NUMERICAL SIMULATIONS OF FAST MHD WAVES IN A CORONAL PLASMA

II. *Impulsively Generated Linear Waves*

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Abstract. The temporal evolution of impulsively generated waves in solar coronal loops is studied in the framework of linearized low β MHD by means of numerical simulations. Loops are approximated by smoothed slabs of gas density in an otherwise uniform magnetic field. The simulations show that there is an energy leakage from the slab, associated with the propagation of wave packets which exhibit both periodic, quasi-periodic and decay phases. The quasi-periodic phase possesses the strongest amplitudes and is thus most likely to correspond with observed time scales.

1. Introduction

The solar corona is viewed as made up of myriads coronal loops, the transverse dimensions of which are shorter than longitudinal ones (e.g., Golub, 1990). Such loops support a wide variety of pulsations (e.g., Tapping, 1978; Roberts, Edwin, and Benz, 1984, and references therein; Aschwanden, 1987; Pasachoff, 1990). There have been a number of recent observational reports of coronal oscillations. For example, Ren-Yang *et al.* (1990) and Qi-Jun *et al.* (1990) found quasi-periodic oscillations with characteristic time scales 1.4–1.6 s in the spike radiation of the outburst observed on May 16, 1981, and concluded that the observed oscillations correspond to MHD waves propagating inside and outside a loop. Ren-Yang *et al.* and Qi-Jun *et al.* concluded that their observations were of a similar form to that predicted by the theory of Roberts, Edwin, and Benz (1983, 1984). Moreover, Pasachoff (1990) has shown that the results matched the calculations of that time showing that there could be short-period (1 s) waves and the models of trapped fast MHD waves suggested by Roberts, Edwin, and Benz (1984). In the event of the type IV burst which took place on April 3, 1980, regular (periodic), irregular (quasi-periodic), and decay phases were shown (Kurths and Karlický, 1989). Again, the theory developed by Roberts, Edwin, and Benz (1984) has been used to interpret this event. Moreover, a doubly-periodic phase has been observed and it has been supposed that the disturbance was generated by two impulses (Kurths and Karlický, 1989).

Short-period oscillations, with periodicities of ~ 1 s, have long been known from radio observations (see Krüger, 1979, for a general review). The occurrence of short-period (0.5–3.0 s) oscillations in Type IV radio bursts is well known (e.g., Tapping, 1978; Trottet *et al.*, 1981). Similar short period oscillations have also been reported in microwaves and in hard X-rays (Takakura *et al.*, 1983; Kane *et al.*, 1983). Decimetric quasi-periodic radio pulsations have been found to be signatures of energetic electrons trapped in coronal loops (Aschwanden, Benz, and Kane, 1990). Magnetosonic waves may also be involved in the acceleration of electrons in flaring loops (de La Beaujardiere and Zweibel, 1989). Long-period (~ 1 min) radio pulsations have been related by Trottet, Pick, and Heyvaerts (1979). Oscillations with periods of 43 s, 80 s, and 5 min have recently been reported by Koutchmy, Zhugzhda, and Locans (1983).

Švestka (1990) has applied the theory developed by Roberts, Edwin, and Benz (1984) to explain a quasi-periodicity (of about 18 min) of flaring arches on 6 November, 1980 and found good agreement with experimental data. Pulsations, which originate from an active region lying at the footpoint of a large coronal loop, of period 24 min have been detected by Harrison (1987) and thought to be caused by a standing or travelling magnetohydrodynamic waves.

The purpose of this paper is to investigate numerically impulsively generated waves in smoothed density enhancements, representative of coronal loops. We are particularly interested in the time scales associated with impulsive waves, and the energy leakage that may occur from a loop. As far as we are aware, no numerical simulations of impulsively excited fast waves in an inhomogeneous medium have been studied previously.

The general equations for fast magnetoacoustic waves that have no y-dependency and $V_y \equiv V_z \equiv 0$ in a zero-β plasma with a constant magnetic field are laid out in Paper I (Murawski and Roberts, 1993), and lead to the wave equation

$$\frac{\partial^2 A}{\partial t^2} = V_{\mathrm{A}}^2(x)\nabla^2 A \tag{1.1}$$

for a vector magnetic potential $\mathbf{A} = A(x, z, t)\hat{\mathbf{y}}$ and Alfvén speed $V_{\mathrm{A}}(x)$. The magnetic field is given by

$$\mathbf{B} = \nabla \times \mathbf{A} = \left(-\frac{\partial A}{\partial z},\ 0,\ \frac{\partial A}{\partial x} \right), \tag{1.2}$$

with the unperturbed magnetic field being uniform ($\mathbf{B} = \mathbf{B}_0 = B_0\hat{\mathbf{z}}$).

Roberts, Edwin, and Benz (1984) studied the behaviour of fast waves in a coronal density inhomogeneity of slab or cylinder form, and included the effects of a finite gas pressure. Considering a top hat density profile, they showed that an impulsively generated fast magnetosonic wave exhibits three phases in its temporal signature. Imagine that a sudden pressure or velocity disturbance occurs at time $t = 0$ at the location $z = 0$ in the centre of the density enhancement. Then, subsequently, pressure variations at a location $z = h$ along the duct begin at a time $t = h/V_{\mathrm{A}e}$, where $V_{\mathrm{A}e}$ denotes the Alfvén speed in the environment of the density duct. This is the start of the *periodic phase*. The periodic phase continues until a time $t = h/V_{\mathrm{A}}$, where V_{A} is the Alfvén speed within the duct, when it suddenly gives way to the *quasi-periodic phase*. This phase lasts till a time $t = h/c_g^{\min}$ is the minimum value of the group velocity of the fast wave (for a detailed discussion see Roberts, Edwin, and Benz, 1984). Time scales τ_p of the periodic phase are generally short, being of the order of

$$\tau_p = \frac{4a}{V_A}\left(1 - \frac{\rho_e}{\rho_0}\right)^{1/2} \tag{1.3}$$

for a slab of width $2a$ and density ρ_0 in an environment of density ρ_e.

The quasi-periodic phase is of larger amplitude than the earlier periodic phase. Its time scale ($\tau_{\min} < \tau_p$) is associated with a minimum in the group velocity. Its duration τ_d is determined by the gas density enhancement:

$$\tau_d = h\left(\frac{1}{c_g^{\min}} - \frac{1}{V_A}\right). \tag{1.4}$$

Finally, the event passes the location $z = h$ and a *decay phase* ensues. Roberts, Edwin, and Benz (1984) argue that distinctive signatures, produced by a solar flare, reconnection events or local MHD instability, are in fact evident in the observed radio pulsations.

This description of the three phases of an impulsively generated fast wave propagating in a density duct is based upon the case of a top hat profile. Here we consider numerically the case of a smooth gas density profile $\rho_0(x)$, that is sharply peaked around a slab boundary. Specifically we take (as in Paper I) the profile

$$\rho_0(x) = \begin{cases} \rho_0, & |x| \le a, \\ \rho_e + (\rho_0/\rho_e - 1)\rho_e/\cosh^s(x-a), & x > a, \end{cases} \tag{1.5}$$

representing a peak density of ρ_0 at $x = 0$ confined within $|x| = a$ and falling rapidly to ρ_e outside the slab (see Figure 2(a) of Paper I).

2. Numerical Results

The numerical code utilizes the fast Fourier transform method in space and the second-order Runge–Kutta method in time. See Paper I for a description. For both the x- and z-directions, 128 Fourier modes have been used. The simulation region was defined by $-32 \le x < 32$ and $-32 \le z < 32$, using the half-width a of the slab as a unit of length. The temporal step length has been chosen as small as possible to preserve numerical accuracy. As a check on numerical results, we have calculated the integrals of motion, the mass M and the energy E:

$$\begin{aligned} M &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho \, dx \, dz, \\ E &= \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\rho_0 V^2 + \frac{1}{\mu}\left(\frac{\partial A}{\partial x}\right)^2 + \frac{1}{\mu}\left(\frac{\partial A}{\partial z}\right)^2\right] dx \, dz. \end{aligned} \tag{2.1}$$

Errors in the calculations were less than 2%.

To discuss the numerical results we introduce the slab energy $e(t)$, defined as follows:

$$e(t) = \int_{-32}^{32-\Delta z} \left\{ \int_{-1}^{1} \left[\rho_0 V^2 + \frac{1}{\mu}\left(\frac{\partial A}{\partial x}\right)^2 + \frac{1}{\mu}\left(\frac{\partial A}{\partial z}\right)^2 \right] \mathrm{d}x \right\} \mathrm{d}z \,, \tag{2.2}$$

where Δz is the step length in the z-direction.

In this paper we consider the initial profile

$$A(x, z, t = 0) = \frac{A_0}{\cosh^2(x - x_0)\cosh^2(z - z_0)} \,, \tag{2.3}$$

where A_0 is the amplitude of a magnetic potential A centred around the point (x_0, z_0).

Consider first the case of an initial disturbance that is centred on the density inhomogeneity, so that $x_0 = z_0 = 0$. We take a density ratio of $\rho_0/\rho_e = 5$. The initial pulse is shown in Figure 1(a). It disperses almost symmetrically both in the x- and z-directions and a dip in its centre is created in the early stages of its time evolution (Figure 1(b)). The dip is, however, anti-symmetric due to the different values of Alfvén speeds inside and outside the slab; perturbations outside the slab begin to propagate with the external Alfvén speed, V_{Ae}, which is larger than the Alfvén speed V_A of the oscillations inside the slab. Symmetry in our diagrammatic representation of $A(x, z; t)$ is violated by trapped oscillations in the slab (Figure 1(c)). Trapped oscillations are evident in the two large amplitude humps and valleys which propagate along the slab in opposite direc-

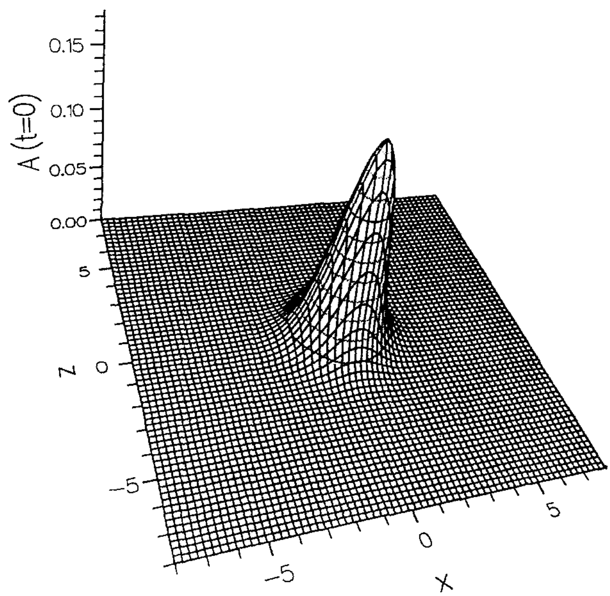

Fig. 1a.

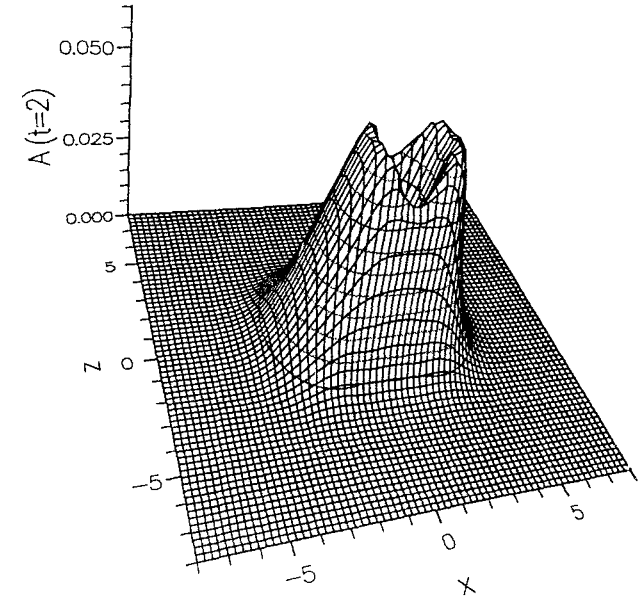

Fig. 1b.

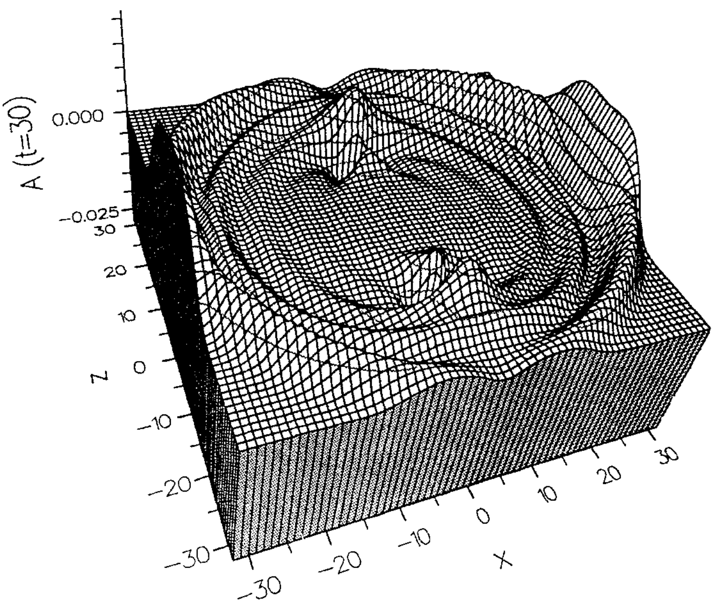

Fig. 1c.

Fig. 1. Magnetic potentials $A(x, z, t)$ of impulsively generated (at $x = z = 0$) waves propagating in a smoothed slab $\rho(x)$ with a gas density ratio $\rho_0/\rho_e = 5$ at times (a) $t = 0$, (b) $t = 2$, (c) $t = 30$. The time is in units of the Alfvén transit time, a/V_A. Notice in (c) the two large amplitude pulses propagating along the axis $x = 0$ and corresponding to trapped waves. Outwardly propagating waves are represented by circular wavy patterns.

tions. Outwardly propagating ripples have circular patterns and propagate faster than trapped waves.

The phenomenon of wave trapping is also apparent in profiles of energy density. Initially, the energy density is well localized around the point $x = z = 0$ (see Figure 2(a)). Shortly thereafter the process of wave trapping begins (sse Figure 2(b)); here additional

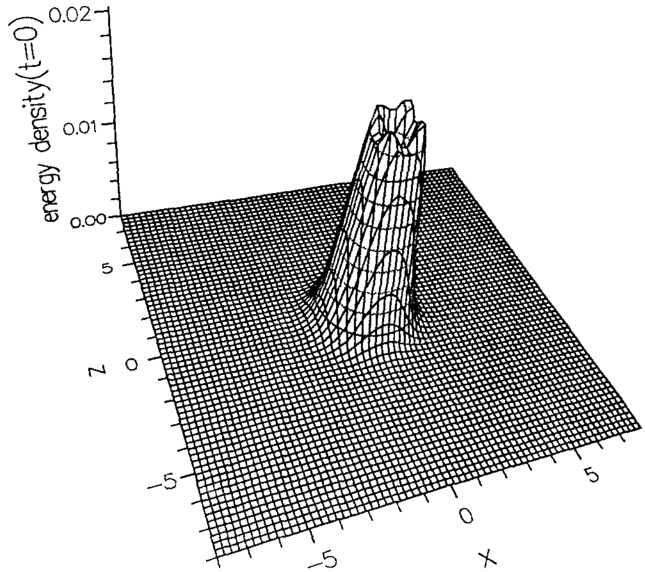

Fig. 2a.

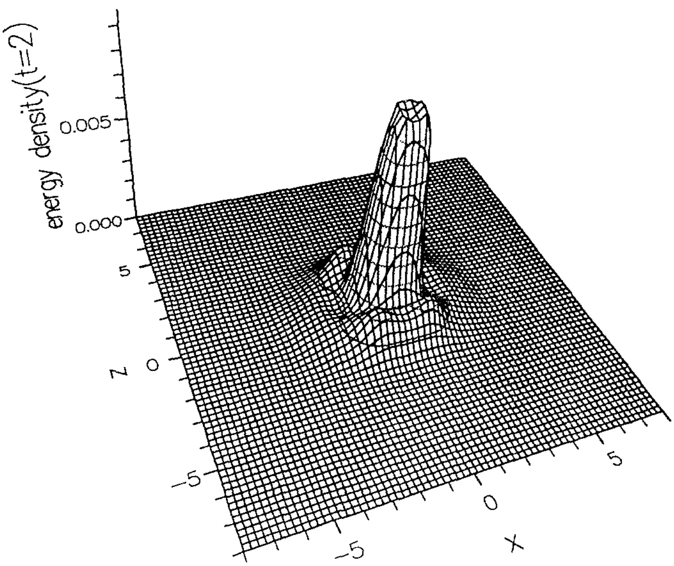

Fig. 2b.

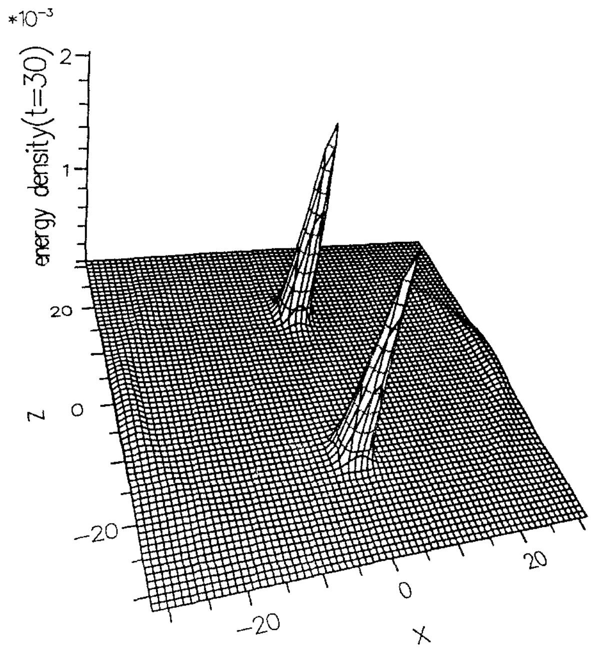

Fig. 2c.

Fig. 2. The wave density profile in impulsively generated (at $x = z = 0$) waves propagating in a smoothed slab with a density ratio $\rho_0/\rho_e = 5$ at times (a) $t = 0$, (b) $t = 2$, (c) $t = 30$. The two pulses apparent in (c) correspond to trapped waves propagating in opposite directions. Circular-like wavy patterns are associated with outwardly propagating waves. Notice that because of $V_{Ae} > V_A$ the trapped waves propagate slower than the outwardly propagating ripples.

small humps propagating along the slab can be seen. The central hump transfers its energy to the already created small humps and as a result they grow in amplitude. This process is more pronounced by $t = 30a/V_A$ (see Figure 2(c)). The central hump has now dispersed and only two pulses, corresponding to trapped waves remain. At the same time energy leakage takes place, corresponding to outwardly propagating waves which appear as small circular patterns.

The combined effects of wave ducting and outward propagation (leakage) from the slab means that only a fraction of the initial energy of the impulsive disturbance remains within the slab. For the circumstances of Figure 2, about 30% of the initial wave energy remains within the slab. This is illustrated in Figure 3 where we have plotted the energy $e(t)$ (defined by (2.2)) as a function of time. Notice the sudden drop of the energy at the initial stage, lasting for about 2 time units, and its subsequent small amplitude oscillations with a period equal to about 10 Alfvén transit times. This phenomenon has been also observed in periodic pulsations (see Paper I).

Figure 3 also shows the behaviour of the potential A, calculated at the point $x = 0$, $z = 8a$. The three phases determined by Roberts, Edwin, and Benz (1983, 1984) for a sausage mode may be distinguished. Oscillation amplitudes in the periodic phase are

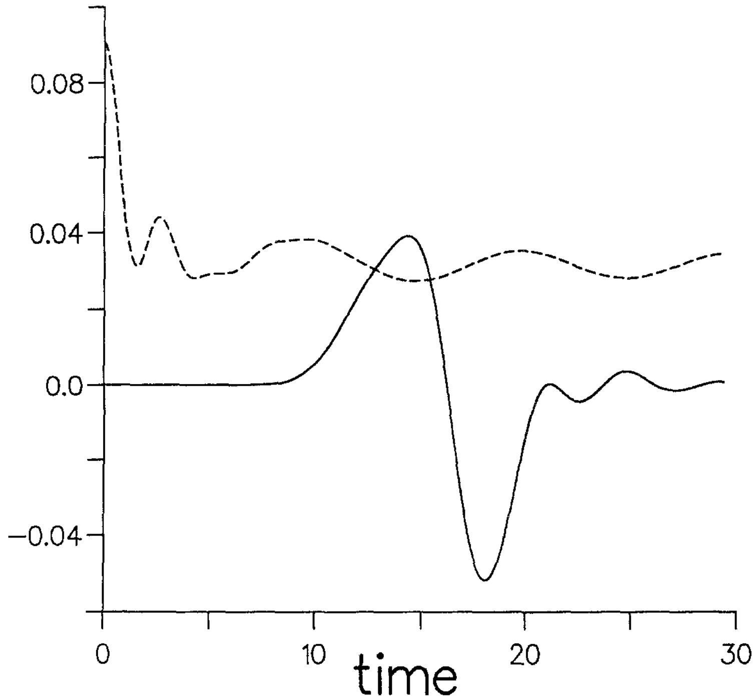

Fig. 3. The time signature of impulsively generated (at $x = z = 0$) waves in a slab with $\rho_0/\rho_c = 5$. Time is in units of the Alfvén transit time, a/V_A. The broken line denotes the slab energy $e(t)$ and solid line denotes $A(x = 0, z = 8a)$. The potential A exhibits three phases of activity with a very small amplitude periodic phase (for $t \leq 8$) represented here by a straight horizontal line. The energy suddenly drops (at $t \simeq 2$) below its initial level to approach finally a stationary state.

so small that the periodic phase is represented by the straight horizontal line for $t \leq 8a/V_A$. Then, a quasi-periodic phase abruptly begins with a large hump and a valley and small amplitude disturbances within it. Such a sudden onset of the quasi-periodic phase is reminiscent of the event observed by McLean *et al.* (1971) and McLean and Sheridan (1973). (A smoother transition from the periodic to the quasi-periodic phase occurs in the case of steeper initial profiles.) The duration of the quasi-periodic phase is about 15 Alfvén transit times. In the corona with $V_A = \sqrt{5} \times 10^3$ km s^{-1} and $2a = 1500$ km, this corresponds to a duration time of about 10 s. There is also a time scale associated with a single pulse. This scale is of the order of 5 Alfvén transit times, corresponding to an oscillation with a period of about 4 s. However, this oscillation time scale varies in the quasi-periodic phase; at its end it is of the order of 2 time units (1–2 s oscillations).

The case of a weaker slab exhibits a similar behaviour. Now, however, disturbances are less efficiently trapped by the slab. About 25% of the initial energy is left in the slab for the case $\rho_0/\rho_e = 2$, illustrated in Figure 4. The time dependence of A allows us to distinguish the three phases; the duration time of the quasi-periodic phase is shorter.

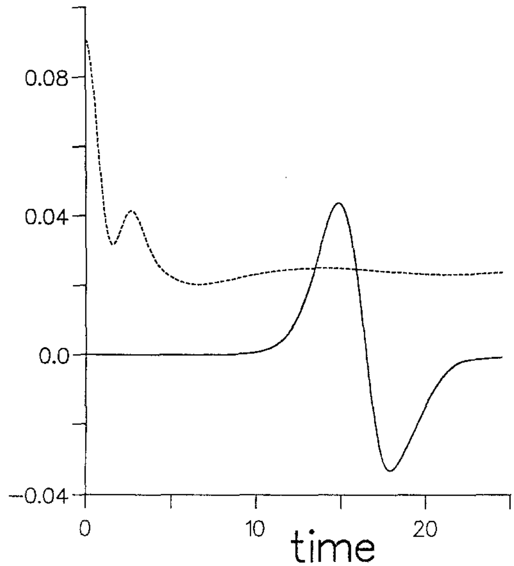

Fig. 4. The time signature of impulsively generated (at $x = z = 0$) waves, with $\rho_0/\rho_e = 2$. The energy $e(t)$ is shown as a broken line with the potential $A(x = 0, z = 8a)$ is a full line. The duration of the quasi-periodic phase is shorter and less energy is trapped within the slab than for the stronger slab (compare with Figure 3).

Here, it is equal to about 10 Alfvén times (Figure 4) which correspond to about 5 s oscillations.

Quite different time signatures characterize waves generated by an impulse located outside the slab. We consider the case of a source located at $x = 3a$, $z = 0$. The time dependence of the potential A shows a much more extended quasi-periodic phase, with a duration time of about 15 Alfvén transit times (see Figure 5) corresponding to 11 s oscillations. Duration of a single pulse is, however, much shorter and of the order of 2 time units ($\frac{3}{2}$ s oscillations). The phenomena associated with impulsively generated waves outside the slab are different in a character to those generated inside the slab. Here, less energy is trapped by the slab. The time dependence of the energy exhibits a maximum at $t \simeq 5a/V_A$ and then the energy declines slowly with some noticable oscillations (Figure 5). Only a part of waves is trapped by the slab and consequently the energy at $t = 30a/V_A$ is slightly larger than at $t = 0$.

The time signatures can also be different for multi-pulses launched at different locations and times. Here we explore two pulses initially located at $x = 0$ and $z = \pm 2a$. Each of them evolves in a similar way to a single pulse as shown in Figure 1. They give, however, different time signature with a prolonged duration of the quasi-periodic phase, equal to about 20 Alfvén transit times corresponding to 15 s oscillations (see Figure 6). This quasi-periodic phase contains two maxima corresponding to two humps propagating along the slab. It is interesting to note a similar event was observed by Kurths and Karlický (1989).

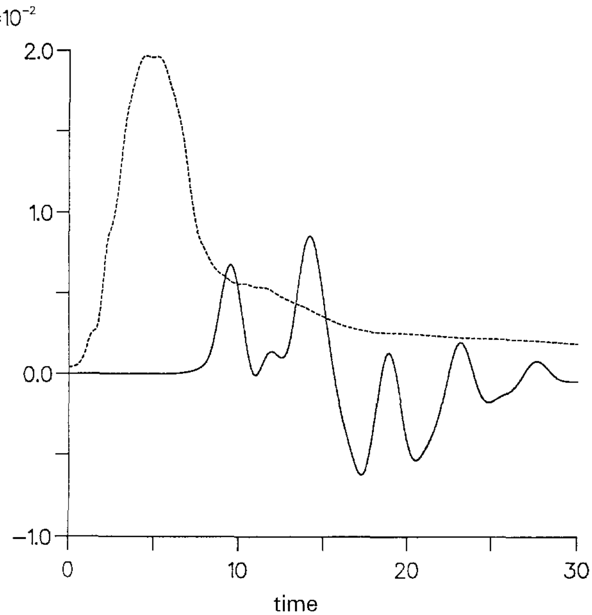

Fig. 5. The time signature disturbance that is impulsively generated at an off-axis position (here $x = 3a, z = 0$), with $\rho_0/\rho_e = 5$. A more complicated and longer lasting quasi-periodic phase characterizes the time signature. The energy $e(t)$ has its maximum at $t \simeq 5$ time units.

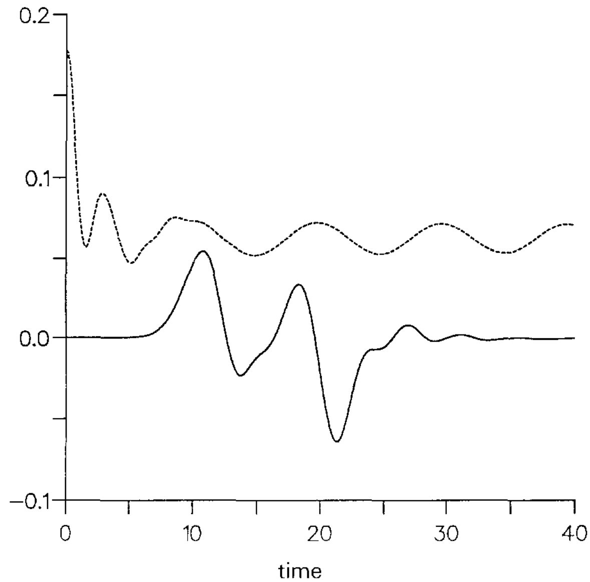

Fig. 6. The time signature of impulsively generated waves in the slab with the density ratio $\rho_0/\rho_e = 5$. A pair of equal initial sources, located at $x = 0, z = \pm 2a$, are considered. The broken and solid lines denote the slab energy $e(t)$ and $A(x = 0, z = 8a)$, respectively. The quasi-periodic phase contains two maxima which correspond to the two pulses that are initially launched.

3. Summary

We have investigated numerically the solution of the wave equation (1.1) for impulsively generated fast waves in smoothed slabs. In the simplest case of a single source placed on the axis of the slab, such waves possess distinctive temporal signatures consisting of the three phases (Roberts, Edwin, and Benz, 1983, 1984). The presented numerical results are in general agreement with the one mode theory because most of the energy of the initial pulse is transferred to such a mode. The numerical results, however, show that the temporal signatures can be quite different for waves excited outside the slab and in the case of a multi-series of impulses generated at different places and times. Our results show also that weaker slabs (with lower gas density ratio) leak more energy from the slab than stronger (high density) slabs.

It is evident from the temporal signatures produced by impulsive sources at various locations, offset from the centre of a slab, that in general we may expect fairly complex signatures to arise in the observations. Indeed, an examination of the various reported observational signatures of radio pulsations (e.g., McLean *et al.*, 1971; Kurths and Karlický, 1989) shows that while there is an overall resemblence to the theoretical picture drawn by Roberts, Edwin, and Benz (1983, 1984) for the simple case of a single impulse at the centre of a slab, there is also a variety of differences. The numerical investigations reported here make clear that such differences are to be expected, given that sources will not, in general, be located at the centre of a slab and now will only a single source always act.

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