ON THE THEORY OF THE LIGHT CURVES OF SUPERNOVAE*

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Abstract. An account of the theory of the light curves of supernovae is presented, based on certain assumptions concerning the passage through the stellar atmosphere of powerful shock waves. The investigation is based on numerical integration of appropriate equations of gas dynamics and radiative heat-conductivity. The calculations substantially involve the ionization and recombination of hydrogen in the envelope of a supernova. Changes are traced in the curves arising from the transition from compact stars with small radius ($\sim 10\, R_{\odot}$), to stars with very extensive envelopes ($\sim 10000\, R_{\odot}$). The light curves for compact stars agree well with observations of the peculiar supernovae in NGC 5457, NGC 6946 and NGC 5236. The characteristics of the light curves with the passage of shock waves through the extended atmosphere coincide within an order of magnitude with observations of the supernovae of type II and type I near their maximum brightness. A powerful heat-wave propagates before the shock-front in the extensive atmosphere which gives rise to a detached supernova envelope in the form of a thin spherical layer. We investigated the condition in an ascending wave of cooling and recombination in the supernova envelope. It is shown that part of the hydrogen may recombine to attain full transparency for radiation passing through it. The observations are compared with the results of the theory of radioactive decay of the elements. This explanation of the light curves by the passage of shock waves requires energies of 10^{50} to 10^{52} ergs, which are in agreement with mechanisms of thermonuclear explosions.

The elaboration, in recent times, of the theory of the destruction of the stars, disintegrating in their nuclear fuel (Fowler and Hoyle, 1964; Colgate and White, 1966; Arnett, 1967; Ivanova *et al.*, 1967, 1969; Arnett, 1969; Hansen and Wheeler, 1969) leads to an inference that supernova (SN) flares are a result of an ejection from the star into interstellar space of matter accelerated by powerful shock waves. However, these papers actually did not consider the connection between the passage of shock waves from the stellar surface and such important observable characteristics as the light curves and spectra of the supernovae.

The first attempt to explain the supernova light curves by the passage of shock waves was produced by Imshennik and Nadyozhin (1964). In this work, as in the following investigations in that direction (Imshennik and Nadyozhin, 1967; Grassberg and Nadyozhin, 1969a, b) a quantitative analysis of the problem was based on the equations of gas dynamics and radiative heat conductivity. Not long thereafter, more general investigations (Imshennik and Morozov, 1964, 1969; Morozov, 1966) were based on the equation of radiative transfer and the equations of gas dynamics. Here the radiation, in the general case, was not supposed to be in equilibrium with the matter.

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The fundamental aim of all these efforts was to provide a general study of transformation of the thermal and kinetic energy of the matter by the passage of powerful shock waves across the external layers of stars into the energy of radiation from the stellar surface. However, the first results (Imshennik and Nadyozhin, 1964) in which this program began to take shape, gave the hope that, on this basis, the observations of the light curves of supernovae could in principle, be explained. That hope was justified in subsequent papers (Grassberg and Nadyozhin, 1969a, b), where models of pre-supernovae were set out with large radii of about 10^4 $R_\odot$.

Another idea was developed in the work of Colgate and White (1966) and Colgate and McKee (1969). These authors supposed that, in the process of gas-dynamic expansion of the supernova envelope, there arises an additional energy source in the β-disintegration of unstable isotopes, carried by a shock-wave from the core of the star. The subsequent decay of this energy source explains the light curve of a supernova. The proposal of a disintegration of radioactive elements had appeared earlier (cf. Borst, 1950; Burbidge *et al.*, 1956). In the work of Colgate and McKee (1969) the latest results of the theory of nucleo-synthesis were used, and provided values for the transport of radiative energy and radiative heat-conductivity against the background of a rapid expansion of the stellar envelope. Somewhat different from the two approaches to the problem referred to above is the work of Morrison and Sartory (1969), in which these authors ignored the gas-dynamic side of the process, but investigated the effects connected with fluorescence through interstellar matter surrounding the supernova ($n \sim 1$ to $10/\text{cm}^3$). The present position with regard to the theory and observations of supernovae is that all three of the foregoing avenues deserve further development. In the present article we shall give a partial account in the work of Imshennik and Nadyozhin (1964, 1967) and Grassberg and Nadyozhin (1969a, b) of the theory of supernova light curves, and compare it with both the observations as well as with the results of other theoretical approaches.

1. Theoretical Light Curves of Supernovae

The characteristics of the light curves are determined by numerical integration of the equations of gas dynamics with radiative heat-transfer. The equations are given in the work of Imshennik and Nadyozhin (1964), with also a short description of the method of calculation. In all results it is assumed for simplicity that the star consists of pure hydrogen, the degree of ionization of which is determined by Saha's equation. The opacity is calculated from the amount of electron scattering, free-free absorption, photo effects and the absorption due to the negative hydrogen ion. The dependance of the hydrogen absorption on temperature and density is presented in Figure 1. In initial stellar models perturbation sets in at the centre of the star which gives rise to a strong shock-wave penetrating through the envelope with a velocity of a few thousand km/sec. The given perturbation is equivalent to an explosion in the centre of the star.

Figures 2–7 show the curves of light for various models, and also the time-variation

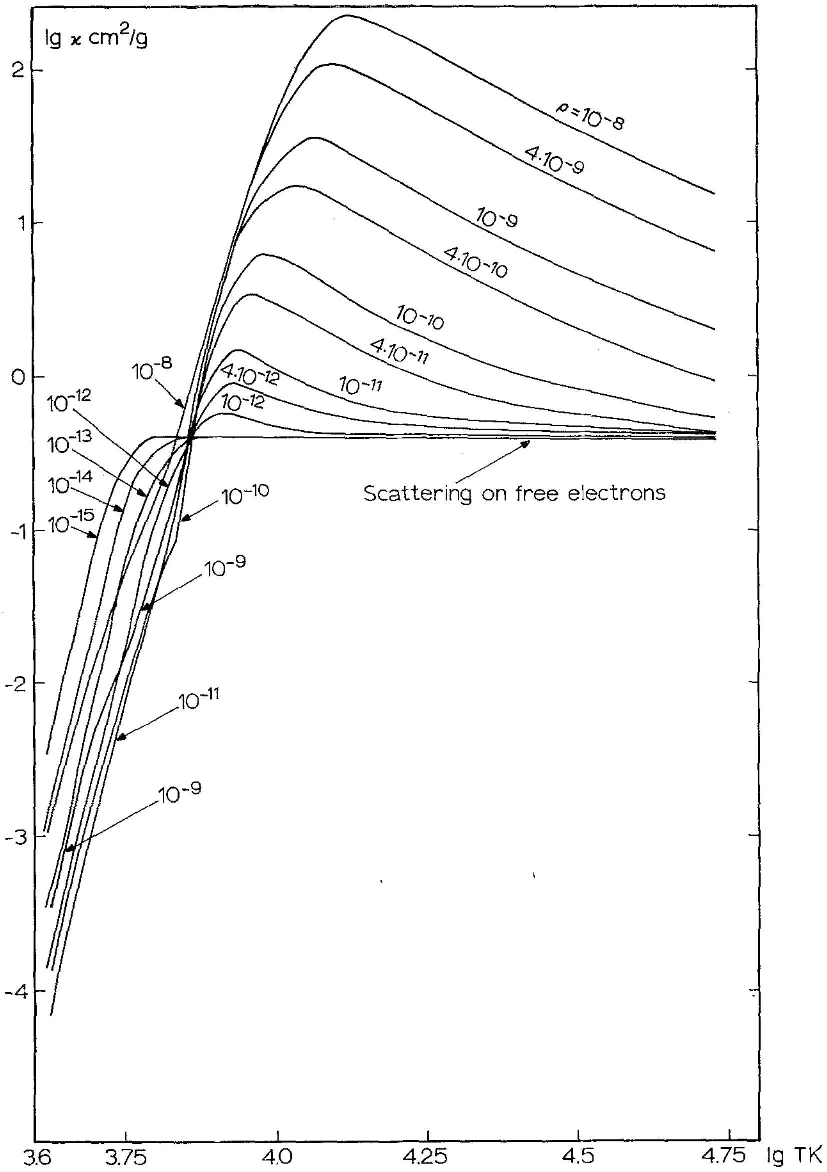

Fig. 1. The opacity of hydrogen.

of the velocity of matter passing through the photosphere of the supernova. The depth of the photosphere implies, conventionally, some standard which corresponds to an optical depth of $\tau = \frac{2}{3}$. Table I contains certain values for various alternatives. The first column of the table shows an arbitrary number corresponding to the model; the columns which follow contain the initial radius of the model, R_0; the mass, M_0; the energy of explosion, E_0; the kinetic energy of the envelope over $\sim$50 days after

TABLE I

Energy characteristics of different models

Model No.	$R_0/R_\odot$	$M_0/M_\odot$	E_0 (ergs)	E_k (ergs)	E_r (ergs)	Notes (initial model, ejection character)
1	9	15	1.05×10^{50}	3.9×10^{49}	3.2×10^{46}	Polytrope $n = 3$; ejected envelope $\Delta M = 3.87\ M_\odot$, Figure 3
2	9	15	0.99×10^{51}	0.91×10^{51}	2.7×10^{47}	Polytrope $n = 3$, full disintegration
3	500	30	3.3×10^{50}	3.1×10^{50}	3.6×10^{48}	'Yellow giant' $R_0 = 500\ R_\odot$ with envelope in mechanical and thermal equilibrium, full disintegration; Figures 4, 5
4	5000	30.9	3.5×10^{51}	3.4×10^{51}	5.9×10^{49}	'Yellow giant' surrounding transparent cool atmosphere with mass $0.9\ M_\odot$, Figures 6 to 9
5	10000	30.9	1.7×10^{51}	1.6×10^{51}	7.9×10^{49}	Similar to Model No. 4 but twice as extended, Figure 10
6	10000	3.09	1×10^{50}	0.9×10^{50}	4.9×10^{48}	Less massive model than No. 5 by a factor 10, on account of a similar drop in density
7	5000	30.9	1.4×10^{52}	$\sim 1.4 \times 10^{52}$	4.5×10^{50}	More intense explosion than in Model No. 4

the explosion, E_k, and finally E_r, the energy radiated up to that moment of time (i.e., the integral of the bolometric light curve).

As a fundamental parameter influencing the light curve, appears the initial radius of the model R_0. The magnitude of the maximum brightness of the curve and its form depends also on the initial energy E_0, and the mass of the star. The results are least sensitive to the initial temperatures of the outer layers and their degree of ionization.

Figure 2 shows the light curve and characteristics of the photosphere of the compact model $M_0 = 15\ M_\odot$ with $R_0 = 9\ R_\odot$. The emergence of the shock wave is accompanied by a peak (lasting for a few minutes) with a radiation maximum $M_{\text{bol max}} \simeq -20^m$. The effective temperature reaches 5×10^5 K, and the matter in that same outer layer of the star acquires a velocity of about 3×10^4 km/sec. As a result of subsequent

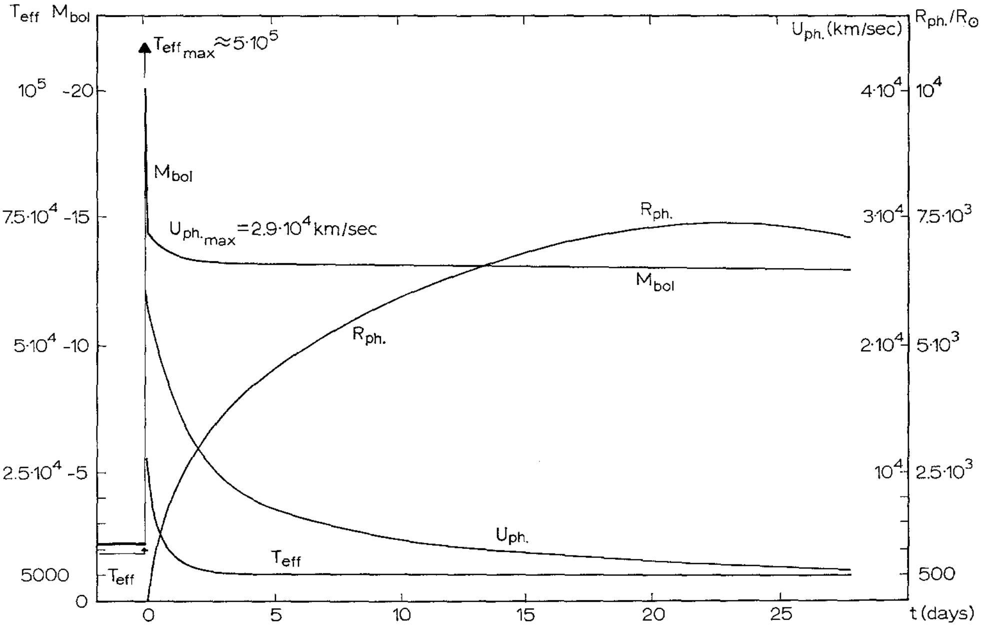

Fig. 2. Model No. 2 $R_0 = 9\ R_\odot$, the bolometric light curve, effective temperature T_{eff}, radius of the photosphere R_{ph} and the speed of matter emerging through the photosphere U_{ph}.

expansion the gas quickly cools down. For example, after three days the dimensions of the photosphere enlarge to $R_{ph} \approx 3 \times 10^3\ R_\odot$, the brightness stops changing, with $M_{\text{bol}} \approx -13^m$, and the effective temperature decreases to $T_{\text{eff}} \approx 5000$ K. The supernova envelope cools off by mixing of the internal matter with that of lower temperatures – thus setting up waves of cooling and recombination (CW). After 50 days the CW reaches the central parts of the mass ejected from the star, and the brightness falls off sharply (Figure 3). The basic energy of this model is radiated away in a stage of constant brightness. In all, the star loses some 2.7×10^{47} ergs (Table I) of radiation in fifty days. Of the radiation in the narrow peak when $T>5\times10^4$ K only 10^{46} ergs are emitted. Figure 3 shows the influence on the light curve of the initial energy E_0.

It is important to observe that, at the same time, we have a difference in the masses of the ejected envelopes: for a value of $E_0=1.05\times10^{50}$ ergs, which is less than the gravitational energy of the model 1.42×10^{50} ergs, the mass of the ejected envelope turns out to be $3.87\,M_\odot$; and for $E_0=0.99\times10^{51}$ ergs there occurs a full disintegration of the star. With the disengagement of an envelope containing a part of the star mass,

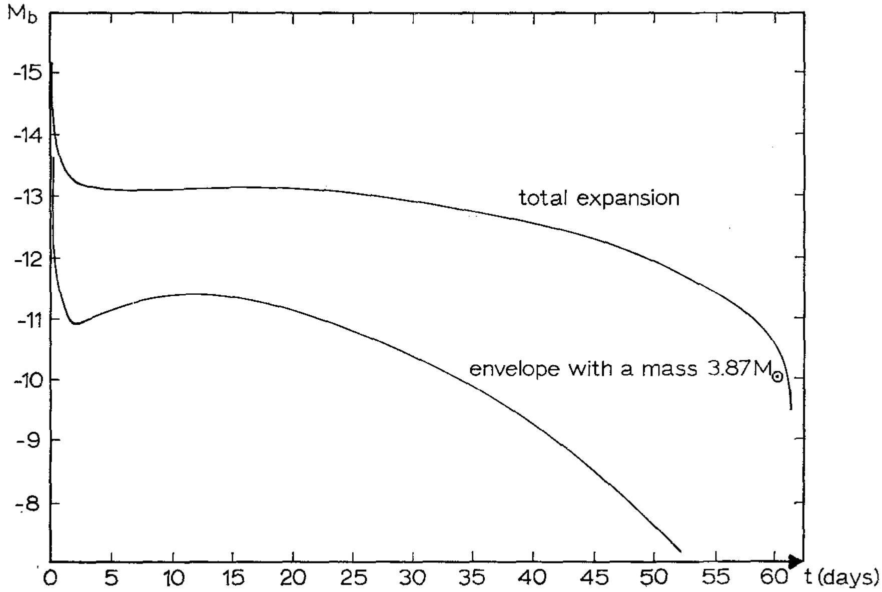

Fig. 3. The influence of strong explosions on the light curve with the compact model $R_0=4\,R_\odot$, $M_0=15\,M_\odot$.

the interior boundary of the shell suffers a gradual diminution of velocity and begins to fall back to the centre of the star. The law of density distribution appears to be quite different from the case of full disintegration; and in this case circumstances are different, especially in the late stages of the light curve (for complete disintegration the light curve has a more extended 'plateau'). As may be seen from Figure 3, the decrease of the energy E_0 by a factor 10 leads to a reduction in M_{bol} of $1^{\mathrm{m}}.5$ at the earlier times (at the time of about 10 days).

Figure 4 shows the results with the more extensive model with $R_0=500\,R_\odot$, and $M_0=30\,M_\odot$. The difference from all other initial model envelopes is to be found not only in the hydrostatic, but also in the radiation, balance. In spite of the increase of R_0 by 50 times the height of the peak of the brightness is almost unchanged, but its duration grows to about one day. The maximum effective temperature drops to 9×10^4 K. A cooling wave is formed only 18 days after the explosion (Figure 5), and that implies a constancy of brightness for $t>18$ days. Notwithstanding, a slow

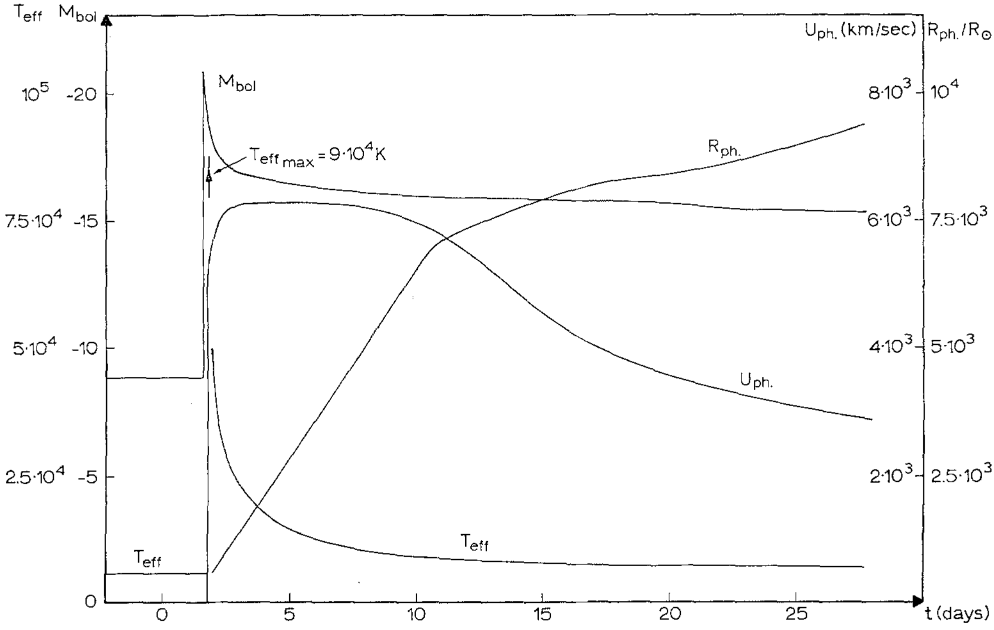

Fig. 4. The explosion of a 'yellow giant'. Model No. 3 $R_0 = 500\ R_\odot$, $M_0 = 30\ M_\odot$.

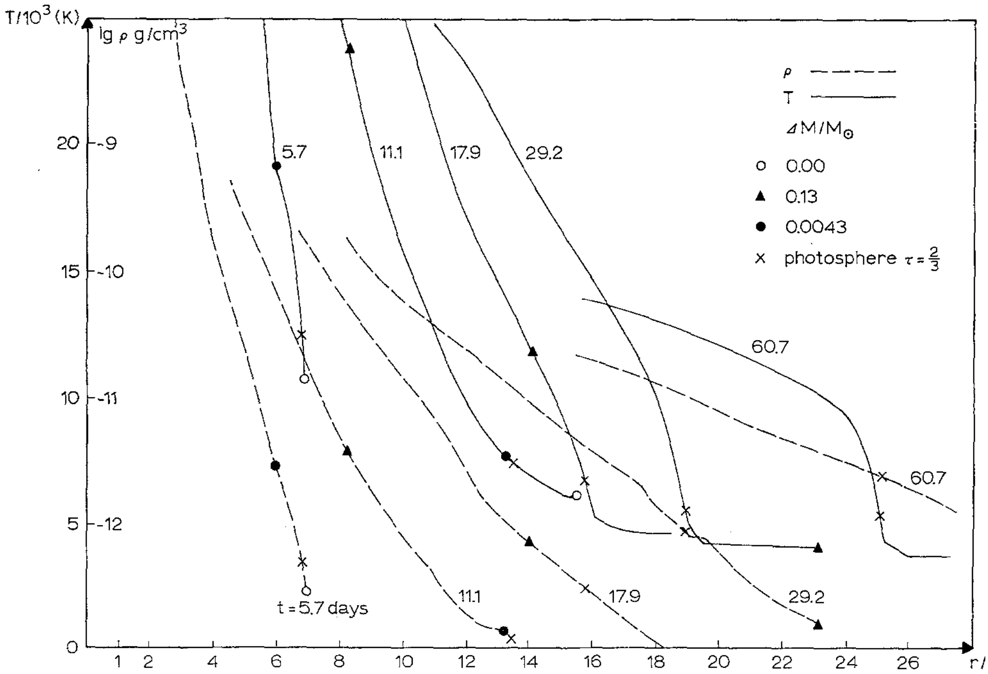

Fig. 5. The formation of CW in a supernova envelope (Model No. 3, $R = 500$). The temperature distribution (continuous line) and the density (dotted line) are shown in Eulerian coordinates for different moments of time. Some Lagrangian points and also the position of the photosphere are marked.

fall in brightness occurs at a much earlier time than the formation of the CW – at $t \sim 5$ days. It is easy to visualize a situation in which the brightness cannot generally alter.

In a regime of free expansion, with contribution to the internal energy from radiation, at each Lagrangian point the following relations are fulfilled: $\varrho \sim t^{-3}$, $r \sim t$, $\varrho \sim T^3$, $T \sim t^{-1}$. From this the stream of energy determining the stellar brightness, is given by

$$L = -\frac{4}{3} \frac{\pi ac}{\kappa \varrho} r^2 \frac{\partial T^4}{\partial r} \approx \frac{\text{const}}{\kappa},$$

where a denotes the Stefan constant; c, the velocity of light; ϱ, the density; T, the temperature; and κ, the absorption coefficient of stellar matter per unit mass. This equation discloses that L may change in time but only on account of a change in the opacity. With pure scattering, the brightness of the star is constant. From a glance at Figures 1 and 5 we may conclude that all the results of the foregoing assumptions are closely fulfilled. This shows that the constancy of the brightness is not necessarily evidence for the formation of cooling waves.

An exploratory investigation was undertaken of a still more extensive model with radius $R_0 = 5000\ R_\odot$. This model was constructed by adding to the initial model with $R_0 = 500\ R_\odot$ an extensive cool atmosphere with density falling off with the cube of the distance. Such an atmosphere is not found to be either in hydrostatic or in thermal equilibrium, but this, however, does not influence our results (for details see Grassberg and Nadyozhin, 1969b). The light curve and characteristics of the photosphere of this model are shown in Figure 6. A substantial role is played by effects connected with

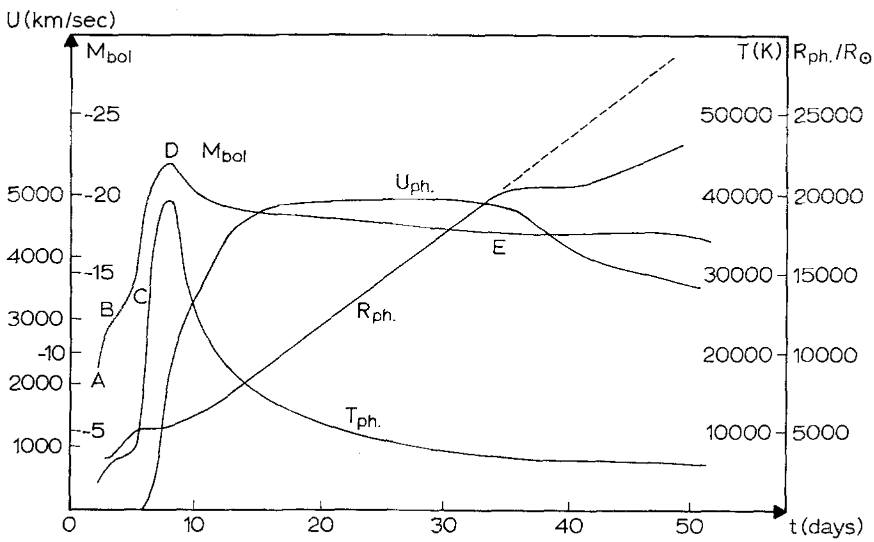

Fig. 6. The extensive Model No. 4, $R_0 = 5000\ R_\odot$, $M_0 = 30.9\ M_\odot$.

radiative heat conductivity. In front of the shock wave a powerful hot 'tongue' is formed. This is marked with streaks on one of the distributions in Eulerian co-ordinates of the temperature (Figure 7). In the outer parts of the envelope the relation between the radiation pressure and gas pressure $P_r/P_g \geqslant 4.45$ becomes critical, for which the discontinuity disappears even if the viscosity is not considered (Zeldovich and Reiser, 1966).

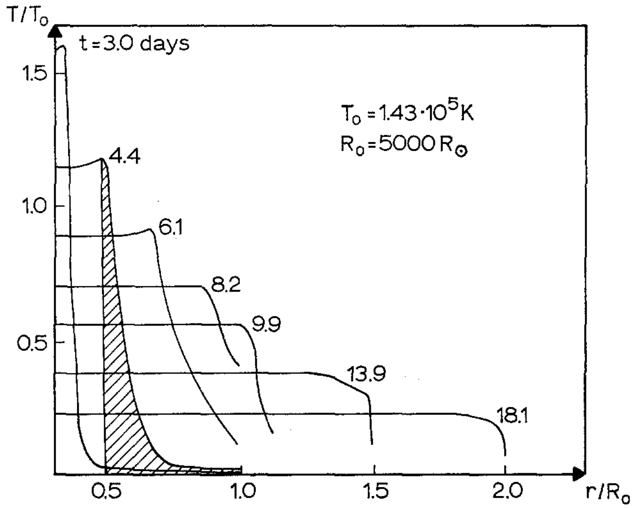

Fig. 7. The distribution of the temperature in Eulerian coordinates in the supernova envelope (Model No. 4). The shaded part of the distribution is for the time $t = 4.4$ days and indicates a hot 'tongue'.

Usually this discontinuity indicates a shock wave front. Its absence would imply that an isothermal stationary region builds up against the front. We may also estimate the breadth ΔR_T of the 'hot tongue' as follows. In so far as the radiation pressure significantly exceeds the gas pressure, the temperature conductivity is given by $c/3\ \kappa\varrho$. The breadth of the settled heat 'tongue' must then be, of course, of the order of

$$\Delta R_T = c/3\kappa\varrho D, \tag{1.1}$$

where D is the speed of the shock front. Substituting in (1.1) typical values of $D = 5000$ km/sec, $\kappa = 0.4$ cm^2/gm, $\varrho = 10^{-12}$ gm/cm^3 we have $\Delta R_T \approx 700\ R_\odot = 0.14\ R_0$. Adopting such dimensions as typical, we obtain the 'heat tongues' shown in Figures 7 and 8. The optical depth of the tongue is then $\Delta\tau_T = \Delta R_T \kappa\varrho \approx 20$. Because of the width of the 'heat tongue' compared with the characteristic scale of density variations (by the law $\varrho \sim r^{-3}$, we have $|\mathrm{d}r/\mathrm{d}\ln\varrho| = 0.33\,r$), the tongue cannot remain, strictly speaking, stationary. But much more important is the fact that, for $t = 6.1$ days, the tongue reaches the stellar surface (Figure 7), and rapidly loses energy by radiation outwards. All the rising branch of the light curve (AD on Figure 6) is accounted for by the penetration of the heat 'tongue' through the atmosphere. The gently sloping part BC reflects the first stage of heating-circulation of the ionization wave.

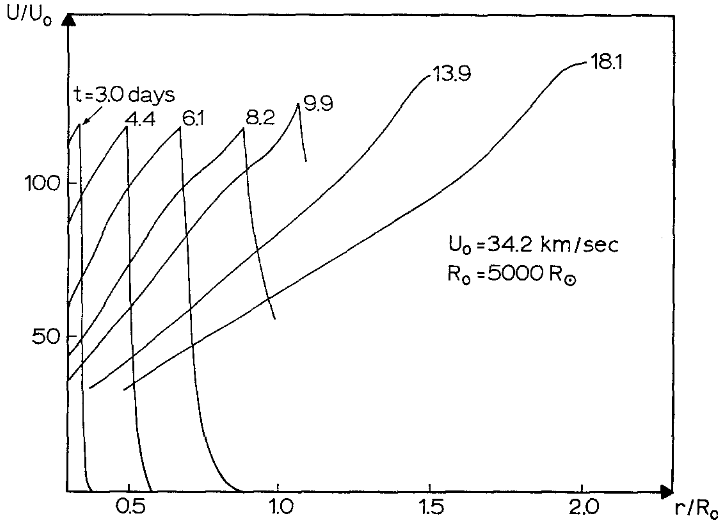

Fig. 8. The distribution of speeds in Eulerian coordinates in the supernova envelope (Model No. 4).

The movement of the 'tongue' sharply changes the law of density-distribution in the supernova envelope in comparison with its initial state. From the moment $t=6.1$ days the expenditure of energy from the shock front in heating up the outer part of the envelope becomes so large that braking of the gas at the shock front begins to set in. Viscous shock gives rise to an isothermal shock. The increase in the density behind such a shock may be thus given by

$$\varrho_2/\varrho_1 = (\mu U^2/RT) + 1\,, \tag{1.2}$$

where ϱ_1 is the density in front of the shock; ϱ_2, that immediately behind the shock; and U, the gas velocity behind the shock (in front of the shock the matter is supposed to be stationary), T is the temperature – constant in the isothermal shockwave, and R and μ denote the universal gas constant and molecular weight. The values for different sizes of the extended model with $R_0 = 5000$ to $10000\ R_\odot$ lead to compressions by a factor of a hundred or even a thousand, and numerical results agree with the relation (1.2) within 10–20%. At $t \approx 8$ days the density peak becomes optically thick, and for this reason the energy flux in the outer layers rapidly weakens. The supply of energy in the heat tongue is low and the brightness of the supernova begins to drop. At the moment of maximum brightness ($t \approx 8.2$ days) the isothermal shockwave situation still extends to 600 $R_\odot$ from the surface. Towards $t \approx 14$ days the shock front peters out, and all energy accumulated in the hot flame ends, and the mass of the whole supernova envelope ($\sim 1\ M_\odot$) proves to be confined to a thin ($\Delta r \approx 150\ R_\odot$) spherical shell (Figure 9, $t = 13.9$ days). The whole of this shell acquires a velocity of about 5000 km/sec. The radius of the photosphere increases more quickly than before. At $t = 35$ days, the outer boundary of the ejected shell begins to cool in the

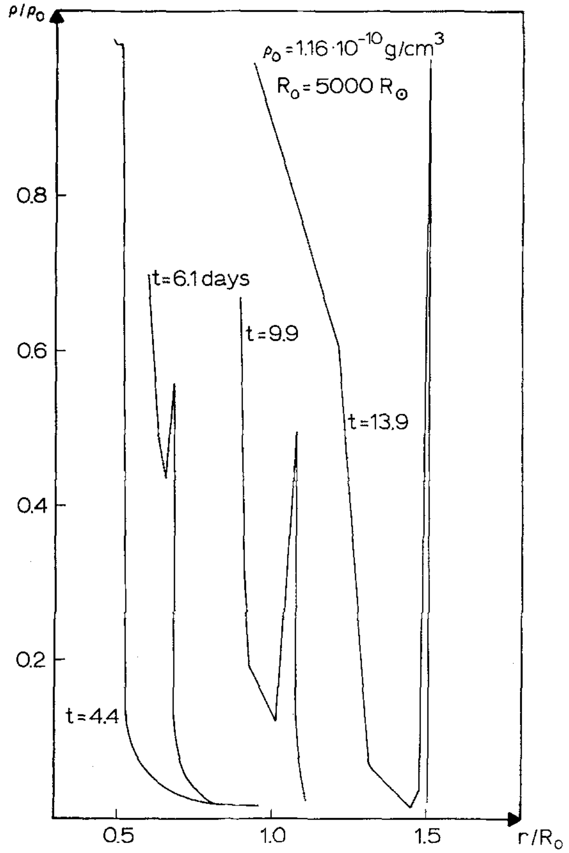

Fig. 9. The distribution of densities in Eulerian coordinates in the supernova envelope (Model No. 4).

CW regime and the increase in photospheric radius is again slowed down. The points on Figure 6 show how the radius would grow if there were no transfer of photosphere deep in the matter. Figures 10 and 11 show light curves and photosphere characteristics for two other extensive models (see Table I). The density of matter in the envelopes of these models is one or two orders smaller than that considered in detail in the previous models. For this reason, the effects of radiative heat conductivity are even more pronounced. An interesting brief decline of the light curve on Figure 11 ($t=19$ days) is produced by the interaction of the CW with an extremely large isothermal jump in density ($\varrho_2/\varrho_1 > 10^3$) (Grassberg and Nadyozhin, 1969b).

The transformation coefficient of energy E_0 of the explosion in the external radiation energy, E_r for all the models under consideration may be found in Table I. In the compact model ($R_0 \approx 10\ R_\odot$) it is only 3×10^{-4}; in the model with $R_0 = 500\ R_\odot$ this coefficient increases to 10^{-2} and it attains 0.02 to 0.05 in the case of the extensive models with $R_0 = 5000$ to $10000\ R_\odot$. The kinetic energy of the matter of an exploding supernova exceeds by a few orders of magnitude the radiative energy which is characteristic of type II supernovae (Shklovsky, 1966). In the supernova of Type I (SNI) these energies do not apparently differ by more than a single order; for this reason, the density of matter in the type I envelopes may be assumed to be significantly less than in those of type II. On the other hand, it is not certain whether a fundamental role is really played by processes not yet taken into account in the equations.

We must still stress that the effects of radiative heat conductivity in the extensive model leads to the separation of a shell in the form of a thin spherical layer. In the compact model the density monotonically increases with the passage – from the outer layers of the star which expand with a velocity greater than that of escape from the gravitational field of the configuration (Nadyozhin and Frank-Kamenetsky, 1964; Imshennik and Nadyozhin, 1964).

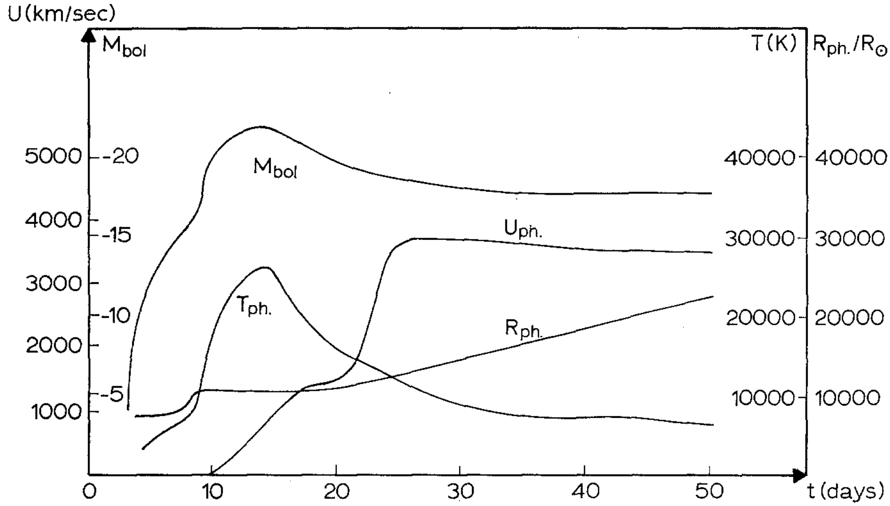

Fig. 10. The extensive Model No. 5, $R_0 = 10000\ R_\odot$, $M_0 = 30.9\ M_\odot$.

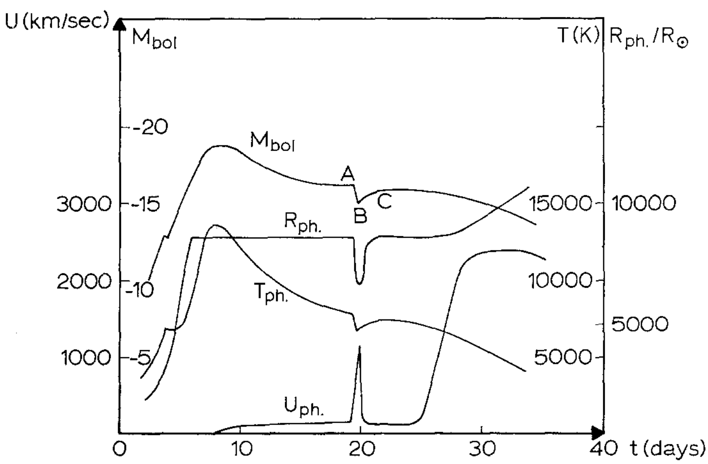

Fig. 11. The extensive Model No. 6, $R_0 = 10000\ R_\odot$, $M_0 = 3.09\ M_\odot$.

2. Recombination and Ionization of Gas in Supernova Shells

As was already shown by Imshennik and Nadyozhin (1964) a central role in radiation of heat in envelopes is played by particle recombination processes. First we may consider the cooling wave, which is so important in the formation of the SN light curve. As was mentioned before, inside the CW front a recombination of matter takes place which we may call surface recombination.

From Figure 1, it is seen that with the lowering of the temperature (at a given density) the coefficient of absorption κ passes through a maximum (with $\varrho \leqslant 10^{-12}$ gm/cm^3 the maximum is practically non-existent) and then it suddenly drops. In Figure 12, in the T-ϱ plane, the dotted line $T_{\kappa\,\max}(\varrho)$ is shown for which κ has a maximum for the given gas density; and the dotted line $T_\kappa(\varrho)$ corresponds to decreased opacities of 2, 10 and 100 times compared with $\kappa_{\max}$. The process of adiabatic expansion of

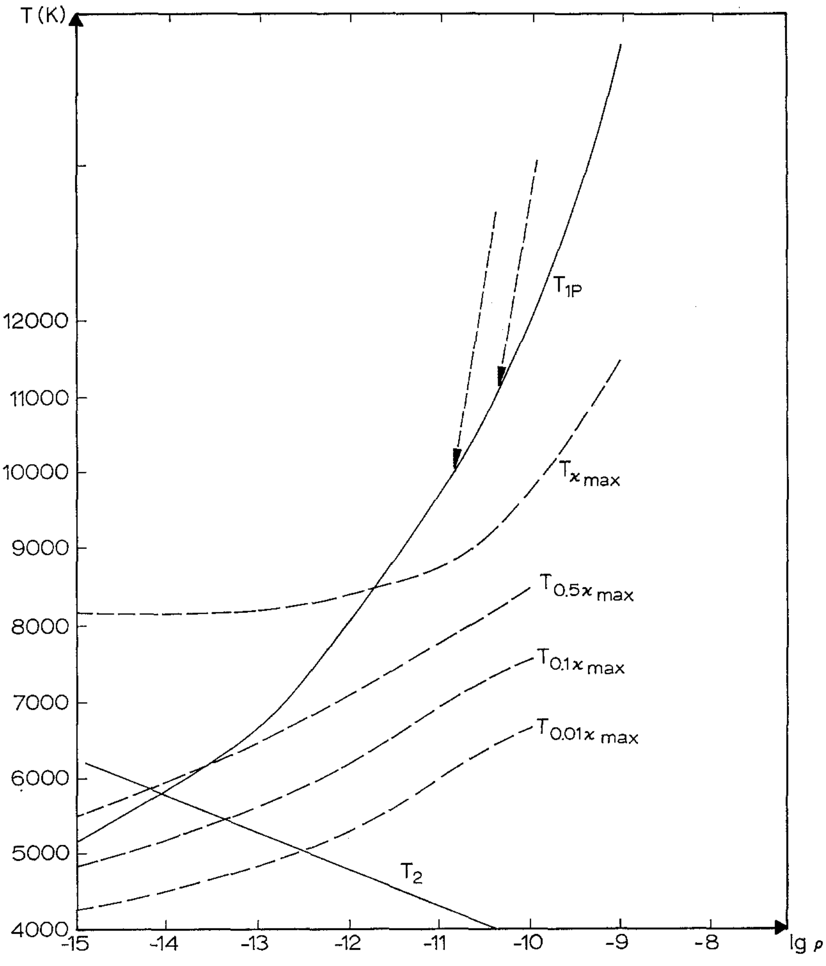

Fig. 12. The conditions giving rise to cooling waves.

matter in the SN envelope (adiabatic processes take place for sufficiently high densities) is accompanied by a cooling which almost exactly follows the law $T \sim \varrho^{1/3}$, due to the predominant role of the radiation. With the beginning of recombination the decline of temperature sharply slows down. The temperature T_{1P}, at which adiabatic cooling compensates for the recombination energy might be estimated from the condition $(P/\varrho^2)\,\mathrm{d}\varrho = R\,(\chi/k)\,\mathrm{d}x$. (The paper of Imshennik and Nadyozhin (1964) gives a rougher estimate for the temperature T_{1P} but this has little influence on the final result.) Since the degree of ionization x differs but little from unity, and $kT \ll \chi$, we have the simple equation to determine the function $T_{1P}(\varrho)$ of the form

$$T_{1P} = \frac{\chi}{k \ln\left[\frac{a m_P^2 \times 0.33}{\varrho^2 \chi^2} T_{1P}^{6.5}\right]}. \tag{2.1}$$

This function is indicated in Figure 12 by a continuous line. It is easily understood that the temperature may decrease for any Lagrangian particle of the gas until it reaches the T_{1P} curve, after which with further expansion it progresses along this curve up to the moment that it undergoes CW.

In the CW front the temperature changes rapidly from the value $T_1 \sim T_{1P}$ to a certain value T_2. CW means that there will be a sharp change in the law of cooling of the gas. From the lower temperature side the gas is transparent and is cooled by radiative flux outgoing from the CW front, but from the hot side the role of radiative heat conductivity is negligibly small. After the passage of the CW front through the gas particles the temperature T_2 is supported on account of the partial absorption of the radiative energy flux from the CW. The transparency temperature T_2 may be estimated from the empirical law $R'\kappa\varrho \approx 1$, where R' is a characteristic measure of the system size. On Figure 12 we give the curve $T_2(\varrho)$ where the characteristic size $R' = R_0 = 10^4\,R_\odot$.

The dependence of T_2 on R' is very weak; a change of R' by an order of magnitude leads to a change in T_2 less than 500 K. Figure 12 indicates that the density, $\varrho \leqslant 10^{-14}$ gm/cm^3, does not generally give rise to the condition of CW. This we may describe as the crossing of the curves of $T_{1P}(\varrho)$ and $T_2(\varrho)$; i.e., transparency begins before the start of recombination with expansion of the gas. But apart from this, a considerable significance attaches to another circumstance. It is true that the temperature before the CW front T_1 differs, strictly speaking, from the temperature T_{1P} as given by the relation (2.1). It is possible, for example, to connect this with the temperature $T_{\kappa\,\max}$; certainly it is not by chance that $T_{\kappa\,\max}$ is close to T_{1P}. For small densities we do not have a sharp difference in opacity for the curves of $T_{\kappa\,\max}$ and T_2 (the opacity changes altogether not more than a few times). For this reason, the CW cannot properly form at densities less than say 10^{-13} gm/cm^3. Here the opacity in the CW front decreases approximately by an order of magnitude. In such a way, an inspection of Figure 12 led us to the conclusion that CW make their appearance at gas densities $\varrho \geqslant 10^{-13}$ gm/cm^3.

The physical peculiarities of a CW in a hypothetical atmosphere have been described by Zeldovich and Reiser (1966). An important difference of the CW in the supernova envelope amounts to this: the basic process inside the front is recombination, and for that reason such a CW is naturally called a recombination wave.

A second important difference is connected with the fact that we are considering here a CW expanding with supersonic speed. Other properties of the CW (Zeldovich and Reiser, 1966) remain the same. For example, the more energetic the CW (we may neglect the internal energy behind the CW (E_2) in comparison with the value (E_1) before the front), the more appropriately we may use the condition of radiative heat-conductivity; moreover, the flux of radiation from the CW front then is given by a simple formula $S_2 = \frac{1}{2} ac\, T_2^4$. The law of preservation of the flux of energy in the supersonic CW for any point inside the wave may be written in the form

$$S = \varrho (U - D)(E_1 - E), \tag{2.2}$$

where D denotes the velocity of the CW front. It is possible to show by considering the full systems of the laws of conservation that the relation (2.2) remains valid with an accuracy up to the order of small quantities $(\Delta\varrho/\varrho)^2$. The brightness of the star is determined through the flux of S_2, across the transparent boundary of the CW, by $L = 4\pi R^2{}_{\mathrm{CW}} S_2$; and, in turn, using the relation of S_2 with the temperature of the transparency T_2, we may express from (2.2) the velocity of the CW relative to the moving gas as

$$D - U = -\frac{acT_2^4}{2\varrho(E_1 - E_2)}. \tag{2.3}$$

The example of $T_2 = 5000\,\mathrm{K}$, in (2.3), may allow us to calculate the difference of the velocities $D - U$ as a function of ϱ and T. On Figure 13 the dashed line shows the curve on which $D - U$ equals 2, 4 and 6 times the velocity of sound $c(\varrho, T)$. The speed of the CW relative to matter in those regions of the curve of $T_{1P}(\varrho)$ exceeds the speed of sound by almost an order of magnitude. But here the relative change of the density is very small: namely, $(\Delta\varrho/\varrho) \sim (c/D - U)^2 \approx 0.01$; and this justifies to a high degree of approximation the validity of the relation (2.2).

Finally Figure 13 shows the curves A and B. On curve A the released recombination energy stands at half the full difference of energies $E_1 - E_2$. From the right of this, as may be easily explained, we have $R(\chi/k) > \frac{1}{2}(E_1 - E_2)$. On curve B, one half of the energy $E_1 - E_2$ gives the change in the internal energy of radiation (from the left of this curve $(aT_1^4 - aT_2^4)/\varrho > \frac{1}{2}(E_1 - E_2)$). The proximity of the curves A and B clearly demonstrates the predominant role of the contribution of radiation to the internal energy. For the cooling wave in the SN envelope where $T_1 \approx T_{1P}$, the contribution of recombination energy is in the region of a half. The formation of CW for the model with $R_0 = 500\, R_\odot$ is indicated on Figure 5. Towards the time of $t = 17.9$ days isothermal transparent region may be well seen, with $T_2 = 5000\,\mathrm{K}$, and there is a temperature 'ledge'. The results of numerical values accord well with the account

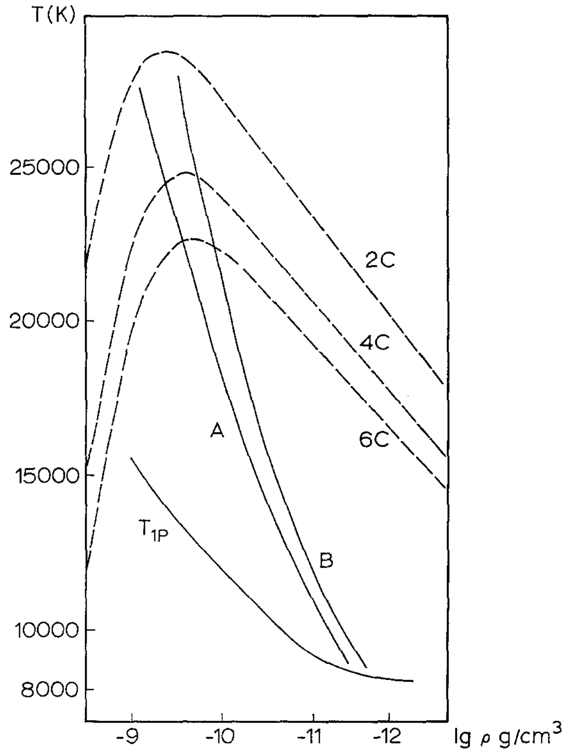

Fig. 13. Physical characteristics of cooling waves (see text).

in this paper of the CW theory. For three moments of time, $t \geqslant 17.9$, using the appropriate curve on Figure 5, we find the quantities ϱ and T_2. Then by Figure 12 we can determine the temperature before the CW front $T_1 = T_{1P}$ (or by the relation (2.1), one or the other):

t (days)	17.9	29.2	60.7
T_2 (K)	5000	4400	4000
ϱ (gm/cm^3)	4×10^{-13}	10^{-12}	2.5×10^{-12}
T_{1P} (K)	7300	8100	8800

We may recognize that the highest temperature, T_1, of the ledge on Figure 5 corresponds with T_{1P} from the theory. The position of the photosphere on Figure 5 also corresponds with the theoretical relation $S_2 = \frac{1}{2}acT_2^4$.

Finally, the origin of the CW at the time $t = 11.1$ days occurred for $\varrho = 10^{-13}$ gm/cm^3 which is also in agreement with our ideas. A description of the CW in the compact model may be found in the paper of Imshennik and Nadyozhin (1964).

The values of the degree of ionization are determined, in general, from Saha's equation. We now wish to estimate the extent to which an application of this equation is justifiable. For the temperatures and densities of interest to us, the basic processes are photo-ionization and photo-recombination. Accordingly, the equation of kinetic ionization takes the form

$$\mathrm{d}x/\mathrm{d}t = \alpha(1 - x) - \beta x^2, \tag{2.4}$$

where x is the degree of ionization; and the coefficients of ionization, α, and recombination, β, without including the number of atoms in the excited states of hydrogen, are given by (cf. Zeldovitch and Reiser, 1966)

$$\left.\begin{aligned} \alpha &= 4.9 \times 10^4 T_* e - \chi/kT_* \quad \sec^{-1}, \\ \beta &= 1.24 \times 10^{13} \frac{\varrho}{\sqrt{T_e}} \quad \sec^{-1}, \end{aligned}\right\} \tag{2.5}$$

where T_e and T_* are the electron temperature of the gas and the radiation in degrees K, and ϱ is in gm/cm^3. The characteristic times of ionization and recombination are given by $t_i = (1/\alpha)(x/1-x)$, $t_r = 1/\beta x$. In the steady state $\beta x^2 = \alpha(1-x)$ and $t_i = t_r = \tau$. Table II contains the values of τ for typical situations, calculated from the relations (2.5) at $T_e = T_* = T$. A comparison with the characteristic time of gas-dynamic expansion t_H shows in all cases the ionization produced is in equilibrium; and the application of the Saha formula is, therefore, justifiable.

Let us now consider the condition of ionization after the CW passage. In the calculations of Imshennik and Nadyozhin (1964) and Grassberg and Nadyozhin (1969a, b) such a transparent gas may be excluded from the consideration (the method of 'ejected' calculating point is described in Imshennik and Nadyozhin, 1964) because it does not exert any influence on the processes in the optically thick inner layers.

TABLE II

Times of the establishment of the ionization equilibrium

N	T(K)	ϱ (gm/cm^3)	τ	t_H	Notes corresponding to the situation
1	5×10^5	10^{-11}	5 sec	100 sec	Peak of brightness in the compact model No. 2, $R_0 = 9\ R_\odot$ (Figure 2)
2	9×10^4	10^{-8}	3×10^{-3} sec	10^4 sec	Brightess peak in model No. 3, $R_0 = 500\ R_\odot$ (Figure 4)
3	10^4	10^{-12}	10 sec	10 days	Before passage of CW through the gas (Figure 5)
4	7×10^3	10^{-12}	10 sec	0,3 day	Passage across the CW front (Figure 5)
5	4×10^3	10^{-13}	1 day	10 days	Movements of the transparent gas cooled by CW in the conditions of free ejection close to the CW (Figure 5)
6	4×10^4	10^{-12}	20 sec	5 days	Extended model No. 4, $R_0 = 5000\ R_\odot$. Maximum brightness (Figure 6)
7	6×10^3	10^{-13}	200 sec	20 days	Extended model No. 4, $R_0 = 5000\ R_\odot$, $t = 50$ days (Figure 6) Photosphere

For different versions on the outer boundary of the CW we have $T_2=4\text{–}5\times10^3$ K, $\varrho=10^{-11}$ to 10^{-12} gm/cm^3, which gives a degree of ionization corresponding to $x=3\times10^{-5}$ to 6×10^{-3}. With increasing distance from the CW front, the degree of ionization of the gas follows the generalized Saha's equation (Ambartsumyan *et al.*, 1952) which may be obtained from (2.4) with $\mathrm{d}x/\mathrm{d}t=0$, after multiplying α with the coefficient of radiation dilution W; and doing so we obtain

$$\frac{x^2}{1-x}=4\times10^{-9}W\frac{\sqrt{T_eT_*}}{\varrho}e^{-\chi/kT_*}. \tag{2.6}$$

It is possible to deduce that the temperature and radius of the photosphere $T_{ph}=T_*$ and R_{ph}, on our model of the CW (Figure 5, $t>30$ days) is almost independent of the time. If, moreover, the coefficient of the dilution of radiation takes the form $W=\frac{1}{4}(R_{ph}/r)^2$, it is easy to show that in the regime of free expansion of the stationary photosphere ($R_{ph}=$const., $T_*=$const.) the right-hand side of (2.6) is proportional to $t\sqrt{T_e}$. In the limiting case of the adiabatic cooling of the electron gas $T_e\sim t^{-2}$, and thus the degree of ionization, far from the photosphere, for transparent hydrogen, remains constant. In reality, the electron gas may be heated up on account of frequent ionization and recombination, with the result that T_e will diminish slowly with the time than as t^{-2}. In the other limiting case of $T_e=$const., the degree of ionization may slowly grow on account of the expansion ($x\sim\sqrt{t}$).

In such a way the degree of ionization, remote from the stationary photosphere of cooled hydrogen, will at least not decrease with the time and possess an order of magnitude of $x=10^{-4}$ to 10^{-2}. In the expansion stages up to the formation of the CW the outermost part of the supernova envelope undergoes recombination under the condition of transparency to the radiation (volume recombination). This may be clearly seen in Figure 5, where we have marked a few Lagrangian points corresponding to different masses, ΔM, measured from the surface. Up to the formation of the CW at 11.1 days, more than 0.0043 $M_\odot$ of hydrogen under the condition of almost full ionization becomes transparent. The density, temperature, and degree of ionization in the particles emerging from the photosphere at $t=11.1$ days are then $\varrho=1.6\times10^{-13}$ gm/cm^3; $T=8000$ K, $x=0.95$. At this point the time at which the ionization equilibrium (~50 sec) is reached is significantly less than the characteristic expansion time (~10 days); and, for that reason, we may employ the generalized Saha's equation (2.6). At the time $t=30$ days when the photosphere becomes stationary particles will be at a distance of $r=35.6\ R_0$ and will form gas of the density $\varrho=8\times10^{-15}$ gm/cm^3 with the coefficient of radiation dilution $W=\frac{1}{4}(19/35.6)^2=0.08$. Substituting this value together with $T_*=5500$ K, $T_e=4000$ K in (2.6) we obtain the degree of ionization $x=0.07$.

In such a way, hydrogen in the outer part of the supernova envelope with mass $\sim0.005\ M_\odot$ undergoes recombination during the time interval $\Delta t=20$ days in the regime of volume radiation. The full energy produced as a result of recombination, amounts to 13.54 eV$\times0.005\ M_\odot/m_P=1.3\times10^{44}$ erg. The average brightness due to volume recombination will be 1.3×10^{44} erg/20 days$=2 \times 10^4 L_\odot$, and too small to have influence on the form of the bolometric light curve. To influence the light curve there would have to be recombination of $\geqslant 1 M_\odot$ of hydrogen. The illumination of recombination energy in the emission lines of the Lyman and Balmer series may show, in particular, in the supernova spectra.

The effect of volume recombination may become significantly large in the very strong explosion of the extensive model ($R \sim 5000 R_\odot$, following the line in Table I), because, owing to the low density of matter the mass of the optically thin and ionized outer part of the envelope appears to be significantly greater. In the envelopes of the compact model ($R_0 \approx 10 R_\odot$) the density is too high and volume recombination is immaterial.

3. Comparison with Observations. Discussion of the Results

At the maximum observed brightness, the absolute photographic magnitude of supernovae, corrected for interstellar absorption is close to $M_P = -19^m$ for SNI and $M_P = -17^m$ to -18^m for SNII (Minkowski, 1964). Supernovae of the second type show a strong ultraviolet continuum close to maximum and colour temperatures of $20–40 \times 10^3$ K (Minkowski, 1939; Arp, 1961). The bolometric correction at this stage may attain -3 magn. (Shklovsky, 1966; Arp, 1961). In SNI a strong ultraviolet continuum is absent. The velocity of expansion of SNII envelopes is of the order of 3000 to 12000 km/sec (Zwicky, 1964, 1965). The speed of expansion of the SNI remnant – the Crab Nebula – is about 1000 km/sec.; however, it is not certain whether the SNI envelopes may also have significantly greater speeds (Minkowski, 1964, 1968; McLaughlin, 1963). Apparently small velocities are not distinguishing feature for SNI, but the kinetic energy of the ejection of the type I envelopes is less than that of the SNII (Minkowski, 1964).

In Table III we present basic theoretical values for the moment of maximum brightness in the cases of the extended models (Nos. 4 to 7), and for the moment

TABLE III

Characteristics at maximum brightness (Nos. of models correspond to Table I)

No. of model	$R_0/R_\odot$	$M_0/M_\odot$	M_{bol}	M_v	T_{eff} (K)	U_{ph} (km/sec)	$R/R_\odot$
1	9	15	$-11^m.3$	$-11^m.1$	5000	1200	2200
2	9	15	$-13^m.2$	$-12^m.8$	4400	4100	6800
Imshennik and Nadyozhin (1964)	20	50	$-14^m.6$	$-14^m.4$	4900	6500	10300
3	500	30	$-15^m.7$	$-15^m.7$	7500	5400	7200
4	5000	30.9	$-21^m.9$	$-18^m.8$	40000	4900	5000
5	10000	30.9	$-21^m.7$	$-19^m.6$	26000	3700	10000
6	10000	3.09	$-18^m.8$	$-17^m.9$	14000	2400	8700
7	5000	30.9	$-25^m.1$	$-19^m.8$	83000	17000	~ 6000

$t = 15$ days in the cases of the compact models including the model with $R_0 = 500\ R_{\odot}$ (Nos. 1 to 3). The visual absolute magnitudes M_v are calculated on the assumption of a Planckian distribution amongst the frequencies.

The results obtained for the extensive models show that the effective temperatures and velocities correspond significantly to supernovae of type II. However, the maximum of brightness of the theoretical models appears to be a few times larger (approximately by one magnitude), than those which are actually observed.

On Figure 14 the continuous lines indicate three characteristic theoretical light curves (Nos. 2, 3, 5 on table 3) plotted on the visual magnitude scale. The dashed lines show light curves for different supernova II types: NGC 4725, NGC 4559 (Zwicky, 1965; Minkowski, 1964) NGC 2841 (1957a) (Zwicky and Karpovich, 1965) and the dot-dash line gives the light curves for Supernovae type I – IC 4182 (Zwicky, 1965; Minkowski, 1964). From Figure 14 it may be seen that the breadth of the light curve of the extensive models is close to the maximum of $M_v = M_{max} + 1$, is equal to about 10 days. This all, then, is about 2–3 times smaller than the observed light curves for SN types II and I. In its form the theoretical light curve with $R_0 = 10000\ R_{\odot}$ is most of all similar to the light curve for the Supernova NGC 2841, but that supernova has an absolute magnitude at maximum of only $-14\overset{m}{.}8$ (Zwicky and Karpovich,

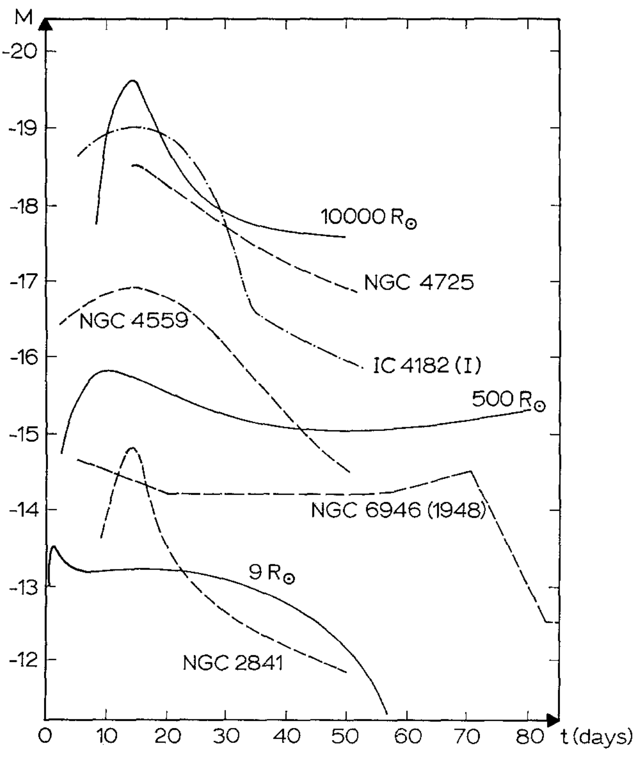

Fig. 14. Comparison of theoretical light curves (continuous curves) with observations (dotted curves). Owing to the scarcity of observations there may be ambiguities.

1965). Pskovsky (1967a) gives a rather different estimate: namely, $-15\overset{m}{.}8$ and a smaller velocity for the envelope (1000 to 2000 km/sec.)

Three supernovae of type II, NGC 5457, NGC 6946 (1948) and NGC 5236 exhibited anomalous light curves. The brightness of these supernovae was practically constant for a 50 to 100 days (plateau); and afterwards occurred a sharp drop of 2^m to 3^m with a following slow decline. The absolute magnitude in the phase of constant light were given as $-15\overset{m}{.}2$, $-14\overset{m}{.}7$ and $-13\overset{m}{.}7$ (Minkowski, 1964). This light curve fits in very well with the theoretical light curve with $R_0 = 500\ R_\odot$ in Figure 14 and, generally, with the compact model case No. 1 to 3 in Table III (the point-like peak is practically impossible to observe). Actually, the absolute visual magnitudes of the compact models, radiating in the long-wave regime according to Table III, lies within the limits $-11\overset{m}{.}1$ to $-15\overset{m}{.}7$ and the constancy of brightness lasts 30 to 100 days (see Figure 3). We do not exclude that, as a result of selective effects amongst the frequencies of the flares, the actual frequency of anomalous supernovae may be considerably greater.

The well-known break ('shoulder') in the SN II light curve has absolute magnitude -13^m to -15^m which is characteristic for the CW. If we take account also of the property of the CW to stabilize the brightness then we may assume that the shoulder phase indicates the existence of CW in the supernova envelope. In this connection it would be very interesting to obtain a light curve in the shoulder phase in different colours and to estimate the effective temperature, which should be close to 4000 to 5000 K. With regard to the short-period 'failure' of the light curve for the extended model with small mass (Model No. 6 in Table III, Figure 11), we mention the interesting supernova discovered by Rosino (1961) (see also Bertola, 1964), where an analogous failure was observed, although a little deeper and of longer duration.

On the spectra of supernovae we have to say little at the present time. If it is true that the 'shoulder' of the light curve corresponds to CW, then at the phase of maximum brightness, up to the shoulder, emission in the Lyman and Balmer lines of hydrogen in supernovae with large expansion velocities should be observed. The emission is due to recombination in the outermost parts of the envelope in the regime of volume radiation. Towards the beginning of the 'shoulder' phase the emission should change to absorption because, with the formation of the CW hydrogen, above the photosphere, is almost fully recombined. The condition of observing the emission lines strongly depends on the intensity of the continuum – i.e., in the final resort on the temperatures in the supernova envelopes. Information about these temperatures from the observations is so scarce that even for SN I it is impossible to conclude with certainty at the possibility of very high temperatures (Pskovski, 1968). The derived values of the effective temperatures of supernovae monotonically decrease after the light maximum, which is in accord with the observed reddening in SN II in between 30 to 100 days (Arp, 1961; Minkowski, 1964; Pskovski, 1967b) and for SN I in the period from 40 to 60 days (Mihalas, 1962; Minkowski, 1964; Bloch *et al.*, 1964; Pskovski, 1967b). The subsequent decrease in reddening is possibly connected with the change of the method of radiation transfer in the envelope in the late stages of expansion. In particular, for SN I the presence of a hot neutron star (Fintsi, 1965) may be essential.

A part of the results shown above is based on suppositions about the existence around the star of a very extensive atmosphere ($\sim 10000\,R_\odot$). Such an atmosphere may be formed either by the passage of the star along the evolutionary track of a supergiant, or by a slow ejection of matter from the star (Bisnovatyi-Kogan and Zeldovich, 1968) in the pre-supernova stage. Certain evidence relevant to the existence of an extended atmosphere is provided by Poveda and Woltjer (1968). If, for some reason, the star does not develop an extended atmosphere, then the light curve must be analogous to those of the anomalous supernovae NGC 5457, NGC 6946, and NGC 5236.

It is interesting to compare the results obtained above with the corresponding data in the work of Colgate and McKee (1969). For the sake of definiteness we may compare the data for the model with $R_0 = 10000\,R_\odot$ and $M_0 = 30.9\,M_\odot$ (the curve of light is shown on Figure 14) with the data of the version with the ejected envelope of mass $1.5\,M_\odot$, considered by Colgate and McKee. A comparison of parameters is presented in Table IV. First of all we conclude that the results of Colgate and McKee correspond better with a supernova type I (this is also pointed out by the authors), whereas the data obtained in the present work conform better to the supernova II type. The work of Colgate and McKee leads, no more than our own, to any fundamental difference between supernovae of both types. For example, Colgate and McKee examine a model with an additionally massive ($M = 7.25\,M_\odot$) hydrogen-helium envelope, while in our present work we constructed an extended model of small mass (No. 6 of Table III).

The important differences in theories underlying the SN models amount to the following: (1) Higher energy production per unit mass (typically by about two orders

TABLE IV

Comparison with the work of Colgate and Mackee (1969)
GIN – the present paper; CMK – the results of Colgate and Mackee (1969)

	Kinetic energy of the envelope	Mean energy per unit mass	Velocity in the photosphere, at the time of maximum brightness	Maximum photospheric velocity
GIN	1.6×10^{51} erg	2.4×10^{16} erg/gm	10^8 cm/sec	4×10^8 cm/sec
CMK	5×10^{51} erg	1.6×10^{18} erg/gm	2×10^9 cm/sec	2×10^9 cm/sec
	Effective temperature at maximum light	Full radiation energy	Character of the drop in the light curve	Radioactive material
GIN	26000K	7.9×10^{49} ergs	The maximum is in the form of a 'hump', with an extended 'plateau' lasting about 100 days	Absent
CMK	6500K	10^{49} ergs	The drop after the maximum is almost exponential	Ni^{56} $0.35\,M_\odot$

of magnitude) in the studies of Colgate and McKee in comparison with our data. (2) At the moment of maximum brightness the effective temperature by Colgate and McKee is 2 to 4 times lower. (3) The ratio of radiative energy to kinetic energy of the envelope in our values for the extensive model $\geqslant 10^{-2}$, whereas for Colgate and McKee it is $\leqslant 10^{-3}$. (4) Although we consider a model different from that given by Colgate and McKee we agree with a clean thermonuclear mechanism of the supernova explosions.

By carrying out this comparison, real differences are apparent in the two types of models of the supernovae, which obviously should appear in the observations. Measurements of the continuous spectra of supernovae in the ultraviolet are of special importance. If the effective temperatures are really so high as the colour temperatures of 20 to 40×10^3 K, then, in theory, radioactive disintegration will be sufficient to increase the mass of radioactive material by an order of magnitude, which seems improbable. The radioactive source in the extended models would but insignificantly change the light curve except at the late stages, so that the full energy of the disintegration, $0.35\,M_\odot$ Ni^{56}, equals 1.8×10^{49} erg, or a few times less than the radiation energy E_r (Table I). On the other hand, for the extended models with small mass (No. 6 of Table III) this energy even exceeds E_r, and thus a noticeable influence of the radioactive sources may be expected in the models with small mass and initial radius.

The results obtained in the present work are based on the assumption of the passage through the atmosphere of powerful shock waves. A more detailed elaboration of the mechanism of explosion of the supernovae leads to such a conclusion. Recently Sparks (1969) pointed out also the following interesting circumstance. If below the surface of the star sufficient energy can be accumulated in a time which is significantly greater than the characteristic hydrodynamic time of the star, and significantly less than the time of its thermal relaxation, then the topmost part of the envelope is ejected with a velocity exceeding that of escape. In the case of a nova explosion, Sparks (1969) showed that the rate of rise and duration of the light changes are determined by the rate of energy production.

In such a way it is possible to have sufficiently long-lasting light changes even in the compact models. It is not certain which mechanism takes place for certain supernovae on the evolutionary track preceding the formation of an iron nucleus.

A certain spread of the sharp peak on the light curve of the compact models may be due to the instability of the shock front propagating through media of diminishing density (Gurevich and Rumyantsev, 1969).

The idea of explaining the light curve of the supernovae by fluorescence apparently meets with serious energy difficulties (this was also pointed out by Colgate and McKee 1969). According to Morrison and Sartory (1969) the energy in the primary light impulse must be of the order of 10^{52} ergs. If account is taken of the fact that, in the compact models (implicitly considered by Morrison and Sartory), the coefficient of transformation of the explosive energy into radiation is less than 10^{-3} (Table I) then the full energy of explosion attains the enormous amount of 10^{55} erg$=5\,M_\odot c^2$.

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References

Ambartsumyan, V. A., Mustel, E. R., Severnyi, A. B., and Sobolev, V. V.: 1952, *Teoreticheskaya Astrophysika*, Gostechizdat, Moscow.
Arnett, D.: 1967, *Can. J. Phys.* **44**, 2553.
Arnett, D.: 1969, *Astrophys. Space Sci.* **5**, 180.
Arp, H.: 1961, *Astrophys. J.* **133**, 883.
Bertola, F.: 1964, *Ann. Astrophys.* **27**, 298.
Bisnovatyi-Kogan, G. S. and Zeldovich, Y. B.: 1968, *Astron. Zh.* **45**, 2.
Bloch, M., Chalonge, D., and Dufay, J.: 1964, *Ann. Astrophys.* **27**, 315.
Borst, L. B.: 1950, *Phys. Rev.* **78**, 807.
Burbidge, G. R., Hoyle, F., Burbidge, E. M., Christy, B. F., and Fowler, W. A.: 1956, *Phys. Rev.* **103**, 1145.
Colgate, S. A. and McKee, Ch.: 1969, *Astrophys. J.* **157**, 623.
Colgate, S. A. and White, R. H.: 1966, *Astrophys. J.* **143**, 626.
Fintsi, A.: 1965, *Phys. Rev. Letters* **15**, 599.
Fowler, W. A. and Hoyle, F.: 1964, *Astrophys. J. Suppl.* **91**, 9.
Grassberg, E. K. and Nadyozhin, D. K.: 1969a, *Astron. Zh.* **46**, 745.
Grassberg, E. K. and Nadyozhin, D. K.: 1969b, *Nauchyiye Informatsii Astron. Sovieta USSR Acad. Sci.* **13**, 96.
Gurevitch, L. E. and Rumyantsev, A. A.: 1969, *Astron. Zh.* **46**, 1158.
Hansen, C. and Wheeler, J.: 1969, *Astrophys. Space Sci.* **3**, 464.
Imshennik, V. S. and Morozov, Y. I.: 1969, *Astron. Zh.* **46**, 800.
Imshennik, V. S. and Morozov, Y. I.: 1964, *Probl. Math. Tech. Fiziki* **2**, 8.
Imshennik, V. S. and Nadyozhin, D. K.: 1967, 'On Supernova Flares', published in *Relativistic Astrophysics*, Nauka, Moscow (ed. by Y. B. Zeldovich and I. D. Novikov).
Ivanova, L. N., Imshennik, V. S., and Nadyozhin, D. K.: 1969, *Nauchyiye Informatsii Astron. Sovieta USSR Acad. Sci.* **13**, 3.
McLaughlin, D. B.: 1963, *Publ. Astron. Soc. Pacific* **75**, 133.
Mihalas, D.: 1962, *Publ. Astron. Soc. Pacific* **74**, 116.
Minkowski, R.: 1939, *Astrophys. J.* **89**, 156.
Minkowski, R.: 1964, *Ann. Rev. Astron. Astrophys.* **2**, 247.
Minkowski, R.: 1968, 'Nebulae and Interstellar Matter', in *Stars and Stellar Systems* **VII** (ed. by Aller and Middlehurst), University of Chicago Press, Ch. 11.
Morrison, P. and Sartori, L.: 1969, *Astrophys. J.* **158**, 541.
Nadyozhin, D. K. and Frank-Kamenetski, D. A.: 1964, *Voprosyi Kosmogonii* **10**, 154.
Poveda, A. and Woltjer, L.: 1968, *Astron J.* **73**, 65.
Pskovski, Y. P.: 1967a, *Astron. Zh.* **44**, 82.
Pskovski, Y. P.: 1967b, *Astron. Zh.* **44**, 708.
Pskovski, Y. P.: 1968, *Astron. Zh.* **45**, 1160.
Rosino, L.: 1961, *IAU Circular*, No. 1782 (1961).
Shklovski, I. S.: 1966, *Supernova Stars*, Nauka, Moscow.
Sparks, W. A.: 1969, *Astrophys. J.* **156**, 569.
Zeldovich, Y. B. and Reiser, Y. P.: 1966, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, Nauka, Moscow.
Zwicky, F.: 1964, *Ann. Astrophys.* **27**, 300.
Zwicky, F.: 1965, 'Stellar Structure' in *Stars and Stellar Systems* (ed. by L. H. Aller, and D. B. McLaughlin), Univ. of Chicago Press, p. 367.
Zwicky, F. and Karpowich, M.: 1965, *Astron J.* **70**, 564.