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Transference of Squeezing in Coupled Oscillators

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We examine the possibility of obtaining the transference of the squeezing effect between two coupled oscillators, one of them described by a quadratic Hamiltonian in terms of the ladder operators, the other one being a linear harmonic oscillator, plus an interaction term. We obtain an exact solution for the time evolution of our coupled system which allows us to find the variances for one- and two-mode oscillations. It is shown that the squeezing generated in one of the oscillators may or may not spread to the other oscillator, depending on the choice of the involved parameters. Other interesting features exhibited for the one- and two-mode oscillations are also discussed.

1. INTRODUCTION

Squeezed states for the electromagnetic field and oscillators have been widely studied, both theoretically (Huen, 1976; Walls, 1983; Loudon and Knight, 1989; Stenholm, 1986) and experimentally for light fields (Slusher *et al.*, 1985; Robinson, 1985, 1986; Wu *et al.*, 1986), and, more recently, for oscillators (Cirac *et al.*, 1993). These states have reduced values in one of the quadratures $\hat{a}_1 = (\hat{a} + \hat{a}^+)/2$ or $\hat{a}_2 = (\hat{a} - \hat{a}^+)/2i$, where $\hat{a}$ and $\hat{a}^+$ stand for the annihilation and creation operators for photons, or the lowering and raising operators for the harmonic oscillator. Since a single mode of the electromagnetic field behaves like a simple harmonic oscillator (HO) with unit mass, the discussion of the production of squeezing for the HO has also immediate relevance in the generation of this effect for light fields.

The purpose of this paper is to examine the possibility of generating the squeezing effect in a linear HO by its transference from a coupled

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quadratic oscillator (QO), that is, a linear oscillator with quadratic self-interacting terms in $\hat{a}$ and $\hat{a}^+$. As is well known (Huen, 1976; Walls, 1983; Loudon and Knight, 1989; Stenholm, 1986), this quadratic Hamiltonian is a candidate to exhibit the squeezing effect and we will show that this reduction of quantum fluctuation generated in the QO may be transferred to one of the quadrature components of the HO, thus becoming also squeezed, out of a coherent state.

We will employ the definitions for quadrature-phase amplitudes as $\hat{x} = (\hbar/2)^{1/2}(\hat{a} + \hat{a}^+)$, $\hat{p} = (\hbar/2)^{1/2}(\hat{a} - \hat{a}^+)/i$, the shot noise level being $(\hbar/2)^{1/2}$. We obtain exact closed solutions for the time evolution of $\hat{x}(t)$ and $\hat{p}(t)$ for the QO and also of $\hat{X}(t)$ and $\hat{P}(t)$, with an analogous definition, for the HO. This allows us to compute their variances as a function of time, which display interesting features.

In a recent paper, Agarwal and Gupta (1989) studied a combined system constituted by an atomic oscillator interacting with a squeezed light field. They found exact solutions for the (non-Hamiltonian) dynamical equations for the combined system. From these solutions they were able to study relaxation of the atomic oscillator for arbitrary bandwidth of the squeezed radiation and also its effects on the vacuum-field Rabi splitting. Since these authors assumed the squeezed light as constituting a bath for the atomic oscillator, they employed consistently the density state formalism through the Wigner function to describe the combined system. In our case, the QO generating the squeezing does not constitute a bath for the HO, hence our approach is quite different from theirs. Our combined system has a Hamiltonian dynamics, thus allowing us to use the pure state formalism. In this way we have employed the Heisenberg picture in exactly solving our coupled system.

Besides the investigation of one-mode oscillations, we also have studied the two-mode oscillations. Here, contrary to the traditional investigations on coupled systems, where one-mode squeezing is absent, both the coordinates of two-mode oscillations may become squeezed, simultaneously or not, depending on the values of the involved parameters.

This paper is organized as follows: in Section 2 we present our model Hamiltonian and establish the notation. In Section 3 we derive the Heisenberg equations of motions and obtain their solutions. In Section 4 we compute the fluctuations in quadrature-phase amplitudes for one-mode oscillations, while in Section 5 we study the squeezing in two-mode oscillations. Finally in Section 6 we present some comments and conclusions.

2. MODEL HAMILTONIAN

Following Gordon *et al.* (1963), we take the model Hamiltonian

$$\hat{H} = \left\{\hbar\Omega\left(\hat{A}^{+}\hat{A} + \frac{1}{2}\right)\right\} + \left\{\hbar\omega\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + \hbar f\left(\hat{a}^{+2} + \hat{a}^{2}\right)\right\}$$
$$- \left\{\hbar\gamma\left(\hat{a}\hat{A}^{+} + \hat{a}^{+}\hat{A}\right)\right\} \tag{2.1}$$

where the first, second, and third brackets stand for the HO, the QO, and the coupling term, respectively. We also have the commutation relations

$$[\hat{a}, \hat{a}^{+}] = [\hat{A}, \hat{A}^{+}] = 1 \tag{2.2}$$

$$[\hat{a}, \hat{A}] = [\hat{a}, \hat{A}^{+}] = [\hat{a}^{+}, \hat{A}] = [\hat{a}^{+}, \hat{A}^{+}] = 0 \tag{2.3}$$

Setting the canonical transformation (e.g., Merzbacher, 1977; Cohen-Tannoudji *et al.*, 1977)

$$\hat{a} = \frac{1}{(2\hbar)^{1/2}}(\hat{x} + i\hat{p}) \tag{2.4}$$

$$\hat{A} = \frac{1}{(2\hbar)^{1/2}}\left(\hat{X} + i\hat{P}\right) \tag{2.5}$$

and their corresponding adjoints, we obtain the Hamiltonian (2.1) in the form

$$\hat{H} = \left\{\frac{1}{2}\Omega\left(\hat{P}^{2} + \hat{X}^{2}\right)\right\} + \left\{\frac{1}{2m}\hat{p}^{2} + \frac{1}{2}k\hat{x}^{2}\right\} - \left\{\gamma\left(\hat{x}\hat{X} + \hat{p}\hat{P}\right)\right\} \tag{2.6}$$

where

$$m = (\omega - 2f)^{-1} \tag{2.7}$$

$$k = (\omega + 2f) \tag{2.8}$$

3. HEISENBERG EQUATIONS OF MOTION AND SOLUTIONS

From equation (2.6) we obtain the Heisenberg equations of motion

$$\dot{\hat{X}} = \Omega\hat{P} - \gamma\hat{p} \tag{3.1}$$

$$\dot{\hat{P}} = -\Omega\hat{X} + \gamma\hat{x} \tag{3.2}$$

$$\dot{\hat{x}} = \frac{\hat{p}}{m} - \gamma\hat{P} \tag{3.3}$$

$$\dot{\hat{p}} = -k\hat{x} + \gamma\hat{X} \tag{3.4}$$

This system of coupled differential equations can be decoupled through the transformation

$$\begin{bmatrix} \tilde{x} \\ \tilde{X} \\ \tilde{p} \\ \tilde{P} \end{bmatrix} = \begin{bmatrix} e^{\lambda/2}\cos\phi/2 & -e^{-\lambda/2}\sin\phi/2 & 0 & 0 \\ e^{\lambda/2}\sin\phi/2 & e^{-\lambda/2}\cos\phi/2 & 0 & 0 \\ 0 & 0 & e^{-\lambda/2}\cos\phi/2 & -e^{\lambda/2}\sin\phi/2 \\ 0 & 0 & e^{-\lambda/2}\sin\phi/2 & e^{\lambda/2}\cos\phi/2 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{X} \\ \hat{p} \\ \hat{P} \end{bmatrix} \tag{3.5}$$

where

$$\lambda = \frac{1}{2}\ln\left[\frac{m(k+\Omega)}{1+m\Omega}\right] \tag{3.6}$$

$$\phi = \arctan\frac{2\gamma}{ke^{-\lambda} - \Omega e^{\lambda}} \tag{3.7}$$

The application of the above transformation, which corresponds to a rotation plus a scale transformation, transforms the coupled Hamiltonian (2.6) into the uncoupled Hamiltonian

$$\hat{H} = \frac{1}{2}\left(\alpha_1\tilde{p}^2 + \beta_1\tilde{x}^2\right) + \frac{1}{2}\left(\alpha_2\tilde{P}^2 + \beta_2\tilde{X}^2\right) \tag{3.8}$$

where

$$\alpha_1 = \Omega e^{-\lambda}\sin^2\frac{\phi}{2} + \frac{1}{m}e^{\lambda}\cos^2\frac{\phi}{2} + \gamma\sin\phi \tag{3.9}$$

$$\beta_1 = \Omega e^{\lambda}\sin^2\frac{\phi}{2} + ke^{-\lambda}\cos^2\frac{\phi}{2} + \gamma\sin\phi \tag{3.10}$$

$$\alpha_2 = \Omega e^{-\lambda}\cos^2\frac{\phi}{2} + \frac{1}{m}e^{\lambda}\sin^2\frac{\phi}{2} - \gamma\sin\phi \tag{3.11}$$

$$\beta_2 = \Omega e^{\lambda}\cos^2\frac{\phi}{2} + ke^{-\lambda}\sin^2\frac{\phi}{2} - \gamma\sin\phi \tag{3.12}$$

Now, the Heisenberg equations of motion obtained from the uncoupled Hamiltonian (3.8) are similar to those given by equations (3.1)–(3.4) with $\gamma = 0$. Hence the solutions of our modified system are given by

$$\tilde{X}(t) = \tilde{X}(0)\cos\omega_2 t + \left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\tilde{P}(0)\sin\omega_2 t \tag{3.13}$$

$$\tilde{P}(t) = -\left(\frac{\beta_2}{\alpha_2}\right)^{1/2}\tilde{X}(0)\sin\omega_2 t + \tilde{P}(0)\cos\omega_2 t \tag{3.14}$$

$$\tilde{x}(t) = \tilde{x}(0)\cos\omega_1 t + \left(\frac{\alpha_1}{\beta_1}\right)^{1/2}\tilde{p}(0)\sin\omega_1 t \tag{3.15}$$

$$\tilde{p}(t) = -\left(\frac{\beta_1}{\alpha_1}\right)^{1/2}\tilde{x}(0)\sin\omega_1 t + \tilde{p}(0)\cos\omega_1 t \tag{3.16}$$

where $\omega_1 = (\alpha_1\beta_1)^{1/2}$, $\omega_2 = (\alpha_2\beta_2)^{1/2}$, whenever $\alpha_i\beta_i > 0$, $i = 1, 2$; otherwise hyperbolic or linear solutions will occur.

Next, the application of the inverse of transformation (3.5) to the solutions given by equations (3.13)–(3.16) results in the solution for our original coupled system. We find

$$\begin{aligned}\hat{X}(t) = &-\left[ad\left(\frac{\alpha_1}{\beta_1}\right)^{1/2}\sin\omega_1 t - bc\left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\sin\omega_2 t\right]\hat{p}(0)\\ &+\left[d^2\left(\frac{\alpha_1}{\beta_1}\right)^{1/2}\sin\omega_1 t + c^2\left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\sin\omega_2 t\right]\hat{P}(0)\\ &-cd[\cos\omega_1 t - \cos\omega_2 t]\hat{x}(0) + [bd\cos\omega_1 t + ac\cos\omega_2 t]\hat{X}(0)\end{aligned} \tag{3.17}$$

$$\begin{aligned}\hat{P}(t) = &\left[bc\left(\frac{\beta_1}{\alpha_1}\right)^{1/2}\sin\omega_1 t - ad\left(\frac{\beta_2}{\alpha_2}\right)^{1/2}\sin\omega_2 t\right]\hat{x}(0)\\ &-\left[b^2\left(\frac{\beta_1}{\alpha_1}\right)^{1/2}\sin\omega_1 t + a^2\left(\frac{\beta_2}{\alpha_2}\right)^{1/2}\sin\omega_2 t\right]\hat{X}(0)\\ &-ab[\cos\omega_1 t - \cos\omega_2 t]\hat{p}(0) + [bd\cos\omega_1 t + ac\cos\omega_2 t]\hat{P}(0)\end{aligned} \tag{3.18}$$

for the HO, and

$$\begin{aligned}\hat{x}(t) = &\left[a^2\left(\frac{\alpha_1}{\beta_1}\right)^{1/2}\sin\omega_1 t + b^2\left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\sin\omega_2 t\right]\hat{p}(0)\\ &-\left[ad\left(\frac{\alpha_1}{\beta_1}\right)^{1/2}\sin\omega_1 t - bc\left(\frac{\alpha_2}{\beta_2}\right)^{1/2}\sin\omega_2 t\right]\hat{P}(0)\\ &+[ac\cos\omega_1 t + bd\cos\omega_2 t]\hat{x}(0) - ab[\cos\omega_1 t - \cos\omega_2 t]\hat{X}(0)\end{aligned} \tag{3.19}$$

$$\begin{aligned}\hat{p}(t) = &-\left[c^2\left(\frac{\beta_1}{\alpha_1}\right)^{1/2}\sin\omega_1 t + d^2\left(\frac{\beta_2}{\alpha_2}\right)^{1/2}\sin\omega_2 t\right]\hat{x}(0)\\ &+\left[bc\left(\frac{\beta_1}{\alpha_1}\right)^{1/2}\sin\omega_1 t - ad\left(\frac{\beta_2}{\alpha_2}\right)^{1/2}\sin\omega_2 t\right]\hat{X}(0)\\ &+[ac\cos\omega_1 t - bd\cos\omega_2 t]\hat{p}(0) - ad[\cos\omega_1 t - \cos\omega_2 t]\hat{P}(0)\end{aligned} \tag{3.20}$$

for the QO, where

$$a = e^{-\lambda/2} \cos \frac{\phi}{2} \tag{3.21}$$

$$b = e^{-\lambda/2} \sin \frac{\phi}{2} \tag{3.22}$$

$$c = e^{\lambda/2} \cos \frac{\phi}{2} \tag{3.23}$$

$$d = e^{\lambda/2} \sin \frac{\phi}{2} \tag{3.24}$$

with λ, ϕ given by equations (3.6), (3.7).

4. FLUCTUATIONS IN QUADRATURE-PHASE AMPLITUDES

From the solutions given in equations (3.17)–(3.20) we obtain the fluctuations in quadrature-phase amplitudes ΔO, defined as

$$\Delta O(t) \equiv \langle \hat{O}(t)^2 - \langle \hat{O}(t) \rangle^2 \rangle^{1/2} \tag{4.1}$$

for an arbitrary operator $\hat{O} = \hat{X}$, $\hat{P}$, $\hat{x}$ or $\hat{p}$ and where the angle brackets stand for expectation values with respect to the eigenfunctions of the corresponding annihilation operators $\hat{a}$ or $\hat{A}$. We obtain

$$\begin{aligned}\Delta X(t) = \left(\frac{\hbar}{2}\right)^{1/2} \Bigg[& d^2(b^2 + c^2) \cos^2 \omega_1 t + c^2(a^2 + d^2) \cos^2 \omega_2 t \\ & + d^2 \frac{\alpha_1}{\beta_1} (a^2 + d^2) \sin^2 \omega_1 t + c^2 \frac{\alpha_2}{\beta_2} (b^2 + c^2) \sin^2 \omega_2 t \\ & + 2cd(ab - cd) \cos \omega_1 t \cos \omega_2 t \\ & + 2cd \left(\frac{\alpha_1 \alpha_2}{\beta_1 \beta_2}\right)^{1/2} (cd - ab) \sin \omega_1 t \sin \omega_2 t \Bigg]^{1/2}\end{aligned} \tag{4.2}$$

$$\begin{aligned}\Delta P(t) = \left(\frac{\hbar}{2}\right)^{1/2} \Bigg[& b^2(a^2 + d^2) \cos^2 \omega_1 t + a^2(b^2 + c^2) \cos^2 \omega_2 t \\ & + b^2 \frac{\beta_1}{\alpha_1} (b^2 + c^2) \sin^2 \omega_1 t + c^2 \frac{\beta_2}{\alpha_2} (a^2 + d^2) \\ & \times \sin^2 \omega_2 t + 2ab(cd - ab) \cos \omega_1 t \cos \omega_2 t \\ & + 2ab \left(\frac{\beta_1 \beta_2}{\alpha_1 \alpha_2}\right)^{1/2} (ab - cd) \sin \omega_1 t \sin \omega_2 t \Bigg]^{1/2}\end{aligned} \tag{4.3}$$

$$\Delta x(t) = \left(\frac{\hbar}{2}\right)^{1/2} \left[a^2(b^2+c^2)\cos^2\omega_1 t + b^2(a^2+d^2)\cos^2\omega_2 t \right.$$
$$+ a^2\frac{\alpha_1}{\beta_1}(a^2+d^2)\sin^2\omega_1 t + b^2\frac{\alpha_2}{\beta_2}(b^2+c^2)$$
$$\times \sin^2\omega_2 t + 2ab(cd-ab)\cos\omega_1 t\cos\omega_2 t$$
$$\left. + 2ab\left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}(ab-cd)\sin\omega_1 t\sin\omega_2 t \right]^{1/2} \tag{4.4}$$

$$\Delta p(t) = \left(\frac{\hbar}{2}\right)^{1/2} \left[c^2(a^2+d^2)\cos^2\omega_1 t + d^2(b^2+c^2)\cos^2\omega_2 t \right.$$
$$+ c^2\frac{\beta_1}{\alpha_1}(b^2+c^2)\sin^2\omega_1 t + d^2\frac{\beta_2}{\alpha_2}(a^2+d^2)$$
$$\times \sin^2\omega_2 t + 2cd(ab-cd)\cos\omega_1 t\cos\omega_2 t$$
$$\left. + 2cd\left(\frac{\beta_1\beta_2}{\alpha_1\alpha_2}\right)(cd-ab)\sin\omega_1 t\sin\omega_2 t \right]^{1/2} \tag{4.5}$$

The results for the variances expressed in equations (4.2)–(4.5) are plotted in Figs. 1–4 [in units of $(\hbar/2)^{1/2}$] for some values of the involved

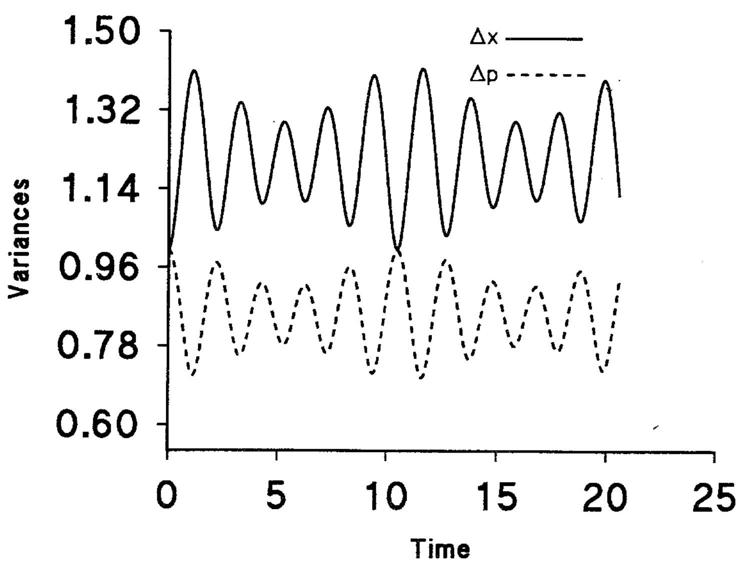

Fig. 1. Variances for the QO as a function of time, for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 0.2$.

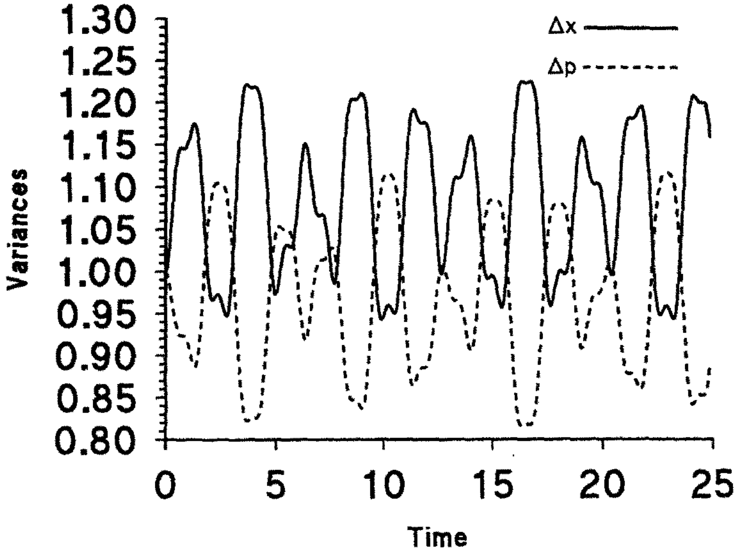

Fig. 2. Variances for the QO as a function of time, for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 2.0$.

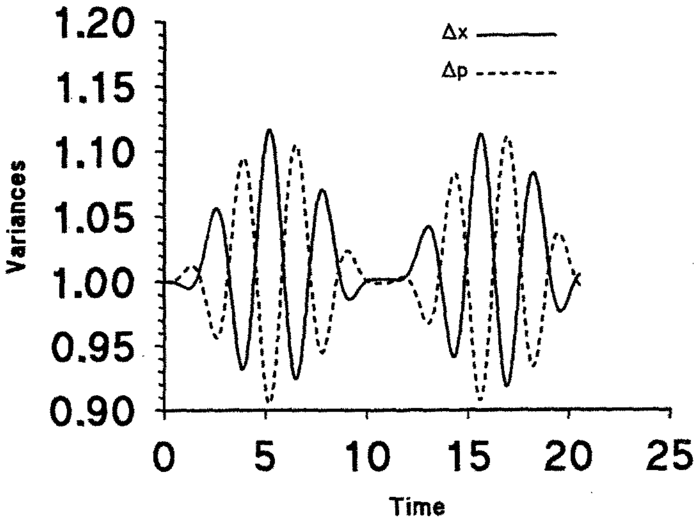

Fig. 3. Variances for the HO as function of time, for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 0.2$.

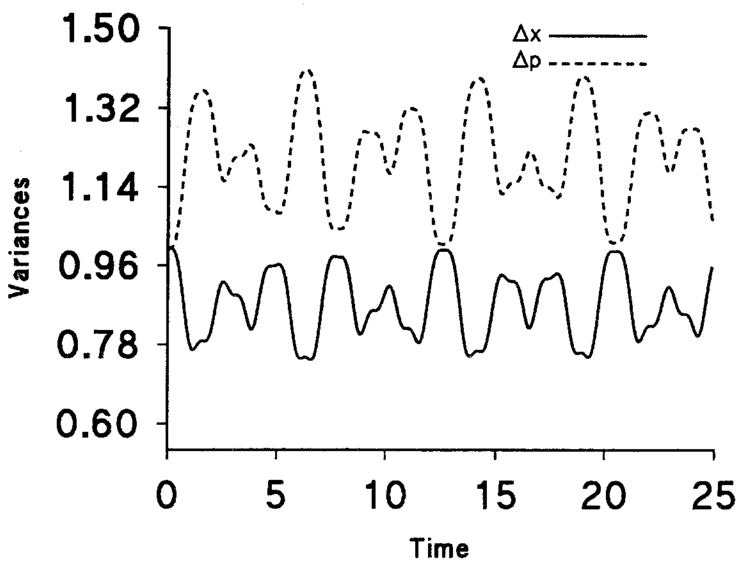

Fig. 4. Variances for the HO as function of time, for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 2.0$.

parameters Ω, ω, f, and γ, or equivalently for m, k, Ω, and γ, and for initial coherent states of both oscillators.

5. SQUEEZING IN TWO-MODE OSCILLATIONS

The results for the variances in Section 4 are those for the one-mode oscillations. In the present section we will investigate the possible occurrence of the squeezing effect for two-mode oscillations in our coupled system. According to some authors (Milburn, 1984; Caves and Schumaker, 1985), in much of the experimental realizations of squeezing phenomena, what is actually produced is two-mode squeezing, rather than the (more traditional) one-mode squeezing. While one-mode squeezing refers to this effect in an individual oscillator, two-mode squeezing occurs in a collective mode, which is a linear combination of variables of both oscillators.

The treatment of two-mode squeezing in the literature usually refers to the case where the effect is absent in the one-mode oscillation (Kim and Noz, 1991; Milburn, 1984; Caves and Schumaker, 1985). In this case two-mode squeezing only will emerge for a highly correlated state of the two oscillators that exhibits the reduced quadrature noise. As we will see here, that is not the case for our coupled system, where squeezing may appear simultaneously in one- and two-mode oscillations. Furthermore, as a consequence, both collective coordinates of the two-mode oscillation (as defined below) may also display the squeezing effect simultaneously, contrary to the traditional result where the occurrence of the effect in one of the collective coordinates excludes the other: reduction in one of them below the shot noise level causes amplification of the other coordinate.

The coordinates for two-mode oscillations are defined in the literature as (see, e.g., Milburn, 1984; Caves and Schumaker, 1985)

$$\hat{x}'(t) = \frac{1}{\sqrt{2}}(\hat{x}(t) + \hat{X}(t)) \tag{5.1}$$

$$\hat{X}'(t) = \frac{1}{\sqrt{2}}(\hat{x}(t) - \hat{X}(t)) \tag{5.2}$$

where $\hat{x}'(t)$ stands for the center-of-mass coordinate (C.M.) and $\hat{X}'(t)$ is the relative coordinate (R.C.).

The above definitions gives the coordinates for the two-mode oscillations in terms of those for one-mode oscillations. Using definition (4.1), and after some simple algebra, we find for the variances

$$\Delta x'(t) = \frac{1}{\sqrt{2}}\{(\Delta x(t))^2 + (\Delta X(t))^2 + 2[\langle x(t)X(t)\rangle - \langle x(t)\rangle\langle X(t)\rangle]\}^{1/2} \tag{5.3}$$

and

$$\Delta X'(t) = \frac{1}{\sqrt{2}}\{(\Delta x(t))^2 + (\Delta X(t))^2 - 2[\langle x(t)X(t)\rangle - \langle x(t)\rangle\langle X(t)\rangle]\}^{1/2} \tag{5.4}$$

By the use of equations (3.17)–(3.20) and some additional simple algebra we easily compute the variances of the two-mode oscillations

$$\begin{aligned}
\Delta x' = \left(\frac{\hbar}{4}\right)^{1/2}\Bigg\{&[(ac - cd)^2 + (db - ab)^2]\cos^2\omega_1 t \\
&+ [(bd + cd)^2 + (ac + ab)^2]\cos^2\omega_2 t \\
&+ [(a^2 - ad)^2 + (d^2 - ad)^2]\frac{\alpha_1}{\beta_1}\sin^2\omega_1 t \\
&+ [(b^2 + bc)^2 + (c^2 + bc)^2]\frac{\alpha_2}{\beta_2}\sin^2\omega_2 t \\
&+ 2[(ac - cd)(bd + cd) + (bd - ab)(ac + ab)]\cos\omega_1 t\cos\omega_2 t \\
&+ 2[(a^2 - ad)(b^2 + bc) + (d^2 - ad)(c^2 + bc)] \\
&\times \left(\frac{\alpha_1\alpha_2}{\beta_1\beta_2}\right)^{1/2}\sin\omega_1 t\sin\omega_2 t\Bigg\}^{1/2}
\end{aligned} \tag{5.5}$$

$$\begin{aligned}\Delta X' = \left(\frac{\hbar}{4}\right)^{1/2} \Bigg\{ & [(ac+cd)^2 + (db+ab)^2] \cos^2 \omega_1 t \\ & + [(bd-cd)^2 + (ac-ab)^2] \cos^2 \omega_2 t \\ & + [(a^2+ad)^2 + (d^2+ad)^2)] \frac{\alpha_1}{\beta_1} \sin^2 \omega_1 t \\ & + [(b^2-bc)^2 + (c^2-bc)^2] \frac{\alpha_2}{\beta_2} \sin^2 \omega_2 t \\ & + 2[(ac+cd)(bd-cd) + (bd+ab)(ac-ab)] \cos \omega_1 t \cos \omega_2 t \\ & + 2[(a^2+ad)(b^2-bc) + (d^2+ad)(c^2-bc)] \\ & \times \left(\frac{\alpha_1 \alpha_2}{\beta_1 \beta_2}\right)^{1/2} \sin \omega_1 t \sin \omega_2 t \Bigg\}^{1/2} \end{aligned} \qquad (5.6$$

where the symbols a, b, c, d, α_i, β_i, and ω_i were defined in Section 3.

These results are plotted, in units of $(\hbar/2)^{1/2}$, in Figs. 5–10, for some values of the parameters Ω, ω, f, and γ, or equivalently for m, k, Ω, and γ.

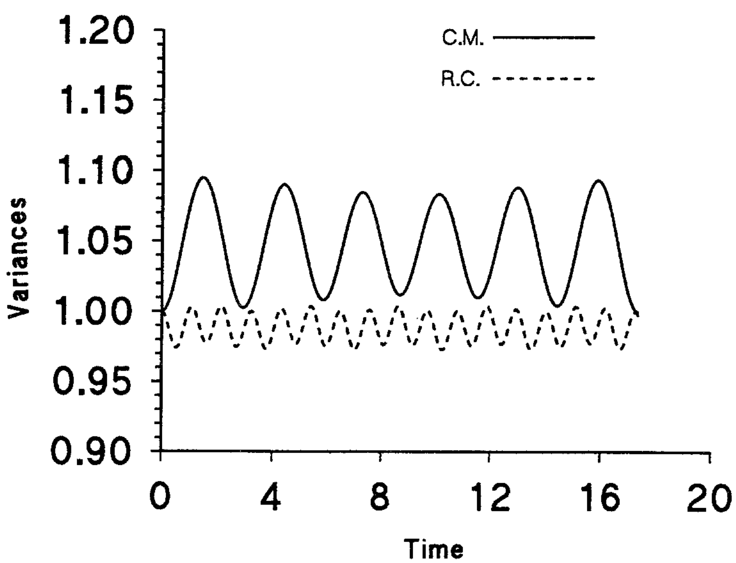

Fig. 5. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 1.5$, $k = 1$, $\Omega = 1$, and $\gamma = 2.0$.

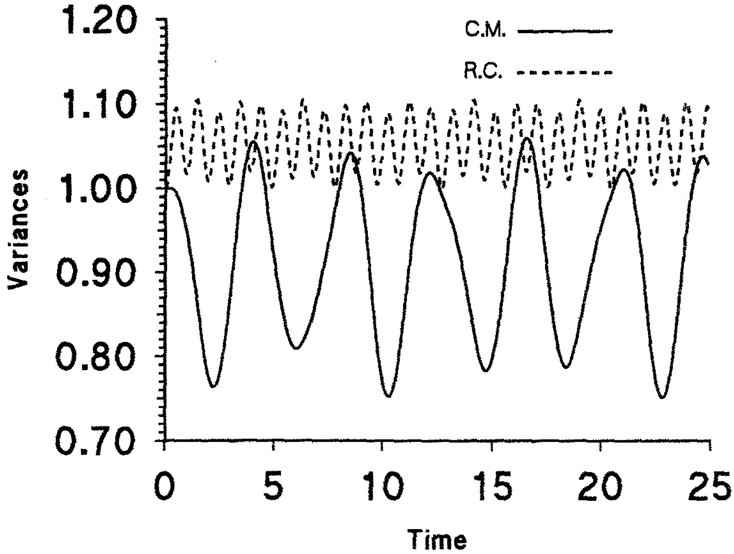

Fig. 6. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 2.0$.

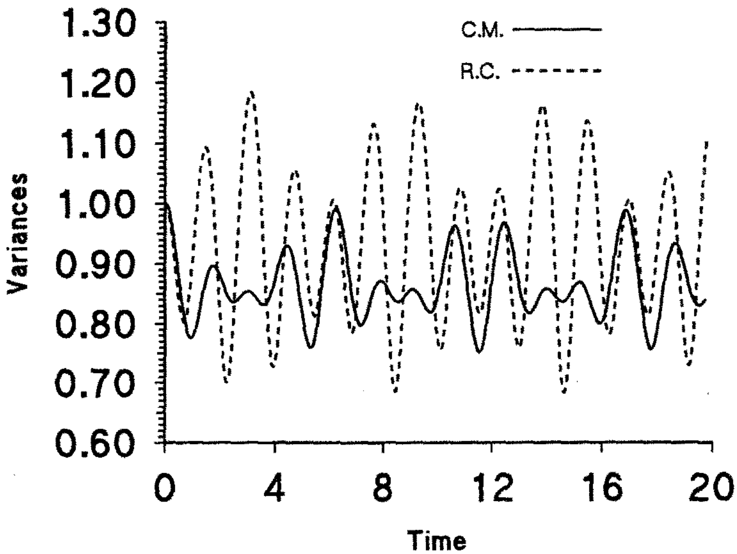

Fig. 7. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 1$, $k = 4$, $\Omega = 1$, and $\gamma = 0.2$.

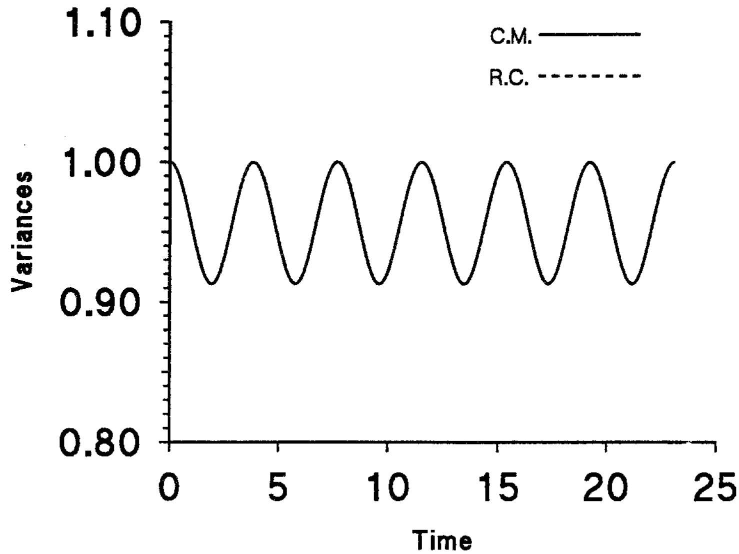

Fig. 8. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 1.5$, $k = 1$, $\Omega = 1$, and $\gamma = 0$.

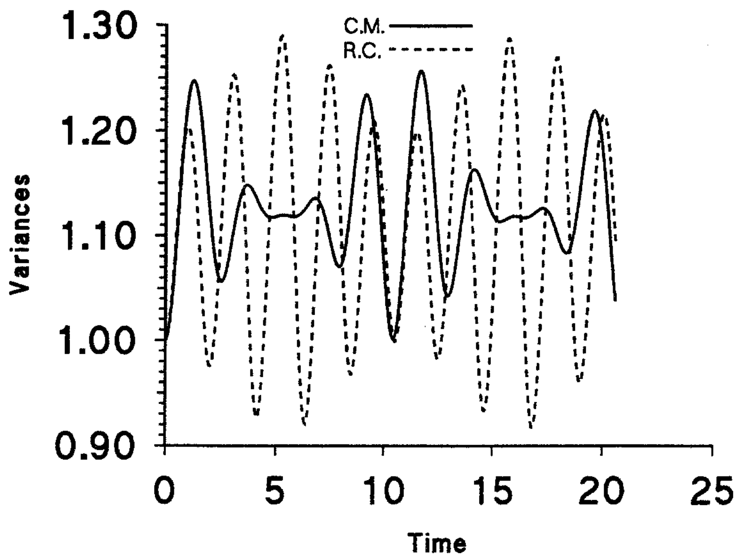

Fig. 9. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 0.2$.

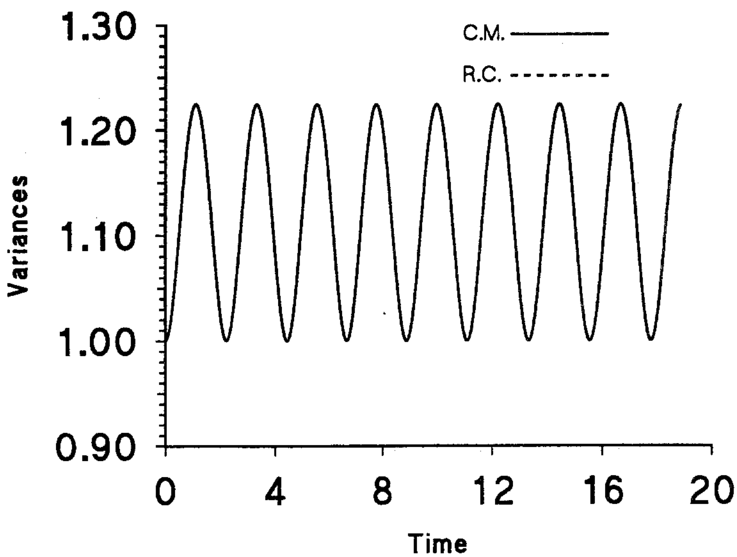

Fig. 10. Variances of center of mass (C.M.) and relative coordinate (R.C.) for parameter values $m = 0.5$, $k = 1$, $\Omega = 1$, and $\gamma = 0$.

6. COMMENTS AND CONCLUSION

We have computed the time evolution of one- and two-mode oscillations of a combined system constituted of a HO coupled to a QO. As is known, the HO does not exhibit a squeezing effect, whereas the QO does, when starting from a coherent state. We have shown that, due to the coupling, the squeezing generated in the QO (Figs. 1 and 2) spreads to the HO (Figs. 3 and 4).

For one-mode oscillation we point out the following features:

(i) If $f = 0$ in equation (2.1), then there is no squeezing in the QO. In this case there is no squeezing effect to be transferred to the HO, and both oscillators remain in their initial coherent state.

(ii) If $f \neq 0$, the QO exhibits a squeezing effect (Figs. 1 and 2). In this case the squeezing is transferred to the HO (Figs. 3 and 4). Note in Figs. 2 and 4 that if the squeezing generated in the QO alternates between both its quadratures, then the squeezing transferred to the HO remains in only one of its quadratures. However, in Figs. 1 and 3, for other values of the parameters, the reverse occurs.

For two-mode oscillations it is known that the squeezing can occur even if $x(t)$ and $X(t)$ are not themselves squeezed (Barnett and Knight, 1987). The minimum requirement for this to be possible is that the two coupled subsystems should be strongly correlated, since $\langle\hat{x}(t)\hat{X}(t)\rangle - \langle\hat{x}(t)\rangle\langle\hat{X}(t)\rangle$ must be sufficiently large and negative (positive) to reduce the variance of $\hat{x}'(t)$ $[\hat{X}'(t)]$ below the threshold for squeezing. In this case, the occurrence of the effect in one of the collective coordinates $\hat{x}'(t)$, $\hat{X}'(t)$ is accompanied by enhancement of noise in the other coordinate. However, if we have that $\hat{x}(t)$ and/or $\hat{X}(t)$ are themselves squeezed, as is the case in our coupled system, then the occurrence of squeezing in both collective coordinates $\hat{x}'(t)$, $\hat{X}'(t)$ may be concomitant (cf. Figs. 5, 6, and 8) or not (Figs. 7, 9, and 10). For two-mode oscillations we point out the following features:

(i) In Fig. 5 there is an almost permanent squeezing in one of the collective coordinates, but the effect scarcely occurs in the other coordinate, for all values of the time. In Fig. 6 this role played by the coordinates is reversed.

(ii) In Fig. 7 the squeezing affects both coordinates for almost all times, but the effect is not the same for $\hat{x}'$ and $\hat{X}'$. In Fig. 8, however, besides being permanent in time, the squeezing coincides exactly for both coordinates, the coupling being zero.

(iii) In Fig. 9 there is no squeezing for almost all times, but the enhancement of noise is not the same for $\hat{x}'(t)$ and $\hat{X}'(t)$. In Fig. 10, however, besides being permanent, the enhancement of noise coincides exactly for both coordinates, the coupling being zero.

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