SPHERICALLY SYMMETRIC ELECTROMAGNETIC MASS MODELS WITH COSMOLOGICAL PARAMETER Λ

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Abstract. An exact solution has been obtained for the Einstein-Maxwell field equation with Λ corresponding to a spherically-symmetric charged perfect fluid distribution. Here the cosmological constant Λ is assumed to be a scalar variable depending on the radial coordinate r of the spherical system, viz., $\Lambda = \Lambda(r)$. The solution set thus obtained presents an electromagnetic mass model.

1. Introduction

Related to the electromagnetic mass models, where gravitational mass has a purely electromagnetic origin as suggested by the Lorentz-Abraham theory of electrons, many authors (Tiwari *et al.*, 1984; Gautreau, 1985; Grøn, 1985a; 1986a,b; Tiwari *et al.*, 1986; Ponce de Leon, 1987a,b; 1988) obtained different types of solutions for the charged perfect fluid distribution corresponding to spherically-symmetric space-time

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

Most of these authors to construct electromagnetic mass models, impose the equation of state

$$\rho + p = 0, \qquad (\rho > 0, p < 0), \tag{2}$$

where ρ is the matter-energy density and p is the fluid pressure. It is interesting to note that some authors (Wenda and Shitong, 1985a,b; Guth, 1981; Grøn, 1985b; 1986a; Linde, 1984) have used the idea of negative pressure, viz., the relation (2), either directly or indirectly, in the black hole physics and even in the inflationary cosmolosy. In this context it may be mentioned here that Einstein was the first who introduced cosmological constant Λ which contributes an equation of state like that of (2). The repulsive effects of the negative pressure term induced by the cosmological constant balances the attractive gravitational force of the matter present in the system considered. In this way Einstein (1917) obtained a solution for the static universe though it was unstable to perturbations. After the discovery of the cosmological redshift, Einstein abandoned the concept of cosmological constant considering it as a zero-valued quantity. Nevertheless, many authors like Lemaître, Eddington, Zel'dovich and de Sitter preferred to retain the Λ term as a factor affecting expansion or contraction. As an example it may be mentioned here that the empty-space solution with Λ given by de Sitter (1917) described an

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exponentially-expanding empty space-time. Thus, it is quite obvious to talk of the cosmological constant in the inflation-dominated era of cosmology.

The investigations of Tiwari *et al.* and others, as mentioned previously, do not contain Λ-term. Therefore, our aim here is to study the role of cosmological scalar with regard to electromagnetic mass models, in case it exists. Here we have extended the work of Tiwari *et al.* (1984) with the introduction of a scalar in the field equations corresponding to the static spherically-symmetric space-time (1) in the case of charged perfect fluid distribution. It is shown that electromagnetic mass models can exist in this case if we introduce a cosmological scalar (in place of a constant), viz., $\Lambda = \Lambda(r)$, where r is the radial coordinate of the spherical system.

In Sect. 2 of the present paper the basic field equations and their consequences are given. Sect. 3 provides a set of solution which can be shown to represent a charge-dependent cosmological model and in Sect. 4 some concluding remarks are made.

2. Field Equations and Their Consequences

The Einstein-Maxwell field equations for the case of charged perfect fluid distribution are given by

$$G^i_j = R^i_j - \frac{1}{2}g^i_j R + g^i_j \Lambda = -8\pi[T^{i(m)}_j + T^{i(em)}_j], \tag{3}$$

$$(\sqrt{-g}\, F^{ij}), j = 4\pi\sqrt{-g}\, J^i \tag{4}$$

and

$$F_{[ij,k]} = 0, \tag{5}$$

where $g^i_j\Lambda$ is the cosmolosical term, F^{ij} is the electromagnetic field tensor and J^i the current four-vector. The matter and the electromagnetic energy-momentum tensors are, respectively, given by

$$T^{i(m)}_j = (\rho + p)u^i u_j - p g^i_j \tag{6}$$

and

$$T^{i(em)}_j = \frac{1}{4\pi}(-F_{jk}F^{ik} + \frac{1}{4}g^i_j F_{kl}F^{kl}). \tag{7}$$

Hence, in the present case, i.e., for the static spherically-symmetric charged perfect fluid corresponding to the metric (1), the field equations are given by

$$\mathrm{e}^{-\lambda}(\lambda'/r - 1/r^2) + 1/r^2 - \Lambda = 8\pi\rho + E^2, \tag{8}$$

$$e^{-\lambda}(\nu'/r + 1/r^2) - 1/r^2 + \Lambda = 8\pi p - E^2, \tag{9}$$

$$e^{-\lambda}[(\nu''/2 - \lambda'\nu'/4 + \nu'^2/4 + (\nu' - \lambda'))/2r] + \Lambda = 8\pi p + E^2 \tag{10}$$

and

$$[r^2 E]' = 4\pi r^2 \sigma e^{\lambda/2}, \tag{11}$$

where σ is the charge density and E the electric field strength. The latter one is defined as

$$E = -e^{-(\nu+\lambda)/2}\phi' \tag{12}$$

and where

$$F_{01} = -F_{10} = \phi' \tag{13}$$

The equations of continuity, viz.,

$$T^i_{j;i} = -(1/8\pi)(R^i_j - \frac{1}{2}g^i_j R + g^i_j \Lambda)_{;i} \tag{14}$$

here take the form

$$\frac{d}{dr}(p - \frac{\Lambda}{8\pi}) = -(\rho + p)\frac{\nu'}{2} + \frac{1}{8\pi r^4}\frac{d}{dr}q^2 \tag{15}$$

The conservation equation (15) is the generalization of the Tolman-Oppenheimer-Volkoff (TOV) equation of hydrostatic equilibrium to the case when charge q as well as cosmological parameter Λ are present.

Adding the field equations (8) and (9), we have

$$e^{-\lambda}(\nu' + \lambda') = 8\pi r(\rho + p). \tag{16}$$

Again, (8) reduces to

$$e^{-\lambda} = 1 - \frac{2M}{r}, \tag{17}$$

where

$$M = 4\pi \int_0^r r^2(\rho + E^2/8\pi + \Lambda/8\pi)\, dr. \tag{18}$$

Also (11) can be written as

$$E = \frac{4\pi}{r^2}\int_0^r r^2 \sigma e^{\lambda/2}\, dr = \frac{q}{r^2} \tag{19}$$

Assuming a relation between the metric coefficients g_{00} and g_{11} as

$$g_{00}\, g_{11} = -1 \tag{20}$$

one immediately (from the line element (1)) gets

$$\nu' + \lambda' = 0. \tag{21}$$

Using Eq. (21) in Eq. (16), we have the equation of state as

$$\rho + p = 0, \qquad (\rho > 0, p < 0) \tag{22}$$

Thus, Eq. (15) reduces to

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(p - \frac{\Lambda}{8\pi}\right) = \frac{1}{8\pi r^4}\frac{\mathrm{d}}{\mathrm{d}r}q^2. \tag{23}$$

Again, we assume that

$$p = -\frac{\Lambda}{8\pi}, \qquad (p < 0, \Lambda > 0) \tag{24}$$

and hence

$$\rho = \frac{\Lambda}{8\pi} \tag{25}$$

Also, equation (23) ultimately takes the form

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{1}{16\pi r^4}\frac{\mathrm{d}}{\mathrm{d}r}q^2, \tag{26}$$

which implies at once that the pressure is dependent on the electric charge.

3. A Charge Dependent Cosmological Model

Assuming that

$$\sigma \mathrm{e}^{\lambda/2} = \sigma_0, \tag{27}$$

where σ_0 is the constant charge density at $r = 0$ of the spherical distribution, we get from (19) the electric field strength E and the charge q as

$$E = \frac{q}{r^2} = \frac{4\pi\sigma_0 r}{3}. \tag{28}$$

After using Eq. (28) in Eq. (26) and joining it with Eq. (22), we have

$$\rho = -p = \frac{\pi\sigma_0^2(a^2 - r^2)}{3}, \tag{29}$$

where ρ, the matter-energy density and p, the pressure have the maximum values at the centre of the sphere $r = 0$ and vanish at the surface $r = a$.

Then, the cosmological parameter Λ is given by

$$\Lambda = \frac{8\pi^2\sigma_0^2(a^2 - r^2)}{3}. \tag{30}$$

From Eqs. (20) and (17), one gets

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2M}{r}, \tag{31}$$

where, from Eq. (18), M can be given as

$$M = \frac{8\pi^2\sigma_0^2 r^3(5a^2 - 2r^2)}{45}. \tag{32}$$

The total gravitational mass can be calculated by using the relation

$$m = \frac{q^2}{2r} + M \tag{33}$$

which for the present situation yields

$$m = \frac{64\pi^2\sigma_0^2 a^5}{45}. \tag{34}$$

Thus, from the above solution it follows that all the physical parameters are dependent on electric charge density. The gravitational mass, therefore, is of purely electromagnetic origin.

4. Conclusions

The most important feature of the solution set is that the traditional cosmological constant Λ here is no more a constant rather a scalar having a functional dependence on r as well as σ_0 (the radial coordinate and charge density, respectively). Moreover, the gravitational mass, obtained here, confirms the Lorentz conjecture of extended electron, viz., an electron consists of 'pure charge and no matter'.

In connection with this it may be mentioned here that the relationship (2), which represents positive matter density and negative pressure, is not unphysical according to the contemporary developments. The above relation, because of the pressure being negative, corresponds to a repulsive gravitational force (Isper and Sikivie, 1983; López, 1988).

This is identified with the Poincaré stress of cohesive force in nature (Poincaré, 1905) which is required to maintain stability for the Lorentz extended electron (Lorentz, 1904).

It has been shown that the equation of state (2), may describe the mesons being in the relativistic dense state (Hakim, 1978). Moreover some authors (Davies, 1984; Hogan, 1984; Grøn, 1985b) describe the "matter" satisfying the relation (2) as "false vacuum", "degenerate vacuum" or "vacuum fluid". It is interesting to note that, according to López (1992), this equation of state not satisfying the energy conditions due to the presence of negative pressure suggests the possible existence in nature of some exotic kind of matter.

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