MOND, DARK MATTER AND THE COSMOLOGICAL CONSTANT

(Letter to the Editor)

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Abstract. A possible connection between MOND (Modification of Newtonian Dynamics) proposed as an alternative hypothesis to dark matter in galaxies and clusters and a residual cosmological constant term dominating cosmological dynamics in a $\Omega = 1$ universe is explored.

In recent papers (Sivaram, 1993 a,b), some alternative approaches other than the conventional dark matter hypothesis to explain flat rotation curves of spiral galaxies and cluster dynamics were explored and critically discussed. Also in other recent papers (Sivaram, 1993 b,c), the consequences of MOND for galaxy formation were considered. 'Modification of Newtonian Dynamics' or MOND was initially proposed (Milgrom, 1983) as an alternative hypothesis to account for the flat rotation curves of spiral galaxies sans dark matter. It has had phenomenological success in explaining a wide variety of observations involving galaxies and clusters (Milgrom and Sanders, 1993). The theory however, requires the ad hoc introduction of a critical acceleration a_0 (empirically about 10^{-8} cm s^{-2}) below which Newtonian dynamics is modified as: $a = (GMa_0)^{1/2}/r$, giving rise to a constant rotation speed $V_c \approx (GMa_0)^{1/4}$ at the periphery of large spirals. The ubiquitous occurrence of a_0 in many astrophysical systems was pointed out (Sivaram, 1993b,c). For instance, for a typical spiral galaxy ($M_{\rm gal} \approx 10^{12} M_\odot$, $R \approx 30$ kpc),

$$a \approx \frac{GM_{\rm gal}}{R^2} \approx 10^{-8}\ {\rm cm\,s^{-2}} \approx {\rm a}_0. \tag{1}$$

For typical cluster of galaxies, $M_c \approx 10^{16} M_\odot$, $R_c \approx 3$ Mpc, and for globular clusters $R_G \sim 100$ pc, $M_G \approx 10^6\ M_\odot$,

$$a \approx \frac{GM_{\rm gal}}{R_c^2} \approx \frac{GM_G}{R_G^2} \approx 10^{-8}\ {\rm cm\,s^{-2}} \approx a_0. \tag{2}$$

It was empirically noted by the above authors that in MOND:

$$a_0 \approx cH_0, \tag{3}$$

where H_0 is the Hubble constant, c is the velocity of light thus suggesting a cosmological link.

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Interestingly enough in early works (Sivaram, 1983a,b) it was pointed out that the gravitational self energy of a typical elementary particle (hadron) was shown to be

$$E_G \approx \frac{Gm^3c}{\hbar} \approx \hbar H_0, \tag{4}$$

implying a surface gravity (acceleration) for the particle:

$$a_h \approx \frac{Gm}{r^2} \approx \frac{Gm^3c}{\hbar}\frac{c}{\hbar} \approx cH_0 \approx a_0 \tag{5}$$

in agreement with the empirical relation Equation (3). Also surface gravity for an electron (a_e) or a typical atomic nucleus (a_n) was found to be by the same argument

$$a_e \approx a_n \approx a_h \approx a_0.$$

Equations (4) and (5) also imply the Weinberg empirical relation

$$m_h \approx \left(\frac{\hbar^2 H_0)}{Gc}\right)^{1/3} \tag{6}$$

for a typical meson mass.

It was noted in (Sivaram, 1993b,c), that the above universality of a_0 occurring in all of the above systems implied that the surface density given by

$$\sum = \frac{M}{R^2} \tag{7}$$

is constant for all of the above systems and indeed:

$$\sum = \frac{M}{R^2} = \frac{a_0}{G} = \text{constant} \tag{8}$$

for each of the above macro and micro systems, ranging from the electron to the supercluster. (M and R being the mass and spatial extent, respectively).

Now we note that the gravitational self-energy *density* of a system is:

$$\rho_G \approx \frac{GM^2}{R^4}, \tag{9}$$

which can be expressed in terms of the surface gravity or acceleration ($a = GM/R^2$) as:

$$\rho_G \approx \frac{a^2}{G} \tag{10}$$

and since $a \approx a_0 = cH_0 =$ constant, universally for each of the above systems, it follows from Equations (1), (2), (5), (6) and (10), that the gravitational self energy density is also surprisingly the same for an elementary particle or for a galaxy, a supercluster, etc. and is given universally by:

$$(\rho_G)_{\text{particle}} = (\rho_G)_{\text{gal}} = (\rho_G)_{\text{supercluster}} = \frac{a_0^2}{G}. \tag{11}$$

i.e., we have a universal relation for the gravitational energy density which turns out to be the same for all the above range of scales:

$$\frac{a_0^2}{G} \approx 10^{-8}\,\text{erg}\,\text{cm}^{-3} \approx 10^{-29}\,\text{g}\,\text{cm}^{-3}. \tag{12}$$

Equation (12) gives the impression that ρ_G seems strikingly near the critical density ρ_c associated with a closed universe ($\Omega = \rho/\rho_c = 1$), i.e.

$$\rho_c \approx \frac{3H_0^2}{8\pi G} \approx 10^{-29}\,\text{g}\,\text{cm}^{-3}\,. \tag{13}$$

That this is not surprising and indeed so, can be demonstrated by considering Equation (6) for a typical particle mass and considering its gravitational self-energy density as:

$$\rho_G \approx \frac{Gm^2}{2(\hbar/mc)(\hbar/mc)^3(4\pi/3)}$$

(m being the particle mass and $\hbar/mc$ its spatial extent).

$$\rho_G \approx \frac{3}{8\pi}\frac{Gm^6c^4}{\hbar^4}\,. \tag{14}$$

Now substitute for $m = (\hbar^2 H_0/Gc)^{1/3}$ (i.e. from Equation (6)) into Equation (14) thus giving

$$\rho_G \approx \frac{3H_0^2c^2}{8\pi G} \text{ or } \frac{3H_0^2}{8\pi G}$$

(in mass density).

This is indeed the closure density as defined by Equation (13). Thus it is striking that

$$\rho_G \approx \rho_c \approx \frac{3H_0^2}{8\pi G}\,. \tag{15}$$

Equation (15) implies the remarkable result that the gravitational self energy density of an elementary particle, or a galaxy or a galactic cluster are all the same (ρ_G) and equal to the critical cosmological matter density (ρ_c) given by Equation (15)!

Also for the universe as a whole, i.e. for a $\Omega = 1$, closed universe with $\rho = \rho_c$, the gravitational self energy density again equals the critical matter density, i.e.

$(\rho_G)_{\text{univ}} \approx \rho_c$. Thus in this case for the universe as a whole, the total energy is zero (i.e. a flat universe)

This supports the Machian picture of local physical quantities being determined by global parameters (Sivaram, 1982, 1986a,b). Recently there have been revivals of the suggestion that a residual cosmological constant or Λ-term can dominate cosmological dynamics. As noted earlier, MOND can account for galactic rotation curves and cluster dynamics without the need to invoke dark matter. However, a closed $\Omega = 1$ universe with $\rho = \rho_c$ as favoured by inflation models of the early universe (to account for the flatness and horizon problems) would still need vast amounts of dark matter on cosmological scales (the baryonic matter is constrained from various arguments to be $\leq 0.1\rho_c$) MOND does not directly settle this. Several ongoing searches over the past couple of years to directly detect dark matter have not been successful and besides there are now far too many possible candidates! This has led to the revival of the suggestion that a residual Λ-term can contribute to as much as $\Omega \approx 0.8$, i.e. $0.8\rho_c$ or more so that (Yoshii, 1993a,b)

$$\rho_{\text{everything}} + \rho_{\text{vacuum}}(\Lambda) = \rho_c. \tag{16}$$

The Λ-term, as is well known, is associated (see the discussion in Sivaram (1986a,b)) with a vacuum energy density given by:

$$\rho_{\text{vac}} \approx \frac{\Lambda c^4}{8\pi G}, \quad \text{or mass density}$$

$$\rho_{\text{vac}} \approx \frac{\Lambda c^2}{8\pi G}. \tag{17}$$

A Λ-term (which has the units of curvature cm^{-2}) of magnitude $\Lambda \approx 10^{-57}\ \text{cm}^{-2}$, would imply a:

$$\rho_{\text{vac}} \approx 10^{-29}\ \text{g cm}^{-3} \approx \rho_c. \tag{18}$$

This domination of the cosmological constant Λ-term would imply from Equations (15), (17) and (18) that

$$\rho_G \approx \rho_c \approx \frac{\Lambda c^2}{8\pi G} \approx \rho_{\text{vac}}, \tag{19}$$

which means that the gravitational self energy density of the whole range of systems in Equations (11) including particles and galaxies equals the background cosmological vacuum energy density due to the presence of a cosmological constant term dominating the dynamics of the universe.

Such a background cosmological constant term apart from making up the critical density can also provide a basis for the existence of the fundamental acceleration a_0 assumed ad hoc in MOND.

The force acting on any particle of mass m in the universe because of the existence of a Λ-term is given through the De Sitter metric as: (Sivaram, 1979):

$$F \approx \frac{c^2 \Lambda r}{m} , \tag{20}$$

so that one gets an acceleration (which is mass independent, as Λ acts universally and does not violate the equivalence principle) of:

$$a \approx c^2 \Lambda r. \tag{21}$$

For $\Lambda \approx 10^{-57}\ \mathrm{cm}^{-2}$, $r \approx R_H \approx 10^{28}$ cm (the Hubble radius).

$$a \approx 10^{-8}\ \mathrm{cm\,s}^{-2} \approx a_0. \tag{22}$$

This suggests that the origin of a_0 on a cosmic scale might be traced to a Λ-term of this magnitude which contributes to most of the critical density. A physical fundamental basis for the vacuum energy dominating in a $\Omega = 1$, $\rho = \rho_c$ universe was explored in Sivaram (1986c). Here quantum field theory in curved space was shown to give rise to a zero point energy whose background energy density was shown to imply $\rho = \rho_c$, if the ultraviolet cut off was the Planck length. Again Equations (20) and (21) would imply a force or acceleration on large scales $r \sim 1/\sqrt{\Lambda}$ or $\Lambda \sim 1/r^2$, which is proportional to $1/r$ rather than $1/r^2$ as in Newtonian dynamics. MOND also implies a $1/r$ force law. These consequences of the Λ-term are tantamount to a modification of the poisson equation as:

$$\nabla^2\phi + \Lambda c^2 = 4\pi G\rho$$

and *not*

$$\nabla^2\phi + \Lambda\phi = 4\pi G\rho$$

as wrongly written by most authors including the earliest mention of it by Einstein. This modification can easily be accommodated by using a variational principle.

Thus MOND supplemented by a cosmological constant term can do away with the need for vast amounts of dark matter on cosmological scales also, not just on galactic or cluster scales. This term may also provide a basis for a_0 and explain why the gravitational self energy density of a wide range of systems equals the vacuum (critical) energy density.

This also applies to the universe as a whole, the vacuum energy density (which equals the critical energy density ρ_c) being equal to the gravitational self energy density of the universe. Indeed for all of the above systems whose surface gravity or acceleration equals a_0, this situation holds, i.e. their gravitational self energy density equals ρ_c the critical density as stated in Equations (15) and (19). This also applies to objects the sizes of planetary nebulae and to primordial gas clouds which were the precursors of the solar system (i.e. for $M = M_\odot$, $R \approx 10^4$ AU;

$a \approx GM/R^2 \approx a_0$). Also star forming regions with $M \approx 10^3\, M_\odot$, $R \approx$ a few parsecs have $a \approx a_0$ (Sivaram, 1994). If Λ be considered as providing a cosmic background curvature for space-time, then noting that c^2/G is the superstring tension (Sivaram, 1987; 1993b); Equation (19) can be written:

$$\rho_G \approx \frac{c^2}{G}\Lambda,$$

i.e. gravitational energy density
$\approx$ S.S. Tension $\times$ background curvature,
$\approx$ same for all macro and micro scales, (23)
$\approx$ universal constant $\approx \rho_c$
(cf. Equations (11), (12) and (19)).

In Sivaram (1993b) we had the relation.

Energy of system $\times$ local curvature $\approx$ S.S. Tension $\times\, a_0$
$=$ same for all scales. (24)

Equations (23) and (24) may be connected by noting

$$a_0 \approx c^2\sqrt{\Lambda} \approx c^2(\text{background curvature})^{1/2}. \quad (25)$$

In Sivaram (1990) it was shown that the energy density of a quantum system (like a particle or quantum) was given as:

$$\rho_e \approx \hbar c \times (\text{local curvature})^2 \quad (26)$$

(e.g. for a nucleon this gives $\rho \approx 10^{14}\ \text{g cm}^{-3}$ etc.). The gravitational energy density is given by:

$$\rho_G \approx Gm^2 \times (\text{local curvature})^2 \approx \rho_c \approx a_0^2/G$$
$$\approx c^2/G \times (\text{background curvature}). \quad (27)$$

Equations (24) to (27) suggest interrelation between local and background (global) quantities through relations such as Equations (6), (12) and (14).

It may also be mentioned that a model with a dominating Λ-term, (like in Equations (16) and (19)) has been shown to provide fair fits to the faint number counts, redshift distributions (Yoshii, 1993a,b) and has the additional cosmological advantage of enabling globular cluster ages to be less than the age of the universe!

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