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Connections between Abstract Quantum Theory and Space–Time Structure II. A Model of Cosmological Evolution

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By ur-theoretic and general relativistic arguments, a new cosmological model is introduced which avoids most well-known cosmological "problems."

Nature is spirit who does not know himself to be spirit —Schelling
Modern cosmology in myth which does not know itself to be myth

1. INTRODUCTION

One of the aims of the ur theory (Weizsäcker, 1971, 1985) is to explain the possible connections between cosmology and elementary particle physics (Görnitz, 1986).

In the first part of the present work (Görnitz, 1988, henceforth referred to as I), I treated the global structure of space by ur-theoretic considerations and also introduced a smallest physically accessible length. Thie enterprise led to a cosmography, a description of a cosmic model *at a fixed time.* Of course, there must also be a description of its time development, of the change of the number of urs with time. There has been no *a priori* assumption for this process. Here an attempt will be made to treat this evolution by means of general relativity and to compare the result with observations. This means that general relativity is used as an existing and adequate theory; it is hoped that general relativity can be reconstructed from ur theory.

As in I, Planck–Wheeler units are used. Again only orders of magnitude are sought, so factors of order $10^{\pm 1}$ will be ignored if possible.

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2. COSMOLOGY AND UR THEORY

2.1. Energy Conditions

In I it was shown that an ur corresponds to an energy of $1/R$ if there are $N = R^2$ urs in the universe. So the total energy of all the urs is $U = N(1/R) = R$ and the energy density is

$$\mu = U/R^3 = 1/R^2 \tag{1}$$

In a first approximation we assume that the urs behave like an ideal fluid and that the first law of thermodynamics holds for this system. It is an *isolated* system in the sense of thermodynamics (perhaps the only one) and we get

$$dU + p\,dV = 0 \tag{2a}$$

or

$$(1 + p \cdot 3R^2)\,dR = 0 \tag{2b}$$

In a realistic cosmological model R cannot be constant, i.e.,

$$dR \neq 0$$

i.e., for the pressure as a function of R it follows from (2) that

$$p = -1/(3R^2) = -\mu/3 \tag{3}$$

A negative pressure in physics is always a hint for instability—which an evolving cosmos will show indeed.

Usually one argues against a negative pressure (e.g., Hawking and Ellis, 1973, pp. 137) that "it is reasonable to assume p is non-negative" in a cosmological model. But the restrictions on energy and pressure (Hawking and Ellis, 1973, p. 90), which are reasonable in general relativity, do not restrict p to positive values: There is the dominant energy condition

$$\mu \geq 0, \qquad \mu \geq p \geq -\mu \tag{4}$$

which implies that matter cannot travel faster than light. For the more restrictive, so-called strong energy condition

$$\mu + 3p \geq 0 \tag{5}$$

an equally good physical interpretation has not been given. The strong energy condition [which (3) also satisfies] guarantees the existence of a singularity for the model.

There is no *a priori* need for the ur-theoretic "ideal fluid" to behave like already known types of matter which possess a positive pressure. To my knowledge, a negative pressure in a cosmological model was first invented by McGrea (1951). He explained that such a uniform pressure cannot be directly observed because only the *gradient* of a pressure can have mechanical effects and the gradient of such a uniform pressure vanishes everywhere.

2.2. The Model

In I an $\mathbb{S}^3$ was introduced as the model for cosmic space. The evolution of this cosmos will be described by a changing number of urs. It is mathematically plausible that, beginning with a finite number of urs, the average number of urs will always be growing. This growth expresses the "open future" (Weizsäcker, 1971).

As a model for this cosmos we get $\mathbb{R}^+ \times \mathbb{S}^3$ with a metric

$$ds^2 = dt^2 - R^2(t)[(1-r^2)^{-1}\,dr^2 + r^2\,d\Omega^2)] \tag{6}$$

i.e., a Robert–Walker cosmos with a function $R(t)$ which is so far undetermined. This function can be evaluated by the assumption that our model should not contradict general relativity.

For an isotropic fluid with an energy-momentum tensor given by equation (3), i.e.,

$$T_i^k = \text{diag}(\mu, -p, -p, -p) = \text{diag}(\mu; \mu/3, \mu/3, \mu/3) \tag{7}$$

Einstein's equation

$$G_i^k = -\kappa T_i^k \tag{8}$$

(κ is Einstein's gravitational constant) reduces to

$$\kappa\mu = 3[1 + (dR/dt)^2]/R^2 \tag{9}$$

$$-\kappa p = [1 + (dR/dt)^2 + 2R(dR^2/dt^2)]/R^2 \tag{10}$$

which is equivalent to

$$(dR^2/dt^2) = 0 \tag{11}$$

Equation (11) has the solution

$$R(t) = R(0) + vt \tag{12}$$

So $\mu(t)$ and $p(t)$ are given by

$$\mu(t) = 3\kappa^{-1}(1+v^2)/[R(0)+vt]^2 = 3(1+v^2)/R^2 \tag{13}$$

$$p(t) = -\kappa^{-1}(1+v^2)/[R(0)+vt]^2 = -(1+v^2)/R^2 \tag{14}$$

(In Planck–Wheeler units, κ has the value 1.)

The cosmological time t goes from 0 to ∞. Here $R(0)$ is the cosmic radius at $t=0$. If at this time the cosmic evolution starts with one ur, $R(0)$

should be of the order 1.[2] The constant expansion velocity v will certainly become the velocity of light, i.e., 1 in our units.

The consequences of such a linearly evolving cosmic model will be discussed in Section 3. The energy-momentum tensor with μ and p from (13) and (14), respectively, will be called ${}_{(\mathrm{ur})}\mathrm{T}_i^k$.

2.3. An Effective Energy-Momentum Tensor

All known types of matter have an energy-momentum tensor different from (3), resp. (7). By introducing an effective cosmological constant we can reestablish the generally used energy-momentum tensors. To do this, we decompose ${}_{(\mathrm{ur})}T_i^k$ into a sum of energy-momentum tensors for matter, light, and vacuum:

$$ {}_{(\mathrm{ur})}T_i^k = {}_{(\mathrm{matter})}T_i^k + {}_{(\mathrm{light})}T_i^k + {}_{(\mathrm{vacuum})}T_i^k \tag{15} $$

or

$$ \begin{bmatrix} \mu & & & \\ & \mu/3 & & \\ & & \mu/3 & \\ & & & \mu/3 \end{bmatrix} = \begin{bmatrix} \mu_m & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} + \begin{bmatrix} \mu_l & & & \\ & -\mu_l/3 & & \\ & & -\mu_l/3 & \\ & & & -\mu_l/3 \end{bmatrix} + \begin{bmatrix} \lambda & & & \\ & \lambda & & \\ & & \lambda & \\ & & & \lambda \end{bmatrix} \tag{16} $$

[2]Here a clarifying remark is needed. "Starts with one ur" can only be used in a metaphorical way. Ur-theory is based on the reconstruction of abstract quantum theory (Weizsäcker, 1985; Drieschner, *et al.*, 1987), i.e., on the concept of separable alternatives. If in the earliest times of cosmic evolution only few alternatives are present, its separability is a quite doubtful concept. So we think that the "start" of the cosmic evolution is beyond the range of all known physical theories. This is indicated on one side by the singularity theorems, which show a breakdown of general relativity, and also by the above-mentioned breakdown of quantum theory. It means that our mathematical frames as well as our scientific concepts become inapplicable. It should not be absolutely impossible to think about the problems related to the beginning of the universe. But it seems plausible that a solution of these problems cannot be reached, in fact certainly not in the frame of today's scientific rationality (see, e.g., Weizsäcker, 1943).

Calling δ the energy density ratio between matter and light

$$\mu_m/\mu_l = \delta \tag{17}$$

then (16) results in

$$\mu = (1+\delta)\mu_m = \lambda \tag{18}$$

and

$$\mu = -\delta_m + 3\lambda \tag{19}$$

So we get

$$\mu_m = \mu \cdot 2\delta/(3+4\delta), \qquad \mu_l = \mu \cdot 2/(3+4\delta), \qquad \lambda = \mu \cdot (\delta+2)/(3+4\delta) \tag{20}$$

or with μ from (13)

$$\mu_m = 6\delta(1+v^2)/\{(3+4\delta)[R(0)+vt]^2\} \tag{21}$$

$$\mu_l = 6(1+v^2)/\{(3+4\delta)[R(0)+vt]^2\} \tag{22}$$

$$\lambda = 3(\delta+2)(1+v^2)/\{(3+4\delta)[R(0)+vt]^2\} \tag{23}$$

Introducing an effective Einstein equation at a chosen time t' constructed with an effective cosmological constant

$$_{\text{eff}}\Lambda = -\lambda(t')$$

we find

$$_{\text{eff}}G_i^k + {_{\text{eff}}\Lambda}\delta_i^k = -\kappa[_{(\text{matter})}T_i^k + {_{(\text{light})}T_i^k}] \tag{24}$$

Of course, by a supposed constancy of $_{\text{eff}}\Lambda$, this equation is valid only for cosmological times around the value t'.

2.4. An Interpretation for the Cosmological Constant

The cosmological constant Λ is usually understood as the vacuum energy density of the universe (Deser, 1982). Of course, this interpretation leads to strange properties of the "vacuum" such as a pressure that for each component is as large as the negative energy density. But only in this case will local Lorentz invariance result.

In our units the observational value is $\Lambda \leq 1/R^2$. The smallness of this value is one of the most serious problems of modern cosmology (Hawking, 1982). If Λ is indeed a *constant*, its fine tuning is hard to explain. On the other hand, in the model given above, the right order of Λ follows straightforwardly without any use of such doubtful inventions as the anthropic principle.

For quantum theory a central point is the existence of a ground state and further its dependence on the system's extension. We think that the above interpretation for the cosmological constant reflects the quantum physical ground-state properties. But as a *constant*, Λ cannot be affected by the expansion of the system, i.e., in the present case the expansion of the universe.

In our model the "vacuum" represents the *ground-state* part of the urs in the universe, i.e., that part that is not expressed by massless or massive particles such as light or matter. Because the model is in essence a quantum-theoretic one, λ is found to be a function of R, resp. of time: $\lambda = \lambda(t)$. So it reflects the extension of the system, whereas ${}_{\text{eff}}\Lambda$ as a constant is only a derived and effective quantity.

3. DISCUSSION

Yoshimura (1986) has explained that standard big bang cosmology gives a coherent picture of our universe with a parameter range $t_0 \approx H_0^{-1}$ [t_0 = age of the universe, H_p = Hubble parameter today, $H(t) = (dR/dt)/R$] and $\Omega_0 \approx 1$ (mass-energy density approximately equal to the critical value for a closed universe). This means that our description of a linearly expanding closed universe may be fairly good.

In the usual standard big bang cosmology some fundamental problems are left unsolved:

1. *The baryon asymmetry* (i.e., that there are baryons, but no antibaryons present) "is quantified by the number ratio of baryons to photons $\approx 10^{-10}$."

In I it was shown that this ratio may be explained by statistical considerations alone. But the real and serious problem is not only a possible explanation of this single number, because the general particle–antiparticle puzzle is not restricted to baryons and is connected with the central problem of the sharp rest masses of only a few fundamental particles.

In Görnitz and Weizsäcker (1986) an attempt for a solution of the particle problem of a closed cosmic space is made in ur-theoretic framework. But the problem of the sharp rest masses is not yet solved, neither in ur theory nor, to my knowledge, in any other one.

2. *The horizon problem* is connected with the homogeneity assumed in the standard model, which is also found in the microwave background.

In the standard model there is a particle horizon $\sim ct$ which forbids causal connections of regions of different directions in the sky in earlier times. But in observations those regions show the same temperature. This problem disappears in a model like ours expanding with $\sim ct$ and possessing no cosmological horizons at all.

3. *The oldness or flatness problem* is related to the fact that in the standard big bang cosmology a very complicated fine tuning for the density is needed to avoid a very early recollapse of the universe (oldness problem) and to give to the universe today the flatness given by the empirical value.

In our model there is no possibility for recollapsing and at the present age of about 10^{60} Planck times the space has to be as flat as it is today.

4. *The cosmological constant* has been discussed in Section 2.4 and is no problem in our model.

5. *The formation of structures* indicates the cosmological origin of galaxies and clusters of galaxies. This problem has not yet been investigated in an ur-theoretic framework. But the concept of hypermanifolds as explained in I opens the possibility of explaining the large cosmic structures. It may be possible that these structures were formed earlier than the particles known today.

6. *The initial singularity* as a starting point for any big bang cosmology seems to us not to be a weakness of the theory. On the contrary, it is a clear reminder of the central problem of any attempt at a cosmology—that is, treating the whole universe as an object for humankind.

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