# RELATIVISTIC REDUCTIONS FOR RADIOINTERFEROMETRIC OBSERVABLES

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**Abstract.** The special and general relativistic corrections for the basic radiointerferometric observables – time-delay and fringe frequency – are studied with an accuracy of the order of $5 \times 10^{-10}$ ('post-Newtonian approximation').

## 1. Introduction

Very-long baseline interferometry plays an important role in the different domains of observational astronomy, and, in particular, in positional astronomy (Dravskich and Finkelstein, 1980). Now, by use of the VLBI technique, the exact positions of sources are determined more precisely than by optical techniques, and the accuracy of the existing radio catalogues is higher than the optical catalogue FK-5 (Fanselow *et al.*, 1981). Furthermore, in the near future, it is planned to construct special wide-band phase-stable VLBI networks with resolving powers up to 0ʺ.001 and data processing 'on-line regime' (Dravskich *et al.*, 1981). This system will be able to solve a lot of qualitatively new problems in different domains of natural sciences.

All useful information on a source observed with a interferometer is extracted by studying quantities quadratic in the field. In a correlation interferometer, in which fields are multiplied together, this quantity is the complex response to a point or extended source. The complex amplitude of this response (so-called 'visibility function') is determined by the brightness distribution of the radio objects and the phase-mutual geometry of the source and baseline of the interferometer. In such circumstances, the response amplitude gives important information about the source structure, phase and, in particular, about its position.

In problems involving positional measurements that are performed, as a rule, on point sources, the phase of response is represented in the form

$$\phi_g = \frac{2\pi}{\lambda} (\mathbf{b} \cdot \mathbf{k}), \tag{1}$$

where **b** is the base vector, **k** is the unit vector in the direction of the source, and $\lambda$ is the wavelength of the incoming radiation. Expressed in time units, it amounts to time-delay

$$\tau_g = \frac{\lambda}{2\pi c} \phi_g = \frac{1}{c} (\mathbf{b} \cdot \mathbf{k}), \tag{2}$$

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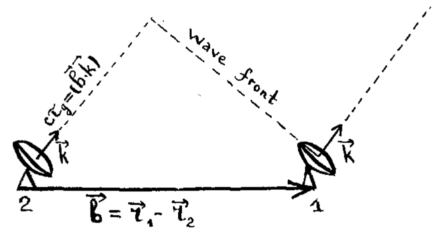


Fig. 1. The definition of base-vector and geometrical-delay.

where $c$ is the velocity of light. This is, in fact, determined by the path difference of the rays from the source to the antennas of the interferometer (Figure 1).

At the source moves through the interferometer diagram due to Earth rotation, since $\tau_g$ is a function of the time, an interference pattern arises with fringe frequency

$$F_g = f \frac{d\tau_g}{dt} = \frac{f}{c} (\mathbf{b} \times \mathbf{\Omega})\mathbf{k}, \tag{3}$$

where $\mathbf{\Omega}$ is the rotational velocity vector of the Earth, and $f$ is the frequency of the radiation being received.

The quantities $\tau_g$ and $F_g$ defined above, have a purely geometrical nature, where many factors corresponding to the real process of the spreading of the plane wave from the source to the radiometers – such as the behaviour of the interferometer in space – and with time, are not accounted for. For an adequate description of the real situation concerning observational $\tau$ and $F$, it is essential to include additional terms, which can be divided into two groups.

In the first group come those corrections which are based on partly verified or not verified theories. The parameters of these theories are still to be improved upon by making use of VLBI observations. In particular, corrections for clock model, precession, nutation, pole motion, tidal movements, global tectonical movements of the Earth's crust, proper motion of the source, and the like, belong to this group. The introduction of such types of corrections complicates the expressions $\tau_{\text{obs}}$ and $F_{\text{obs}}$, and increases the number of parameters to be determined. However, such problems can be solved if one treats together the data for $\tau_{\text{obs}}$ and $F_{\text{obs}}$ as obtained by observing many sources with various declinations in different hour angles (Brosche *et al.*, 1973; Rogers, 1973).

The second group consists of such corrections as can be determined on the basis of reliable theories, or, by indirect measurements. These are, in particular, numerous apparatus corrections, corrections for ionospherical and tropospheric refraction, gravitational refraction, linear and quadratic abberations, relativistic effects in time-scales, etc.

In the present work, we investigate in details such a group of prior-calculated corrections connected with a finite velocity of light and the influence of the gravitational field of the Sun. In view of the high resolving power of the VLBI technique, we study the effects of special and general relativity $\tau$ and $F$ with an accuracy of $5 \times 10^{-10}$ ('post-Newtonian approximation').

## 2. Time-Delay

Let us now consider the definition of $\tau_{\text{obs}}$ in heliocentric coordinate system $S_s$ (Figure 2). The coordinate time of the spreading of a wave between points having heliocentric coordinates $(\mathbf{R}_i^s, t_i^s)$ and $(\mathbf{R}_0^s, t_0^s)$ is given by

$$c(t_i^s - t_0^s) = |\mathbf{R}_0^s(t_0^s) - \mathbf{R}_i^s(t_i^s)| + r_g \ln \frac{R_i^s - \mathbf{R}_i^s \cdot \mathbf{k}_i}{R_0^s - \mathbf{R}_0^s \cdot \mathbf{k}_i}, \tag{3}$$

where the second term of expression (3) represents the influence of gravitational field of the Sun (Brumberg, 1972), $r_g = 2GM_s/c^2$ is the gravitational radius of the Sun, $M_s$ is the mass of the Sun, $G$ is the gravitational constant, and $\mathbf{k}_i$ is the unit vector in the direction of the source from the $i$th telescope. Thus, time-delay is expressed in coordinate

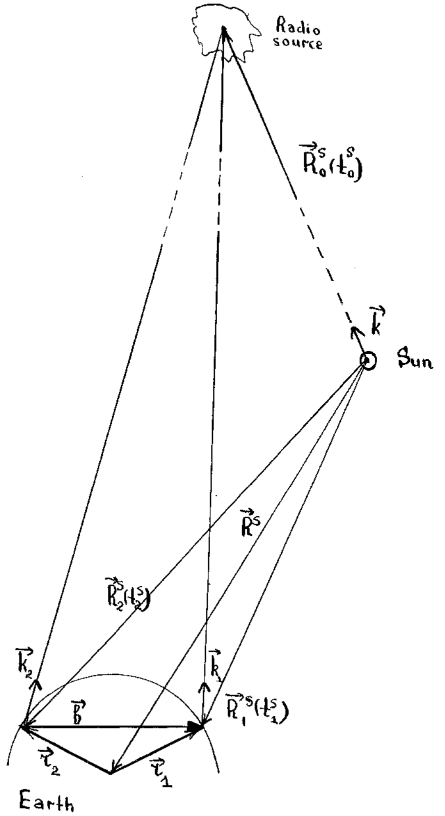


Fig. 2. The definition of time-delay in heliocentric coordinate system $S_s$.

time ($t_2^s > t_1^s$) in the form

$$\tau'_{\rm obs} = t_2^s - t_1^s = \frac{1}{c}\,|\mathbf{R}_0^s(t_0^s) - \mathbf{R}_2^s(t_2^s)| - \frac{1}{c}\,|\mathbf{R}_0^z(t_0^s) - \mathbf{R}_1^s(t_(^s)| +$$

$$+ \frac{r_g}{c}\left\{\ln\frac{R_2^s - \mathbf{R}_2^s\cdot\mathbf{k}_2}{R_1^s - \mathbf{R}_1^s\cdot\mathbf{k}_1} + \ln\frac{R_0^s - \mathbf{R}_0^s\cdot\mathbf{k}_1}{R_0^s - \mathbf{R}_0^s\cdot\mathbf{k}_2}\right\}.$$

As the observable sources are located in galactic or extragalactic distances, where $R_0^s \gg R_i^s$, we have

$$|\mathbf{R}_0^s(t_0^s) - \mathbf{R}_i^s(t_i^s)| \approx R_0^s(t_0^s) - \mathbf{R}_i^s(t_i^s)\mathbf{k},$$

$$R_0 - \mathbf{R}_0\cdot\mathbf{k}_i = R_0(1 - \mathbf{k}\cdot\mathbf{k}_i),$$

$$\mathbf{k}_i \simeq \mathbf{k} + \frac{1}{R_0^s}\cdot\{\mathbf{k}(\mathbf{R}_i^s\cdot\mathbf{k}) - \mathbf{R}_i^s\} - \frac{1}{(R_0^s)^2}\left\{\mathbf{R}_i^s(\mathbf{R}_i^s\cdot\mathbf{k}) + \mathbf{k}\,\frac{(\mathbf{R}_i^s)^2}{2} - \tfrac{3}{2}\mathbf{k}(\mathbf{R}_i^s\cdot\mathbf{k})^2\right\},$$

$$\frac{R_0^s - \mathbf{R}_0^s\cdot\mathbf{k}_1}{R_0^s - \mathbf{R}_0^s\cdot\mathbf{k}_2} \approx \frac{(\mathbf{R}_1^s)^2 - (\mathbf{R}_1^s\cdot\mathbf{k})^2}{(\mathbf{R}_2^s)^2 - (\mathbf{R}_2^s\cdot\mathbf{k})^2},$$

where $\mathbf{k} = \mathbf{R}_0^s/R_0^s$.

Consequently,

$$t_2^s - t_1^s = \tau'_{\rm obs} = \frac{1}{c}\,\{\mathbf{R}_1^s(t_1^s) - \mathbf{R}_2^s(t_2^s)\}\,\mathbf{k} + \tau_{\rm grav}, \tag{4}$$

where the correction related to gravitational refraction $\tau_{\rm grav}$ can be expressed in the form (here and in what follows we omit index '$s$' in terms of order of $1/c^3$)

$$\tau_{\rm grav} = \frac{r_g}{c}\ln\frac{R_1 + \mathbf{R}_1\cdot\mathbf{k}}{R_2 + \mathbf{R}_2\cdot\mathbf{k}} \approx$$

$$\approx \frac{r_g}{c}\ln\frac{1 + \dfrac{\mathbf{R}}{R}\,\mathbf{k} + \dfrac{1}{R}\left\{\left(\mathbf{r}_1\,\dfrac{\mathbf{R}}{R} + \mathbf{r}_1\cdot\mathbf{k}\right) + \dfrac{r_1^2}{2R} - \dfrac{(\mathbf{R}\cdot\mathbf{r}_1)^2}{2R^3}\right\}}{1 + \dfrac{\mathbf{R}}{R}\,\mathbf{k} + \dfrac{1}{R}\left\{\left(\mathbf{r}_2\,\dfrac{\mathbf{R}}{R} + \mathbf{r}_2\cdot\mathbf{k}\right) + \dfrac{r_2^2}{2R} - \dfrac{(\mathbf{R}\cdot\mathbf{r}_2)^2}{2R^3}\right\}}. \tag{5}$$

It can be easily be seen that when the source is located in the 'upper conjunction' with one of the telescopes of interferometer $\tau_{\rm grav} \to \infty$. If $\mathbf{r}_i/r_i$ is not parallel with $\mathbf{k}$,

$$\tau_{\rm grav} = \frac{r_g}{cR}\,\frac{(\mathbf{b}\cdot\mathbf{k}) + \left(\mathbf{b}\cdot\dfrac{\mathbf{R}}{R}\right)}{1 + \dfrac{\mathbf{R}}{R}\,\mathbf{k}}. \tag{6}$$

Now we proceed to transform the first term of the right-hand side of expression (4). Introducing the definition of the base vector as the difference of $\mathbf{R}_i^s$ in some instant of time $\tilde{t}^s$ as

$$\mathbf{b}^s = \mathbf{R}_1^s(\tilde{t}^s) - \mathbf{R}_2^s(\tilde{t}^s)\,, \tag{7}$$

we put $\tilde{t}^s$ in a general expression of the form

$$\tilde{t}^s = (\tilde{t}_1^s + \gamma t_2^s)/(1+\gamma)\,, \tag{8}$$

where $0 \leq \gamma \leq \infty$. The case $\gamma = 0$ corresponds to the definition of base vector $\mathbf{b}^s$ as the difference of $\mathbf{R}_i^s$ in heliocentric instant $t_1^s$ and the case $\gamma \to \infty$ corresponds to the instant $t_2^s$.

It is easy to see that

$$\begin{aligned}\mathbf{R}_2^s(t_2^s) - \mathbf{R}_1^s(t_1^s) = {} & -\mathbf{b}^s + \frac{1}{(1+\gamma)c}\,(\dot{\mathbf{R}}_2^s + \gamma\dot{\mathbf{R}}_1^s)\,(\mathbf{b}^s \cdot \mathbf{k}) - \\ & - \frac{1}{(1+\gamma)^2 c^2}\,(\dot{\mathbf{R}}_2^s + \gamma\dot{\mathbf{R}}_1^s)\,(\mathbf{b}^s \cdot \mathbf{k})\,\{(\dot{\mathbf{R}}_2^s + \gamma\dot{\mathbf{R}}_1^s)\mathbf{k}\} + \\ & + \frac{1}{2c^2(1+\gamma)^2}\,(\ddot{\mathbf{R}}_2^s - \gamma^2\,\ddot{\mathbf{R}}_1^s)\,(\mathbf{b}^s \cdot \mathbf{k})\,. \end{aligned} \tag{9}$$

Substituting (9) in (4), we get

$$\begin{aligned}\tau'_{\text{obs}} = {} & \frac{1}{c}\,(\mathbf{b}^s \cdot \mathbf{k}) - \frac{1}{c^2(1+\gamma)}\,[(\dot{\mathbf{R}}_2^s + \gamma\dot{\mathbf{R}}_1^s)\mathbf{k}]\,(\mathbf{b}^s \cdot \mathbf{k}) + \\ & + \frac{1}{c^3(1+\gamma)^2}\,[(\dot{\mathbf{R}}_2 + \gamma\dot{\mathbf{R}}_1)\mathbf{k}]^2\,(\mathbf{b} \cdot \mathbf{k}) - \\ & - \frac{1}{2c^3(1+\gamma)^2}\,[(\ddot{\mathbf{R}}_2 - \gamma^2\ddot{\mathbf{R}}_1)\mathbf{k}]\,(\mathbf{b} \cdot \mathbf{k})^2 + \tau_{\text{grav}}\,. \end{aligned} \tag{10}$$

Taking into account

$$\mathbf{R}_i^s = \mathbf{R}^s + \mathbf{r}_i^s \tag{11}$$

we proceed to transform Equation (10) from heliocentric coordinate system $S_s$ to geocentric coordinate system $S_e$. First of all, if is essential to express the base vector in system $S_e$ as the difference of $\mathbf{r}_i$ in geocentrical instant

$$\tilde{t} = (t_1 + \gamma t_2)/(1+\gamma)\,. \tag{12}$$

It should be noted that heliocentric instant $t_1^{s'}$ at telescope No. 2 is simultaneous in system $S_e$ to instant $t_1$ at telescope No. 1. It is, therefore, calculated by using the time component of Lorentz transformation

$$t_1^{s'} = t_1^s - \frac{1}{c^2}(\mathbf{b}^s \cdot \mathbf{V}^s), \qquad \mathbf{V}^s = \dot{\mathbf{R}}^s. \tag{13a}$$

Likewise, heliocentric instant $t_2^{s'}$ at telescope No. 1, which is simultaneous in system $S_e$ to $t_2$ at telescope No. 2, is given by

$$t_2^{s'} = t_2^s + \frac{1}{c^2}(\mathbf{b}^s \cdot \mathbf{V}^s). \tag{13b}$$

Consequently, the geocentric instant $\tilde{t}$ at telescope No. 1 corresponds to heliocentric instant

$$\tilde{t}^{(1)s} = (t_1^s + \gamma t_2^{s'})/(1+\gamma) = t^s + \frac{\gamma}{1+\gamma}\frac{\mathbf{b}^s \cdot \mathbf{V}^s}{c^2}; \tag{14a}$$

and for telescope No. 2,

$$\tilde{t}^{(2)s} = (t_1^{s'} + \gamma t_2^s)/(1+\gamma) = \tilde{t}^s - \frac{1}{1+\gamma}\frac{\mathbf{b}^s \cdot \mathbf{V}^s}{c^2}. \tag{14b}$$

According to the space component of the Lorentz transformation, we have

$$\mathbf{b} = \mathbf{B}^s - \frac{\mathbf{B}^s \cdot \mathbf{V}^s}{(V^s)^2}\mathbf{V}^s + \frac{\dfrac{\mathbf{B}^s \cdot \mathbf{V}^s}{(V^s)^2}\mathbf{V}^s - \mathbf{V}^s(\tilde{t}^{(1)s} - \tilde{t}^{(2)s})}{\sqrt{1 - \left(\dfrac{V^s}{c}\right)^2}}, \tag{15}$$

where

$$\mathbf{B}^s = \mathbf{R}_1^s(t^{(1)s}) - \mathbf{R}_2^s(t^{(2)s}); \tag{15a}$$

$$\mathbf{b}^s = \mathbf{b} - \frac{\mathbf{V}_2 + \gamma\mathbf{V}_1}{(1+\gamma)c^2}(\mathbf{b} \cdot \mathbf{V}^s) - \frac{1}{2c^2}\mathbf{V}^s(\mathbf{b} \cdot \mathbf{V}^s). \tag{16}$$

Substituting (11) and (16) in (10), and taking into account $\mathbf{V}_i = \dot{\mathbf{r}}_i$, we obtain

$$\begin{aligned}\tau'_{\text{obs}} = \tau_g \Bigg\{ 1 &- \frac{1}{c}\left[\left(\mathbf{V}^s + \frac{\gamma\mathbf{V}_1 + \mathbf{V}_2}{1+\gamma}\right)\mathbf{k} - \frac{1-\gamma}{2(1+\gamma)}\tau_g(\mathbf{W}^s \cdot \mathbf{k}) - \right.\\ &\left. - \frac{1}{2(1+\gamma)^2}\tau_g(\mathbf{W}_2 - \gamma^2\mathbf{W}_1)\mathbf{k}\right] + \frac{1}{c^2}\left[(\mathbf{V}^s \cdot \mathbf{k})^2 + \right.\\ &\left. + \frac{2}{1+\gamma}(\mathbf{V}^s \cdot \mathbf{k})(\gamma\mathbf{V}_1 + \mathbf{V}_2)\mathbf{k} + \frac{1}{(1+\gamma)^2}((\gamma\mathbf{V}_1 + \mathbf{V}_2)\mathbf{k})^2\right]\Bigg\} - \\ &- \frac{1}{c^3}(\mathbf{b} \cdot \mathbf{V}^s)\frac{\gamma\mathbf{V}_1 + \mathbf{V}_2}{1+\gamma}\mathbf{k} - \frac{1}{2c^3}(\mathbf{V}^s \cdot \mathbf{k})(\mathbf{b} \cdot \mathbf{V}^s) + \tau_{\text{grav}},\end{aligned} \tag{17}$$

where

$$\tau_g = \frac{1}{c}\,(\mathbf{b}\cdot\mathbf{k}) = \frac{1}{c}\,(\mathbf{r}_1(\tilde{t}) - \mathbf{r}_2(\tilde{t}))\mathbf{k} \tag{18}$$

and $\mathbf{W}^s = \dot{\mathbf{V}}^s$, $\mathbf{W}_i = \dot{\mathbf{V}}_i$.

If we consider $\tau'_{\text{obs}}$ with an accuracy of the order of $5 \times 10^{-10}$, then

$$\tau'_{\text{obs}} = \tau_g \left\{ 1 - \frac{1}{c}\left(\mathbf{V}^s + \frac{\gamma\mathbf{V}_1 + \mathbf{V}_2}{1+\gamma}\right)\mathbf{k} + \frac{1}{c^2}\,(\mathbf{V}^s\cdot\mathbf{k}) - \right.$$

$$\left. - \frac{1-\gamma}{2(1+\gamma)}\,\frac{\tau_g}{c}\,(\mathbf{W}^s\cdot\mathbf{k}) \right\} - \frac{1}{2c^3}\,(\mathbf{V}^s\cdot\mathbf{k})\,(\mathbf{b}\cdot\mathbf{V}^s) + \tau_{\text{grav}} \tag{19}$$

and with an accuracy of the order of $5 \times 10^{-9}$, then

$$\tau'_{\text{obs}} = \tau_g \left\{ 1 - \frac{1}{c}\left(\mathbf{V}^s + \frac{\gamma\mathbf{V}_1 + \mathbf{V}_2}{1+\gamma}\right)\mathbf{k} + \frac{1}{c^2}\,(\mathbf{V}^s\cdot\mathbf{k})^2 \right\} -$$

$$\frac{1}{2c^3}\,(\mathbf{V}^s\cdot\mathbf{k})\,(\mathbf{b}\cdot\mathbf{V}^s) + \tau_{\text{grav}}\,. \tag{20}$$

As we have already noted, $\tau'_{\text{obs}} = t_2^s - t_1^s$ is expressed in terms of coordinate time, so it should be transformed by using well-known relation

$$\mathrm{d}s_i = \left\{ 1 - \frac{1}{c^2}\left(\frac{(\dot{\mathbf{R}}_i^s)^2}{2} + \phi_i\right)\right\} \mathrm{d}t^s \tag{21}$$

to $\tau_{\text{obs}} = s_2 - s_1$, where $s_i$ is the proper time and $\phi_i = GM_s/R_i$ is the Newtonian potential.

Since the local clocks of the telescopes have been synchronized in heliocentric instant of time $\tilde{t}^s$, then

$$\tau_{\text{obs}} = s_2 - s_1 = (s_2 - \tilde{s}) + (\tilde{s} - s_1) =$$

$$= \int_{\tilde{t}^s}^{t_2^s} \left\{ 1 - \frac{1}{c^2}\left(\phi_2 + \frac{(\dot{\mathbf{R}}_2^s)^2}{2}\right)\right\} \mathrm{d}t + \int_{t_1^s}^{\tilde{t}^s} \left\{ 1 - \frac{1}{c^2}\left(\frac{(\dot{\mathbf{R}}_1^s)^2}{2}\right)\right\} \mathrm{d}t\,. \tag{22}$$

Hence,

$$\tau_{\text{obs}} = \tau'_{\text{obs}} + \Delta t\,, \tag{23}$$

where

$$\Delta t = -\frac{1}{c^2}\left\{ \frac{\phi_2 + \gamma\phi_1}{1+\gamma} + \frac{1}{2}(V^s)^2 + \frac{\mathbf{V}_2 + \gamma\mathbf{V}_1}{1+\gamma}\,\mathbf{V}^s + \frac{V_2^2 + \gamma V_1^2}{2(1+\gamma)} \right\} \tau_g\,. \tag{24}$$

Finally, we get

$$\Delta t = -\frac{1}{c^2}\left\{\frac{GM_s}{R} + \frac{1}{2}(V^s)^2 + \frac{\mathbf{V}_2 + \gamma\mathbf{V}_1}{1+\gamma}\,\mathbf{V}^s\right\}\tau_g \tag{25}$$

with an accuracy of the order of $5 \times 10^{-10}$, and

$$\Delta t = -\frac{1}{c^2}\left\{\frac{GM_s}{R} + \frac{1}{2}(V^s)^2\right\}\tau_g \tag{26}$$

with the accuracy of the order of $5 \times 10^{-9}$.

## 3. Fringe Frequency

If we start from the definition

$$F_{\text{obs}} = f\,\frac{\mathrm{d}}{\mathrm{d}\tilde{s}}\,(s_2 - s_1) = f\,\frac{\mathrm{d}(s_2 - s_1)}{\mathrm{d}\tilde{t}^s}\,\frac{\mathrm{d}\tilde{t}^s}{\mathrm{d}\tilde{s}} \tag{26}$$

and use (21), it follows that

$$\frac{\mathrm{d}\tilde{t}^s}{\mathrm{d}\tilde{s}} = 1 + \frac{1}{c^2}\left\{\frac{\phi_1 + \gamma\phi_2}{1+\gamma} + \frac{(\dot{\mathbf{R}}_1^s)^2 + \gamma(\dot{\mathbf{R}}_2^s)^2}{2(1+\gamma)}\right\}. \tag{27}$$

Consequently, in view of (23) and (27), $F_{\text{obs}}$ takes the form

$$F_{\text{obs}} = f\,\frac{\mathrm{d}\tau'_{\text{obs}}}{\mathrm{d}t^s} + \Delta F, \tag{28}$$

where

$$\begin{aligned}\Delta F = &-\frac{f}{c^2}\,\tau_g\left\{\frac{\dot{\phi}_2 + \gamma\dot{\phi}_1}{1+\gamma} + \mathbf{V}^s\cdot\mathbf{W}^s + \frac{\mathbf{W}_2^s + \gamma\mathbf{W}_1^s}{1+\gamma}\,\mathbf{V}^s + \right.\\ &\left.+\frac{\mathbf{V}_2^s + \gamma\mathbf{V}_1^s}{1+\gamma}\,\mathbf{W}^s + \frac{\mathbf{V}_2^s\cdot\mathbf{W}_1^s + \gamma\mathbf{V}_1^s\cdot\mathbf{W}_1^s}{1+\gamma}\right\} + \frac{1}{c^2}\,\frac{\gamma-1}{\gamma+1}\,F_g\times\\ &\times\left\{\frac{GM_s}{(R^s)^3}\,(\mathbf{b}\cdot\mathbf{R}^s) + (\mathbf{V}_2^s - \mathbf{V}_1^s)\mathbf{V}^s + (V_2^s)^2 - (V_1^s)^2\right\}\end{aligned} \tag{29}$$

and

$$F_g = f\,\frac{\mathrm{d}\tau_g}{\mathrm{d}t} = \frac{f}{c}\,[\mathbf{V}_1(\tilde{t}) - \mathbf{V}_2(\tilde{t})]\,\mathbf{k}\,; \tag{30}$$

with an accuracy of the order of $5 \times 10^{-10}$,

$$\Delta F \approx 0\,. \tag{31}$$

Now we proceed to obtain an exact expression for $d\tau'_{obs}/dt^s$. A straightforward differentiation of expression (10) yields

$$\frac{d\tau'_{obs}}{d\tilde{t}^s} = \frac{1}{c}\left(\mathbf{V}_1^s(\tilde{t}^s) - \mathbf{V}_2(\tilde{t}^s)\right)\mathbf{k} - \frac{1}{c^2(1+\gamma)}\left[(1+\gamma)(\mathbf{V}^s\cdot\mathbf{k}) + \right.$$

$$\left. + (\mathbf{V}_2^s + \gamma\mathbf{V}_1^s)\mathbf{k}\right]\left[(\mathbf{V}_1^s - \mathbf{V}_2^s)\mathbf{k}\right] - \frac{1}{c^2(1+\gamma)}\cdot\left[((1+\gamma)\mathbf{W}^s + \mathbf{W}_2^s + \gamma\mathbf{W}_1^s)\mathbf{k}\right] \times$$

$$\times (\mathbf{b}^s\cdot\mathbf{k}) + \frac{1}{(1+\gamma)^2c^3}\left[((1+\gamma)\mathbf{V}^s + \mathbf{V}_2 + \gamma\mathbf{V}_1)\mathbf{k}\right]^2\left[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{k}\right] +$$

$$+ \frac{2}{(1+\gamma)^2c^3}\left[((1+\gamma)\mathbf{V}^s + \mathbf{V}_2 + \gamma\mathbf{V}_1)\mathbf{k}\right]\left[((1+\gamma)\mathbf{W}^s + \mathbf{W}_2 + \gamma\mathbf{W}_1)\mathbf{k}\right] \times$$

$$\times (\mathbf{b}\cdot\mathbf{k}) - \frac{1}{c^3(1+\gamma)^2}(\mathbf{b}\cdot\mathbf{k})\left[((1-\gamma^2)\mathbf{W}^s + \mathbf{W}_2 - \gamma^2\mathbf{W}_1)\mathbf{k}\right]\left[\mathbf{V}_1 - \mathbf{V}_2)\mathbf{k}\right] -$$

$$- \frac{1}{2c^3(1+\gamma)^2}(\mathbf{b}\cdot\mathbf{k})^2\left[((1-\gamma^2)\dot{\mathbf{W}}^s + \dot{\mathbf{W}}_2 + \gamma^2\dot{\mathbf{W}}_1)\mathbf{k}\right] + \dot{\tau}_{grav}\,. \tag{32}$$

Furthermore, using the Lorentz transformation to transform $\mathbf{V}_i^s(\tilde{t}^s)$ in $S_s$ to $\mathbf{V}_i(\tilde{t})$ in $S_e$, we find that

$$d\mathbf{R}_i^s(t^{(i)s}) = \frac{\frac{1}{(V^s)^2}(d\mathbf{r}_i(\tilde{t})\mathbf{V}^s)\mathbf{V}^s + \mathbf{V}^s\,d\tilde{t}}{\sqrt{1-\left(\frac{V^s}{c}\right)^2}} + \left[d\mathbf{r}_i(\tilde{t}) - \frac{d\mathbf{r}_i(\tilde{t})\mathbf{V}^s}{(V^s)^2}\mathbf{V}^s\right] \approx$$

$$\approx d\tilde{t}\left\{\frac{1}{2c^2}(\mathbf{V}_i\cdot\mathbf{V}^s)\mathbf{V}^s + \mathbf{V}^s + \mathbf{V}_i\right\},$$

$$d\tilde{t}^{(i)s} = \frac{dt + \frac{d\mathbf{r}_i(\tilde{t})\mathbf{V}^s}{c^2}}{\sqrt{1-\left(\frac{V^s}{c}\right)^2}} = d\tilde{t}\left\{1 + \frac{1}{2}\left(\frac{V^s}{c}\right)^2 + \frac{\mathbf{V}_i\cdot\mathbf{V}^s}{c^2}\right\}, \tag{33}$$

where $\tilde{t}^{(i)s}$ is determined according to (14), we have

$$\mathbf{V}_2^s(\tilde{t}^s) = \mathbf{V}_2(\tilde{t}) - \frac{1}{2}\left(\frac{V^s}{c}\right)^2 - \frac{1}{2}\frac{\mathbf{V}_2\cdot\mathbf{V}^s}{c^2}\mathbf{V}^s - \frac{\mathbf{V}_2\cdot\mathbf{V}^s}{c^2}\mathbf{V}_2 + \frac{1}{1+\gamma}\frac{\mathbf{W}_2}{c^2}(\mathbf{b}\cdot\mathbf{V}^{(s)}),$$

$$\mathbf{V}_1^s(\tilde{t}^s) = \mathbf{V}_1(\tilde{t}) - \frac{1}{2}\left(\frac{V^s}{c}\right)^2 - \frac{1}{2}\frac{\mathbf{V}_1\cdot\mathbf{V}^s}{c^2}\mathbf{V}^s - \frac{\mathbf{V}_1\cdot\mathbf{V}^s}{c^2}\mathbf{V}_1 - \frac{\gamma}{1+\gamma}\frac{\mathbf{W}_1}{c^2}(\mathbf{b}\cdot\mathbf{V}^s). \tag{34}$$

Substituting (34) in (32), we get

$$f\,\frac{\mathrm{d}\tau'_{\text{obs}}}{\mathrm{d}\tilde{t}^s} = F_g\left\{1 - \frac{1}{c(1+\gamma)}\left[(1+\gamma)(\mathbf{V}^s\cdot\mathbf{k}) + (\mathbf{V}_2 + \gamma\mathbf{V}_1)\mathbf{k} + \frac{1}{1+\gamma}\,\tau_g \times\right.\right.$$

$$\left.\times\,((1-\gamma^2)\mathbf{W}^s + \mathbf{W}_2 - \gamma^2\mathbf{W}_1)\mathbf{k}\right] + \frac{1}{(1+\gamma)^2c^2}\times$$

$$\left.\times\left[(((1+\gamma)\mathbf{V}^s + \mathbf{V}_2 + \gamma\mathbf{V}_1)\mathbf{k})^2 - \frac{(1+\gamma)^2}{2}\,(V^s)^2\right]\right\} - \frac{f\tau_g}{c}\left\{(\mathbf{W}^s\cdot\mathbf{k}) + \right.$$

$$+\frac{\mathbf{W}_2 + \gamma\mathbf{W}_1}{1+\gamma}\,\mathbf{k} + \frac{\tau_g}{2(1+\gamma)^2}\,((1-\gamma^2)\dot{\mathbf{W}}^s + \dot{\mathbf{W}}_2 - \gamma^2\dot{\mathbf{W}}_1)\mathbf{k} - \frac{2}{(1+\gamma)^2}\times$$

$$\left.\times\,[((1+\gamma)\mathbf{V}^s + \mathbf{V}_2 + \gamma\mathbf{V}_1)\mathbf{k}]\,[((1+\gamma)\mathbf{W}^s + \mathbf{W}_2 + \gamma\mathbf{W}_1)\mathbf{k}]\right\} -$$

$$-\frac{f}{c^3}\left\{(\mathbf{V}_1\cdot\mathbf{V}^s)(\mathbf{V}_1\cdot\mathbf{k}) - (\mathbf{V}_2\cdot\mathbf{V}^s)(\mathbf{V}_2\cdot\mathbf{k}) + \tfrac{1}{2}[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{V}^s]\times\right.$$

$$\left.\times\,(\mathbf{V}^s\cdot\mathbf{k}) + (\mathbf{b}\cdot\mathbf{V}^s)\,\frac{\gamma\mathbf{W}_1 + \mathbf{W}_2}{1+\gamma}\,\mathbf{k}\right\} + f\dot{\tau}_{\text{grav}}\,. \tag{35}$$

The limit with an accuracy of the order of $5\times10^{-10}$ leads to

$$F_{\text{obs}} = F_g\left\{1 - \frac{1}{c}\left[\mathbf{V}^s\cdot\mathbf{k} + \frac{\mathbf{V}_2 + \gamma\mathbf{V}_1}{1+\gamma}\,\mathbf{k} + \frac{1-\gamma}{1+\gamma}\,\tau_g\,(\mathbf{W}^s\cdot\mathbf{k})\right] + \right.$$

$$\left.+\frac{1}{c^2}\,[(\mathbf{V}^s\cdot\mathbf{k})^2 - \tfrac{1}{2}(V^s)^2]\right\} - \frac{f\tau_g}{c}\left(\mathbf{W}^s\cdot\mathbf{k} + \frac{\mathbf{W}_2 + \gamma\mathbf{W}_1}{1+\gamma}\,\mathbf{k}\right) -$$

$$-\frac{f}{c^3}\,\{(\mathbf{V}_1\cdot\mathbf{V}^s)(\mathbf{V}_1\cdot\mathbf{k}) - (\mathbf{V}_2\cdot\mathbf{V}^s)(\mathbf{V}_2\cdot\mathbf{k}) +$$

$$+\tfrac{1}{2}[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{V}^s]\,(\mathbf{V}^s\cdot\mathbf{k})\} + F_{\text{grav}} \tag{36a}$$

and to

$$F_{\text{obs}} = F_g\left\{1 - \frac{1}{c}\left[\mathbf{V}^s\cdot\mathbf{k} + \frac{\mathbf{V}_2 + \gamma\mathbf{V}_1}{1+\gamma}\,\mathbf{k} + \frac{1-\gamma}{1+\gamma}\,\tau_g(\mathbf{W}^s\cdot\mathbf{k})\right] + \right.$$

$$\left.+\frac{1}{c^2}\,[(\mathbf{V}^s\cdot\mathbf{k})^2 - \tfrac{1}{2}(V^s)^2]\right\} - \frac{f\tau_g}{c}\left\{(\mathbf{W}^s\cdot\mathbf{k}) + \frac{\mathbf{W}_2 + \gamma\mathbf{W}_1}{1+\gamma}\,\mathbf{k}\right\} -$$

$$-\frac{f}{2c^3}\,[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{V}^s]\,(\mathbf{V}^s\cdot\mathbf{k}) + F_{\text{grav}}\,; \tag{36b}$$

with an accuracy of the order of $5 \times 10^{-9}$, where $F_{grav}$, by virtue of (5), is given by

$$F_{grav} = \frac{f}{c} r_g \left\{ \frac{\mathbf{V}^s + \mathbf{V}_1}{R_1 + \mathbf{R}_1 \cdot \mathbf{k}} \mathbf{k} - \frac{\mathbf{V}^s + \mathbf{V}_2}{R_2 + \mathbf{R}_2 \cdot \mathbf{k}} \mathbf{k} \right\}. \tag{37}$$

Obviously, for the situation when the source is located in the 'upper conjunction' with one of the telescopes of the radiointerferometer $F_{grav} \to \infty$. However, if $\mathbf{r}_i / r_i$ is not parallel to $\mathbf{k}$, then

$$F_{grav} = \frac{r_g}{R^s + \mathbf{R}^s \cdot \mathbf{k}} F_g \tag{38}$$

with an accuracy of the order of $5 \times 10^{-10}$.

## 4. Conclusions

We now give the final expression of $\tau_{obs}$ and $F_{obs}$ taking into consideration the post-Newtonian reductions. For a general case in which all quantities, particularly the base vector $\mathbf{b}$ are calculated in the instant of time $t_1$, a simultaneous event of time when the wave is received at telescope No. 1 ($\gamma = 0$).

According to (19) and (25), we obtain

$$\tau_{obs} = \tau_g \left\{ 1 - \frac{1}{c} (\mathbf{V}^s + \mathbf{V}_2)\mathbf{k} - \frac{1}{2c} \tau_g (\mathbf{W}^s \cdot \mathbf{k}) + \right.$$
$$\left. + \frac{1}{c^2} \left[ (\mathbf{V}^s \cdot \mathbf{k})^2 - \frac{GM_s}{R^s} + \tfrac{1}{2}(V^s)^2 + \mathbf{V}_2 \cdot \mathbf{V}^s \right] \right\} -$$
$$- \frac{1}{2c^3} (\mathbf{V}^s \cdot \mathbf{k})(\mathbf{b} \cdot \mathbf{V}^s) + \tau_{grav}, \tag{39}$$

with an accuracy of the order of $5 \times 10^{-10}$; and

$$\tau_{obs} = \tau_g \left\{ 1 - \frac{1}{c} (\mathbf{V}^s + \mathbf{V}_2)\mathbf{k} + \frac{1}{c^2} \left[ (\mathbf{V}^s \cdot \mathbf{k})^2 - \right. \right.$$
$$\left. \left. - \frac{GM_s}{R^s} + \tfrac{1}{2}(V^s)^2 \right] \right\} - \frac{1}{2c^3} (\mathbf{V}^s \cdot \mathbf{k})(\mathbf{b} \cdot \mathbf{V}^s) + \tau_{grav}, \tag{40}$$

with an accuracy of the order of $5 \times 10^{-9}$. The geometric time-delay assumes the form

$$\tau_g = \frac{1}{c} (\mathbf{r}_1(t_1) - \mathbf{r}_2(t_1))\mathbf{k}. \tag{41}$$

The terms of oder of $1/c$ in expressions (39) and (40) are exact analogues of 'classical' annual and diurnal aberration. These are the result of the plane-wave front reaching telescope No. 2 in an instant of time $t_2 = t_1 + \tau > t_1$. As such, the observable value $\tau_{obs}$, as obtained by cross-correlation, is related to the base $\mathbf{b}_{ret} = \mathbf{r}_2(t_1 + \tau) + \mathbf{r}_1(t_1)$ ('retarded base' Cohen and Shaffer, 1971), just as $\tau_g$ is determined by the simultaneous base $\mathbf{b} = \mathbf{r}_1(t_1) - \mathbf{r}_2(t_1)$.

The term $\tau_{grav}$ due to gravitational refraction is, in general, given by expression (5), but when $\mathbf{r}_i/r_i$ is not parallel to $\mathbf{k}$, it is then given by (6). Also this effect must be taken into account for the location of the source in 'lower conjunction' with one of the telescopes of the interferometer ($\mathbf{R}^s/R^s = \mathbf{r}_i/r_i = \mathbf{k}$), because then $\tau_{grav}/\tau_g \approx 2r_g/R^s \approx 2 \times 10^{-8}$.

According to (36) and (37), we get

$$F_{obs} = F_g \left\{1 - \frac{1}{c}\left[\mathbf{V}^s \cdot \mathbf{k} + \mathbf{V}_2 \cdot \mathbf{k} + \tau_g(\mathbf{W}^s \cdot \mathbf{k})\right] + \frac{1}{c^2}\left[(\mathbf{V}^s \cdot \mathbf{k})^2 - \tfrac{1}{2}(V^s)^2\right]\right\} - \\ - \frac{f\tau_g}{c}(\mathbf{W}^s \cdot \mathbf{k} + \mathbf{W}_2 \cdot \mathbf{k}) - \frac{f}{c^3}\{(\mathbf{V}_1 \cdot \mathbf{V}^s)(\mathbf{V}_1 \cdot \mathbf{k}) - \\ - (\mathbf{V}_2 \cdot \mathbf{V}^s)(\mathbf{V}_2 \cdot \mathbf{k}) + \tfrac{1}{2}[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{V}^s](\mathbf{V}^s \cdot \mathbf{k})\} + F_{grav} \tag{42}$$

for an accuracy of the order of $5 \times 10^{-10}$, and

$$F_{obs} = F_g \left\{1 - \frac{1}{c}[\mathbf{V}^s \cdot \mathbf{k} + \mathbf{V}_2 \cdot \mathbf{k} + \tau_g(\mathbf{W}^s \cdot \mathbf{k})] + \\ + \frac{1}{c^2}\left[(\mathbf{V}^s \cdot \mathbf{k})^2 - \tfrac{1}{2}(V^s)^2\right]\right\} - \frac{f\tau_g}{c}(\mathbf{W}^s \cdot \mathbf{k} + \mathbf{W}_2 \cdot \mathbf{k}) - \\ - \frac{f}{2c^3}[(\mathbf{V}_1 - \mathbf{V}_2)\mathbf{V}^s](\mathbf{V}^s \cdot \mathbf{k}) + F_{grav} \tag{43}$$

for an accuracy of the order of $5 \times 10^{-9}$. Here the geometrical fringe-frequency assumes the form

$$F_g = f\,\frac{d\tau_g}{dt_1} = \frac{f}{c}(\mathbf{V}_1(t_1) - \mathbf{V}_2(t_1))\mathbf{k}\,. \tag{44}$$

The term $F_{grav}$ induced by the gravitational field of the Sun is, in general, determined by expression (37), and for the case when $\mathbf{r}_i/r_i$ is not parallel to $\mathbf{k}$, is determined by expression (38).

In conclusion, we give a short consideration to the prior calculation of the main terms (of order of $1/c$) in expressions of $\tau_{obs}$ and $F_{obs}$, which are connected with annual aberration.

In accordance with the recommendations of the IAU, it is necessary to calculate the annual aberration by using the full velocity of the Earth with respect to the barycenter of the solar system. In the orthogonal equatorial coordinate system, radius-vector

TABLE I

Orbital velocity components of the Earth

| Coefficients of sin (S) or cos (C) (in $10^{-7}$ AU day$^{-1}$) | | | Arguments (in parts of circle) | Period (in days) |
|---|---|---|---|---|
| $\dot{x}$ | $\dot{y}$ | $\dot{z}$ | | |
| 171997 S | −157789 C | −68452 C | 0.77693521 + 100.002136 $T$ | 365.2 |
| (2881−7.2 $T$) S | (−2643 + 6.3 $T$) C | (−1147 + 3.5 $T$) C | 0.77270142 + 199.999497 $T$ | 182.6 |
| 71.5 S | −65.6 C | −28.5 C | 0.751206011 + 1336.855231 $T$ | 27.3 |
| 70.1 S | −64.2 C | −28.0 C | 0.625780556 + 8.4293972 $T$ | 4333 |
| 54.3 S | −49.8 C | −21.6 C | 0.768467624 + 299.996857 $T$ | 121.8 |
| 15.9 C | 14.7 S | 6.1 C | 0.487766667 + 3.393872 $T$ | 10762 |
| (15.8 + 20 $T$) C | (15.2 + 18 $T$) S | (4.8 + 8 $T$) S | 0.825780556 + 8.429397 $T$ | 4333 |
| −6 S | 5.5 C | 2.4 C | 0.785402808 − 99.992585 $T$ | 365.3 |
| 3.8 S | −3.5 C | −1.5 C | 0.57371881 + 266.407500 $T$ | 13.7 |
| 3.4 S | −3.1 C | −1.4 C | 0.251561111 + 16.358794 $T$ | 2166 |
| −3.1 C | 2.8 C | 1.2 C | 0.853079136 − 191.570099 $T$ | 190.7 |
| – | 2.6 C | −5.9 C | 0.031252469 + 1342.227848 $T$ | 27.2 |
| −2.6 S | 2.4 C | 1 C | 0.122765201 − 2485.009063 $T$ | 14.7 |
| −2.2 S | −2 C | −0.9 C | 0.628705988 − 162.553368 $T$ | 224.7 |
| 2.2 S | −2 C | −0.9 C | 0.535519478 + 99.946947 $T$ | 365.4 |
| −2.2 S | 2 C | 0.9 C | 0.018350956 + 100.057325 $T$ | 365.0 |
| −1.8 C | 1.7 S | – | 0.034347192 − 225.104594 $T$ | 162.3 |
| −1.7 S | 1.6 C | – | 0.201158333 + 1.189942 $T$ | 30695 |
| 1.6 S | 1.5 C | – | 0.963036081 + 83.133790 $T$ | 439.4 |
| 1.6 C | 1.4 S | – | 0.018350956 + 100.057325 $T$ | 365.0 |
| 1.2 C | 1.4 S | – | 0.535519478 + 99.946947 $T$ | 365.4 |
| −1.2 S | −1.1 S | – | 0.628705988 + 162.5533675 $T$ | 224.7 |
| 1.2 C | 1.1 C | – | 0.402715772 + 108.431533 $T$ | 336.8 |
| 1.1 S | 1.1 S | – | 0.104808333 + 0.606836 $T$ | 60189 |
| 1.1 S | 1 C | – | 0.034347192 − 225.104599 $T$ | 162.3 |
| −1.1 S | −1 C | – | 0.699476600 + 3810.561422 $T$ | 9.6 |
| 1.1 S | −1 C | – | 0.235766173 − 399.994218 $T$ | 91.3 |
| −1 S | −1 C | – | 0.104808333 + 0.606836 $T$ | 60189 |
| −1 S | −0.9 C | – | 0.597284228 − 91.572739 $T$ | 398.9 |
| 0.9 C | 0.9 C | – | 0.5169065137 + 283.138063 $T$ | 129.0 |
| | – | – | 0.975533333 + 6.787744 $T$ | 5381 |

TABLE II

Orbital acceleration components of the Earth

| Coefficients of sin (S) or cos (C) (in $10^{-7}$ AU day$^{-1}$) | | | Arguments (in parts of circle) | Period (in days) |
|---|---|---|---|---|
| $\ddot{x}$ | $\ddot{y}$ | $\ddot{z}$ | | |
| 2959 C | 2715 S | 1178 S | 0.77693521 + 100.002136 $T$ | 365.2 |
| 99 C | 91 S | 39 S | 0.77270142 + 199.999497 $T$ | 182.6 |
| 16 C | 15 S | 7 S | 0.751206011 + 1336.855231 $T$ | 27.3 |
| 3 C | 3 S | 1 S | 0.768467624 + 299.996857 $T$ | 121.8 |
| 2 C | 2 S | 1 S | 0.57371881 + 2662.407500 $T$ | 13.7 |
| – | −1 S | – | 0.031252469 + 1342.227848 $T$ | 27.2 |
| −1 C | −1 S | – | 0.122765201 − 2485.009063 $T$ | 14.7 |
| 1 C | 1 S | | 0.699747600 + 3810.561422 $T$ | 9.6 |

$\dot{\mathbf{R}}^s = \mathbf{V}^s = \dot{x}, \dot{y}, \dot{z})$ has the following projections:

$$\dot{x} = -\left(\dot{X} + \frac{\sum_i m_i \dot{x}_i}{1 + \sum_i m_i}\right), \quad \dot{y} = -\left(\dot{Y} + \frac{\sum_i m_i \dot{y}_i}{1 + \sum_i m_i}\right),$$

$$\dot{z} = \left(\dot{Z} + \frac{\sum_i m_i \dot{z}_i}{1 + \sum_i m_i}\right), \tag{45}$$

where $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are the derivatives of the equatorial coordinates of the Sun, $\dot{x}_i$, $\dot{y}_i$, $\dot{z}_i$ are the derivatives of the coordinates of the $i$th planet, and $m_i$ is the mass of the $i$th planet expressed in solar units mass.

Values of $\dot{x}$, $\dot{y}$, $\dot{z}$ can be calculated directly by using the expansion in trigonometric series (Grubanov, 1972)

$$(\dot{x}, \dot{y}, \dot{z}) = \sum_{i=1}^{n} (a, b, c) T^{\alpha_i} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \sum_{j=1}^{12} \beta_j A_j ,$$

where $a$, $b$, $c$ are numerical coefficients of the terms of the series for $\dot{x}$, $\dot{y}$, $\dot{z}$, respectively; $T$ is the time in Julian centuries; $\alpha_i$ is the integer power $0 \leq \alpha_i \leq 3$, $A_j$ is the set of fundamental arguments in the theory of planet motion; and $\beta_j$ is the integer multiple.

In the series from Gubanov (1972) for prior calculation of $\tau$ and $F$ with an accuracy up to $5 \times 10^{-10}$, it is enough to retain terms of the order of $10^{-7}$ AU day$^{-1} \approx 15$ cm s$^{-1}$. The contracted form of such series from Gubanov (1972) is given by Table I. From convenience of calculation, the arguments were transformed to linear form, where $T$ must be expressed in Julian centuries from the moment of January 1900, $0^h 12^m$ ET.

The projections of velocity calculated in this manner relates to the instant equatorial coordinate system. However, according to the recommendation of the General Assembly of the IAU (1976) aberration corrections should include term depending on eccentricity of the Earth orbit $e$ and the longitude of solar perihelion $\Gamma$. Therefore, the projections of vector $(\dot{x}, \dot{y}, \dot{z})$ can be calculated using the formulas given in Gubanov (1972) as

$$\dot{x}' \approx \dot{x} + 0.000283 - 0.000002\,T,$$

$$\dot{y}' \approx \dot{y} + 0.000051 + 0.000008\,T,$$

$$\dot{z}' \approx \dot{z} + 0.000022 + 0.000003\,T. \tag{46}$$

It is necessary to know $\ddot{\mathbf{R}}^s = (\ddot{x}, \ddot{y}, \ddot{z})$ with an accuracy of $\approx 10^{-7}$ AU day$^{-2} \approx 10^{-4}$ cm s$^{-2}$ in order to obtain $F_{\text{obs}}$ with an accuracy of $5 \times 10^{-10}$. The values of $\ddot{\mathbf{R}}^s = (\ddot{x}, \ddot{y}, \ddot{z})$ with such accuracy are given in Table II. With such accuracy, it can be seen that $\ddot{\mathbf{R}}^s \approx \ddot{\mathbf{R}}/s$.

The value of $\mathbf{R}^s$ is essential in the determination of $\tau_{\text{obs}}$ and $F_{\text{obs}}$, which are introduced by the gravitational field of the Sun ('gravitational refraction' and transformation from coordinate to proper time). It is evident that for the sources located outside the ecliptic plane, or at good-enough angular distances from the Sun, it is sufficient to suppose $e = 0$, and

$$\mathbf{R}^s \approx R^s \left\{ -\cos \lambda_s,\ -\sin \lambda_s \cos \varepsilon,\ -\sin \lambda_s \sin \varepsilon \right\} , \tag{47}$$

where $\lambda_s$ is the true longitude of the Sun, and $\varepsilon$ is the inclination of the ecliptic plane to the equator.

Lastly, the vector $\mathbf{k}$ can be represented in the form

$$\mathbf{k} = \left\{ \cos \delta \cos (s_0 - \alpha), \cos \delta \sin ((s_0 - \alpha), \sin \delta \right\} , \tag{48}$$

where $s_0$ is Greenwich stellar time and $(\alpha, \delta)$ are equatorial coordinates of the source. It is sufficient to know the approximate value of the source coordinates with an accuracy of the order of several tenth fractions of arc seconds in prior calculations of relativistic terms in $\tau_{\text{obs}}$ and $F_{\text{obs}}$ having the accuracy of $5 \times 10^{-10}$.

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