SELF-SIMILAR SOLUTIONS IN THE THEORY OF FLARE-UPS IN NOVAE, I

J. B. SINGH and P. R. VISHWAKARMA

Department of Mathematics, S.G.R. Postgraduate College, Dobhi, Jaunpur, (U.P.), India

(Received 5 April, 1983)

Abstract. The problems of central stellar explosions under the assumption that the temperature gradient is zero in the rear flow field have been studied. Also, the similarity solutions of the field variables for the homothermal flows of a self-gravitating gas behind the spherical shock-wave propagating in a non-uniform atmosphere at rest are obtained. The total energy of the wave is taken to be time-dependent obeying a power-law.

1. Introduction

The observational data shows that the unsteady motion of a large mass of the gas followed by sudden release of energy results in flare-ups in novae and supernovae. Carrus *et al.* (1951), Sedov (1959), Purohit (1974), Singh (1982), and Singh and Vishwakarma (1983) have discussed the self-similar adiabatic or isothermal flows in self-gravitating gas. They have obtained numerical solutions assuming that the total energy of the wave is either constant or increases with time.

A qualitative behaviour of the gaseous mass may be investigated with the help of equations of motion and equilibrium, taking gravitational forces into account. The total energy of the flow increases with time because of the pressure exerted on the gas by an expanding surface when the wave is driven by fresh erupting solar plasma for some time. Here we have found the extent for three types of model: the first having the total energy of explosion constant, the second having constant velocity of propagation of shock waves, and the third having neither constant total energy of the wave nor constant velocity of propagation of shock waves.

When the flows are associated with high temperature such as novae burst the assumption of adiabaticity seems to be not valid because of intense heat exchange. Instead, one may propose that the temperature gradient is zero in the flow. In the present work we attempt to study such a flow of self-gravitating gas behind the shock wave and assume that the disturbance is headed by an isothermal shock. In this case, radiation effects are already implicitly present.

2. Equations of the Problem

If we neglect viscosity and thermal conductivity, the basic differential equations governing the isothermal flow in self-gravitating gas are given by

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0 \,, \tag{1}$$

Astrophysics and Space Science **95** (1983) 99–104. 0004–640X/83/0951–0099$00.90

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0 , \tag{2}$$

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2 , \tag{3}$$

$$\frac{\partial T}{\partial r} = 0 , \tag{4}$$

where r, t, u, ρ, p, m, T are the radial distance from the centre, time, velocity, density, pressure, mass contained in a sphere of radius r, and the temperature of the fluid particles, respectively; G represents the gravitational constant. Equation (4) with the help of perfect gas law ($p = \Gamma \rho T$) can be replaced by

$$\frac{p}{p_2} = \frac{\rho}{\rho_2} , \tag{5}$$

where the suffix 2 denotes the quantities just behind the shock. Initial flow variables immediately ahead of the shock by the suffix 1, are

$$u_1 = 0 ; \qquad \rho_1 = A r_2^{-w} , \qquad (2 \leq w \leq 2.5) ,$$

$$m_1 = \frac{4\pi A}{(3 - w)} r_2^{3-w} ; \qquad p_1 = \frac{2\pi A^2 G}{(w - 1)(3 - w)} r_2^{2-2w} , \tag{6}$$

where r_2 is the shock radius and A and w are constants. These are the solutions of the equilibrium equations.

The disturbance is headed by an isothermal shock and jump conditions at it are:

$$\begin{aligned} &\rho_1 U = \rho_2 (U - u_2) = m_s , \\ &p_2 - p_1 = m_s u_2 , \\ &T_1 = T_2 , \\ &m_1 = m_2 ; \end{aligned} \tag{7}$$

where U denotes the shock velocity.

The present self-similar model, including a driven wave produced by a flare energy release E that is time-dependent, has been adopted and it is given by

$$E = B t^q , \qquad (0 \leq q \leq 1) \tag{8}$$

where B and q are constants.

3. Similarity Solutions

By a standard dimensional analysis of Sedov (1959), the non-dimensional variable η is defined by

$$\eta = (\alpha A G)^{-1/w} r t^{-\delta} , \tag{9}$$

where

$$\delta = \frac{2}{w} = \frac{2+q}{5-w}\,, \tag{10}$$

which discloses that

$$q = \frac{2}{w}\,(5-2w)\,; \tag{11}$$

and the limits of q and w are

$$0 \leq q \leq 1 \quad \text{and} \quad 2 \leq w \leq 2.5\,. \tag{12}$$

The other transformations for flow variables are

$$\begin{aligned} u &= \frac{r}{t}\,V(\eta)\,, \qquad & \rho &= \frac{1}{Gt^2}\,R(\eta)\,, \\ p &= \frac{r^2}{Gt^4}\,P(\eta)\,, \qquad & m &= \frac{r^3}{Gt^2}\,M(\eta)\,, \end{aligned} \tag{13}$$

α is a constant to be determined by the condition that η assumes the value 1 at the shock front. With these transformations, the basic equations are written as

$$\eta\left[\frac{\mathrm{d}V}{\mathrm{d}\eta} + (V-\delta)\,\frac{1}{R}\,\frac{\mathrm{d}R}{\mathrm{d}\eta}\right] - 2 + 3V = 0\,, \tag{14}$$

$$\eta\left[(V-\delta)\,\frac{\mathrm{d}V}{\mathrm{d}\eta} + \frac{1}{R}\,\frac{\mathrm{d}P}{\mathrm{d}\eta}\right] + V(V-1) + M + 2\,\frac{P}{R} = 0\,, \tag{15}$$

$$\eta\,\frac{\mathrm{d}M}{\mathrm{d}\eta} - 4\pi R + 3M = 0\,, \tag{16}$$

$$\eta^2 P = Q\delta^2 R\,. \tag{17}$$

The transformed shock conditions are

$$V(1) = (1-Q)\delta\,, \tag{18}$$

$$R(1) = \frac{2(w-1)(3-w)}{\pi w^2}\,, \tag{19}$$

$$P(1) = \frac{2\delta^2(w-1)(3-w)Q}{\pi w^2}\,, \tag{20}$$

$$M(1) = \frac{8(w-1)Q}{w^2}\,, \tag{21}$$

$$Q \quad = \frac{C^2}{U^2} ; \tag{22}$$

where $C^2 = p_1/\rho_1$ is the square of the isothermal sound ahead of the shock. Substitute the values, the parameter takes the form

$$Q = \frac{\pi w^2}{2(w-1)(3-w)\alpha} . \tag{23}$$

Once Q is fixed, α is known beforehand for a given density distribution ahead of the shock. The ratio γ of specific heats, does not envolve either through the basic equations or through boundary conditions in this problem. Equations (14)–(17) with the boundary conditions (18)–(23) give the solution of our problem.

4. Numerical Solutions and Results

The case $w = 2.5$ corresponds to blast wave problem while $w = 2.0$ gives the problem of uniformly expanding shock-wave in a medium with zero temperature gradient. For other values of w in between 2 and 2.5, neither the total energy of the wave is constant nor shock wave expands uniformly. We restrict the range for Q to $0 \le Q \le 1$. The

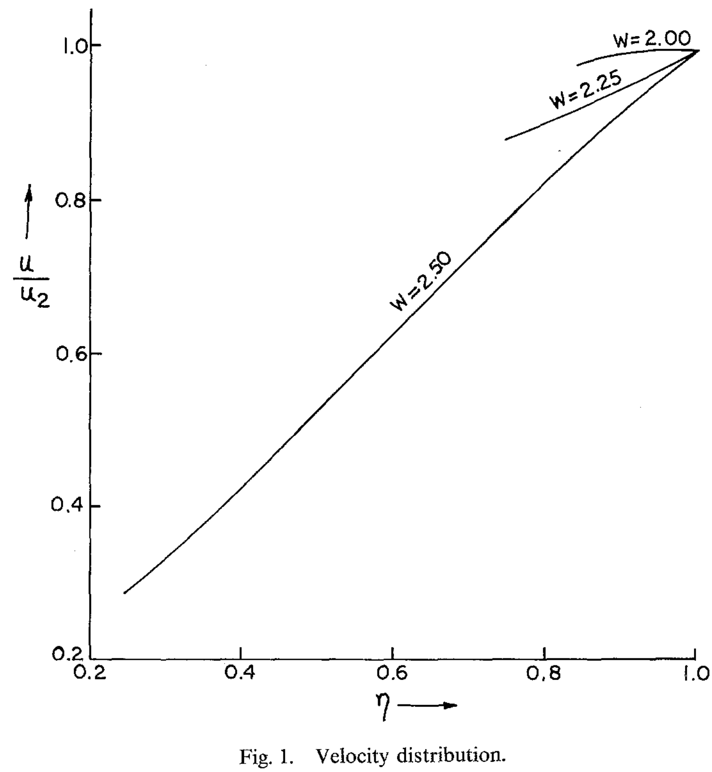

Fig. 1. Velocity distribution.

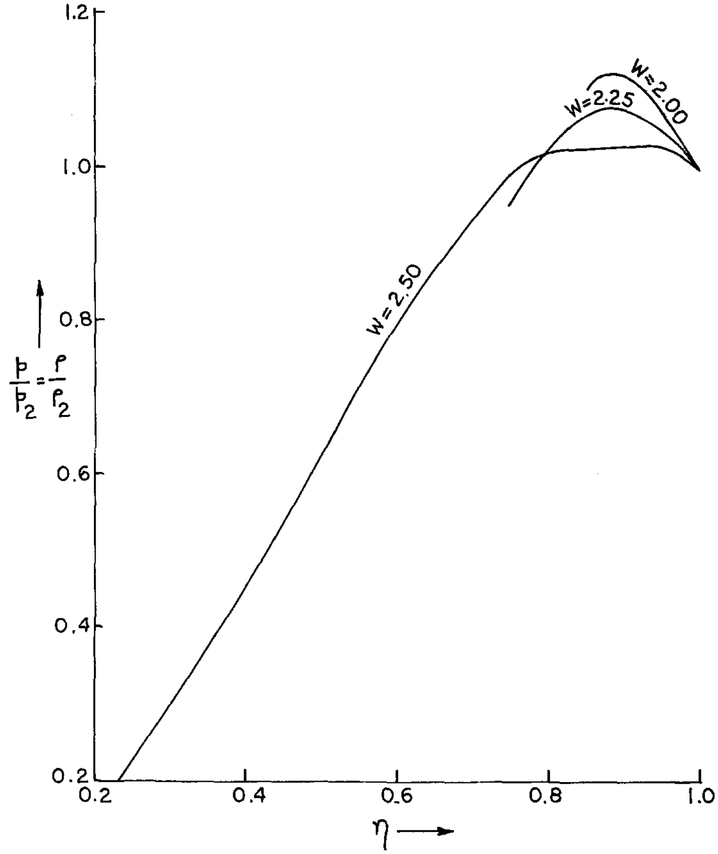

Fig. 2. Density distribution and pressure distribution.

kinematic condition at the inner expanding surface is

$$V(\bar{\eta}) = \delta\,, \tag{24}$$

where $\bar{\eta}$ is the value of η at the inner expanding surface and its relation with shock radius is $\bar{r} = \bar{\eta} r_2$, where $\bar{r}$ is the Eulerian coordinate of inner expanding surface. The kinematic condition demands that the velocity of the fluid particle at the expanding surface is equal to the velocity of the surface itself.

For exhibiting the numerical solutions it is convenient to write the field variables in the non-dimensional form as

$$\frac{u}{u_2} = \frac{\eta V(\eta)}{V(1)}\,, \tag{25}$$

$$\frac{\rho}{\rho_2} = \frac{p}{p_2} = \frac{R(\eta)}{R(1)}\,, \tag{26}$$

$$\frac{m}{m_2} = \frac{\eta^3 M(\eta)}{M(1)} . \tag{27}$$

The numerical integration is carried out on DES-system 1090 computer by RKGS program installed at IIT, Kanpur for the three cases $w = 2.0, 2.25, 2.5$ which are of practical interest in Astrophysics. The values of other parameters are $Q = 0.15$, $q = 0$, $\frac{4}{9}$, 1. The nature of the field variables is illustrated through the Figures 1–3.

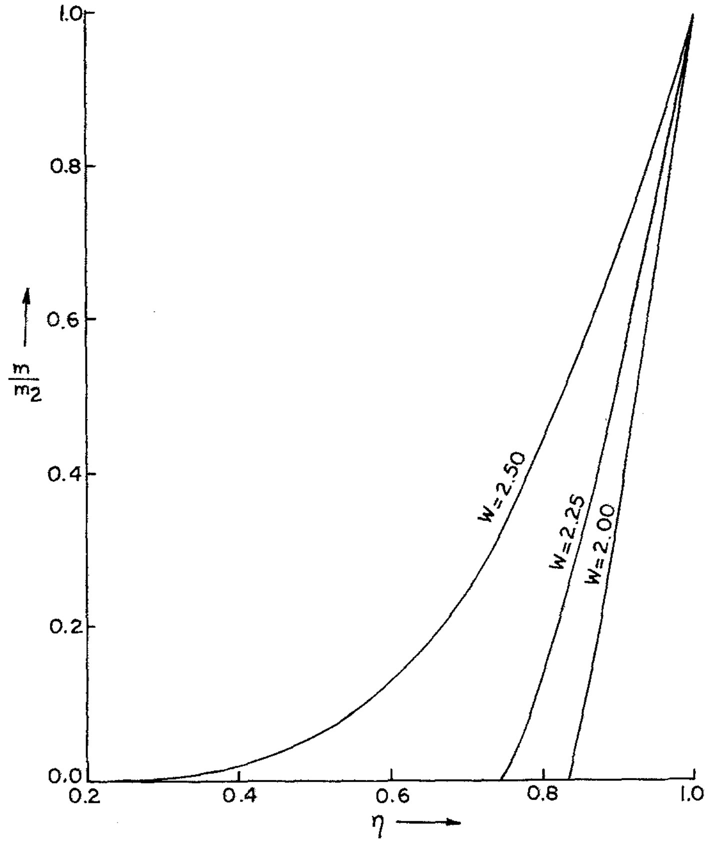

Fig. 3. Mass distribution.

References

Carrus, P. A., Fox, P. A., Haas, F., and Kopal, Z.: 1951, *Astrophys. J.* **113**, 495.
Purohit, S. C.: 1974, *J. Phys. Soc. Japan* **36**, 288.
Sedov, L. I.: 1959, *Similarity and Dimensional Methods in Mechanics*, Academic Press, London.
Singh, J. B.: 1982, *Astrophys. Space Sci.* **87**, 347.
Singh, J. B. and Vishwakarma, P. R.: 1983, *Astrophys. Space Sci.* **95**, 105 (this issue).