $\nu = 0$ we call B, and finally those for which $\nu = -1$ we call C. The neutrino (ν) and positron (e_+) wave functions are then to be represented by six vortex states, with the lower indices giving the value for the electric charge of these vortex states:

$$(\nu) = \begin{cases} A_0 \\ B_0 \\ C_0 \end{cases} \qquad (e_+) = \begin{cases} A_{1/3} \\ B_{1/3} \\ C_{1/3} \end{cases} \tag{11.7}$$

A second set of six vortex states is obtained by replacing the neutrino and positron by their antiparticles. We claim that the first six vortex states can reproduce all the six u and d quarks of the first family. How the neutrino and positron are composed of these vortex states is already shown in (11.7). With the indexes r, g, b (red, green, blue) identifying what is called the color, we have for the three colors of the u quark

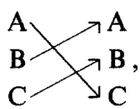
$$u_r = \begin{cases} A_{1/3} \\ B_{1/3} \\ C_0 \end{cases} \qquad u_g = \begin{cases} A_{1/3} \\ B_0 \\ C_{1/3} \end{cases} \qquad u_b = \begin{cases} A_0 \\ B_{1/3} \\ C_{1/3} \end{cases} \tag{11.8}$$

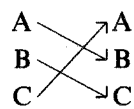
For the three d quarks we have

$$d_r = \begin{cases} A_0 \\ B_0 \\ C_{-1/3} \end{cases} \qquad d_g = \begin{cases} A_0 \\ B_{-1/3} \\ C_0 \end{cases} \qquad d_b = \begin{cases} A_{-1/3} \\ B_0 \\ C_0 \end{cases} \tag{11.9}$$

Because the vortices are substates of the leptons, color confinement then simply means that only those vortex configurations which can be combined into leptons are able to assume the form of free particles. Mesons are made up from quark–antiquark configurations, each containing three vortices and three antivortices.

If the vortices interact, they do this by the exchange bosons. But because they are confined within a thin sheet, the bosons are massive, very much like an electromagnetic wave in a waveguide where the photons are massive with a longitudinal component in addition to their transverse component. It is then possible to explain the eight gluons of the standard model. The gluons are bosons transmitting angular momentum. To change an A vortex into a B vortex, a B vortex into a C vortex, or vice versa, requires a change in the angular momentum $\Delta L = \pm 1$, and to change an A vortex into a C vortex, or vice versa, a change by $\Delta L = \pm 2$ is needed. These changes can be made by just two angular momentum operators with $L = 1$ and $L = 2$, having $\sum (2L+1) = 2+1+4+1 = 8$ states, equal to the number of the gluons in QCD. The transitions in QCD identified by red–green ($r-g$), red–blue ($r-b$) and green–blue ($g-b$), lead to changes in the angular momentum of the vortices in the following way:

$r \to g = g \to b$: 

, $r \to b$: 

(11.10)

These transitions require four changes $\Delta L = \pm 1$ and two changes $\Delta L = \pm 2$, in total 6 changes. In addition, there is the change for which $\Delta L = 0$,

$$r \to r = g \to g = b \to b: \quad \begin{matrix} A \to A \\ B \to B \\ C \to C \end{matrix} \tag{11.11}$$

realized with the two $L_z = 0$ components of the angular momentum operators for $L = 1$ and $L = 2$, and which for this reason must be counted twice. Together with the six changes involving $\Delta L = \pm 1, \pm 2$, one has a total of eight possible changes involving the exchange of angular momentum. The eight gluons of QCD are not to be identified with these eight angular momentum-transmitting bosons, but rather with certain combinations of them. A color-changing gluon would be always a superposition of a spin-1 and a spin-2 boson. A transition leaving the color unchanged would involve the superposition of spin-2 or spin-1 bosons. The color charge is thereby reduced to angular momentum, and through angular momentum, quantization to the zero-point fluctuations of the Planck masses like the other charges.

The same decomposition into vortex states done here for the first family can be repeated for the higher generations. And the weak interaction phenomenon is explained by the exchange of bosons made up from spin-1 and spin-2 angular momentum transitions between the vortices of the leptons given by (11.7).

We remark that our model can be compared with the rishon model, with the rishons turning out to be vortex states. The three hypercolor charges of the rishon model are the three angular momentum states $L = 1, 0, -1$ of the vortices. The prescription of the rishon model that only those configurations are possible which are color neutral with regard to the hypercolor is explained by the requirement that the vortex states must add up to zero angular momentum.

The axial current interaction of the standard model occurs only for configurations involving in our model vortex substructures in which the leptons split up in a triplet of vortices. Very much as a bound particle can execute radial zero-point oscillations, a vortex can execute azimuthal zero-point oscillations. With the moment of inertia $\Theta \simeq m_P r_P^2$ and angular velocity ω, the magnitude of these zero-point fluctuations is determined by the uncertainty principle for rotational motion

$$\Theta\omega \simeq \hbar \tag{11.12}$$

leading to an energy density equal in order of magnitude to the energy density of the radial oscillations. The vortices thereby become the source of virtual vector field waves which by order of magnitude again must have the coupling strength $g^2 \sim \hbar c$. However, because the virtual rotational fluctuations have the character of an axial vector, the current to which these interactions couple must be an axial vector as well.

The origin of non-Abelian gauge theories can be explained as a result of vortex–vortex interaction through their radial and rotational zero-point fluctuations. These vortices behave like small electrically charged magnets, and the total vortex–vortex interaction must therefore be a superposition of electric- and magnetic-type interactions. The static electric force exerted by a magnet 1 on a magnet 2 is

$$F_e = -e\nabla\Phi_1 \tag{11.13}$$

where Φ_1 is the scalar electric potential produced by the electric charge on magnet 1. To compute the static magnetic force, we consider two magnets of moments $\mathbf{m}_1$, $\mathbf{m}_2$ separated by the distance r, with the magnetic fields produced by them expressed through their vector potentials:

$$\mathbf{A}_1 = \frac{\mathbf{m}_1 \times \mathbf{e}_r}{r^2}$$
$$\mathbf{A}_2 = -\frac{\mathbf{m}_2 \times \mathbf{e}_r}{r^2} \tag{11.14}$$

where it is assumed that $\mathbf{e}_r$ is perpendicular to $\mathbf{m}_1$ and $\mathbf{m}_2$, and where $\mathbf{e}_r$ is a unit vector along r. The magnetic force on $\mathbf{m}_2$ by $\mathbf{m}_1$ is then given by

$$F_m = \nabla(\mathbf{m}_2 \cdot \text{curl}\, \mathbf{A}_1) = 6\mathbf{A}_1 \cdot \mathbf{A}_2 \mathbf{e}_r \tag{11.15}$$

For three interacting vortices, three vector potentials are likewise needed. The nonlinear quadratic term in (11.15) is typical for terms occurring in Yang–Mills theories. The expressions for F_e and F_m are valid for static fields. Relativistic invariance requires that the forces must in general be expressed by relativistically invariant Yang–Mills field theories.

12. THE ORIGIN OF THE HIGGS FIELD

Because the vortex resonance energy (4.7) gives the elementary particles a finite rest mass, it suggests that this resonance energy be identified with the mass of the hypothetical Higgs particle. With the above given value $R/r_P \simeq 6 \times 10^3$, this would mean that the Higgs particle has the mass $\hbar\omega_v \lesssim 10^{12}$ GeV (far too large for the SSC).

To explore this hypothesis in more detail, we write down the relativistic nonlinear wave equation derived from the Higgs Lagrangian

$$\Box\phi + \kappa^2\phi - 4\lambda\phi^3 = 0 \tag{12.1}$$

where $\lambda > 0$ and $\kappa = mc/\hbar$, with m equal to the mass of the Higgs meson. Because our model is exactly nonrelativistic, we have to make the transition of (12.1) to its nonrelativistic approximation. In the nonrelativistic limit we have to put

$$-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \rightarrow \frac{2im}{\hbar}\frac{\partial\phi}{\partial t} + \kappa^2\phi \tag{12.2}$$

by which (12.1) becomes

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\phi - mc^2\phi + \frac{2\lambda\hbar^2}{m}\phi^3 \tag{12.3}$$

As in the Higgs model, (12.3) has the static global solution

$$\phi_0^2 = \kappa^2/2\lambda \tag{12.4}$$

If $\phi_0^2 = 1/2r_P^3$ as suggested by our model, one finds, with $m = m_P(r_P/R)^2$,

$$\lambda = \kappa^2 r_P^3 = r_P(r_P/R)^4 \simeq 2.5 \times 10^{-48}\,\text{cm} \tag{12.5}$$

In the standard model the weak vector boson mass can be expressed by the "weak magnetic vector potential" **A** of the WSG theory as follows (with the vacuum gauge **A** = 0):

$$m_W c^2 = eA \tag{12.6}$$

We suggest that the "weak magnetic field" of WSG theory is the cause of the lepton wave functions splitting up into the vortices representing the wave functions of the quarks, implying that

$$m_W c^2 = 2^{1/4} g(\hbar c/G_F)^{1/2} \simeq 85\,\text{GeV} \tag{12.7}$$

where g is the semiweak coupling constant. If the vortices into which a lepton splits up are line vortices in a sheet of thickness δ, with the vortices ending at the two surfaces of the sheet, the "weak magnetic field" in the sheet must be of the order $H \sim A/\delta$. And because of $m_W c\delta \simeq \hbar$, one finds from (12.6) an expression for H:

$$H = (m_W c^2)^2/e\hbar c \simeq 10^{26}\,[\text{esu}] \tag{12.8}$$

corresponding to a huge "weak magnetic field" of $\sim 10^{26}$ G. As in the fractional quantized Hall effect, it leads to a Lorentz force strong enough to produce a vortex structure. According to (12.8), the magnetic field needed must be at least of the order

$$H \sim \hbar c/er^2 \tag{12.9}$$

where r is the Larmor radius. For relativistic velocities r is of the order

$$r \sim m_W c^2/eH \tag{12.10}$$

Eliminating r from (12.9) and (12.10) gives the same value of H as (12.8). Furthermore, from

$$H \sim 4\pi ne\delta \tag{12.11}$$

one finds for the number density of charges in the sheet

$$n \simeq (m_W c/\hbar)^3 \simeq 10^{51}\,\text{cm}^{-3} \tag{12.12}$$

[provided each intermediate vector boson has the unit charge $e \sim (\hbar c)^{1/2}$].

The intermediate vector bosons of the WSG theory are explained in the model as vortex–antivortex pairs from the first generation. This implies

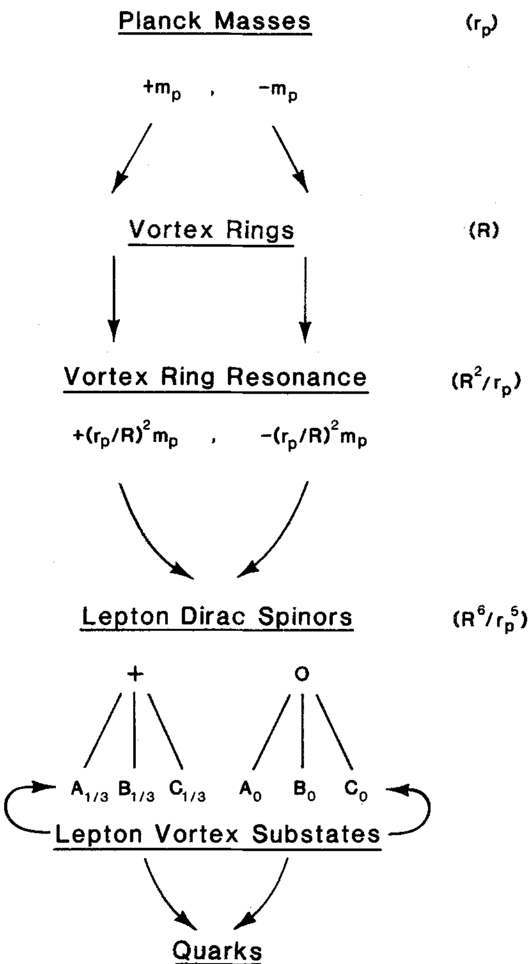


Fig. 3. The different hierarchies and scales.

Table I. Meson States and Their Corresponding Higher Families

Lepton–antilepton = positronium, . . . , higher families
Quark–antiquark = π^+ meson, . . . , higher families
Vortex–antivortex = W, Z bosons, . . . , higher families

the existence of heavier intermediate vector bosons made up from vortices of the higher generations. Assuming that the mass ratios of the vector bosons are about equal to the mass ratios of the leptons, the vector boson of the second generation should have a mass of $\sim 2 \times 10^4$ GeV.

In Fig. 3 we display the different hierarchies and scales, and in Table I the different meson states. Because mesons are derived from the leptons, positronium has been included as a meson state.

13. THE STANDARD MODEL AS AN ASYMPTOTIC LIMIT

It is often argued that the extremely good agreement between theory and experiment in quantum electrodynamics (QED) provides strong evidence in support of the special theory of relativity. However, it was shown by Weinberg (1987) that this argument is questionable. Very generally, the Lagrange density describing the electron and photon must be an infinite series of the form

$$\begin{aligned}\mathscr{L} = & -\bar{\psi}\left(\gamma^\mu \frac{\partial}{\partial x^\mu} + m\right)\psi - \frac{1}{4}\left(\frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\right)^2 \\ & + ie_0 A_\mu \bar{\psi}\gamma^\mu\psi \\ & - e_1\left(\frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}\right)\bar{\psi}\sigma^{\mu\nu}\psi - e_2\bar{\psi}\psi\bar{\psi}\psi + \cdots\end{aligned} \tag{13.1}$$

In QED only the first three terms, which are the kinetic energy of the electron, the kinetic energy of the photon, and the interaction energy between both through the coupling constant e_0, are taken into account. In the form (13.1), e_0 is nondimensional, but the coupling constants $e_1, e_2, \ldots$ with which the following terms are multiplied have dimensions of the inverse power of a mass, e_1 the dimension mass^{-1}, e_2 the dimension mass^{-2}, and so on. The very good agreement of theory and experiment in QED can be readily understood if this mass is very large, for example, equal to the GUT mass ($\sim 10^{16}$ GeV) or Planck mass ($\sim 10^{19}$ GeV), because one can then completely ignore all the higher-order terms. With the remaining three terms in the Lagrange density, the theory can be renormalized, which would not be possible if the higher-order terms are included. One can therefore say that QED in particular, and the standard model in general, are low-energy asymptotic approximations. At the very high energies at which the higher-order terms cannot be neglected, one would either have to find a way to sum up all the higher-order terms or look for another theory of which the Lagrange density (13.1) is a low-energy limit.

The Planck aether model is an attempt to follow the second alternative. In it there would be complete symmetry between the positive and negative Planck masses, canceling each other out, were it not for the quantum potential. Through it this symmetry is broken. The quantum potential leads to zero-point oscillations of the Planck masses bound in the vortices, setting up a Newtonian-type gravitational force. This force couples the vortex filaments, leading to Maxwell's and Einstein's equations, and to Dirac spinors. It is through the nonlinearity of the gravitational field, resulting in a residual positive mass for two interacting masses of opposite sign but equal in magnitude, that the vortex lattice of the Planck aether assumes a positive mass and with it all elementary particles, including photons and gravitons. If this positive energy is compensated by the negative gravitational interaction energy, the total energy remains zero and with it the cosmological constant.

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