LONG AND SHORT TERM VARIATION OF THE 10.7 cm SOLAR FLUX. THE PHOTOSPHERIC GRANULES AND THE ZÜRICH NUMBERS

JOHN XANTHAKIS and CONSTANTINE POULAKOS
Research Center for Astronomy and Applied Mathematics, Academy of Athens, Greece

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Abstract. Based on diurnal values of the total radio-flux density at 10.7 cm as well as on corresponding Zürich numbers the relation giving the radio-flux as function of R is established. The behaviour of low and high values of R and F_{2800} during the time interval 1957–1976 is studied. Preliminary conclusions drawn by other investigators are confirmed. A prediction of the total radio-flux for the 22nd solar cycle is given.

1. Introduction

The existence of a relationship between the Zürich numbers or the sunspot area and the radio-flux values has been known since long ago (Covington, 1948; Allen, 1957; Covington and Harvey, 1960; Das Gupta and Basu, 1964; Casteli *et al.*, 1965; Covington, 1976; Dodson and Hedeman, 1975). In this report we present results of a statistical investigation of the total solar radio-flux at 10.7 cm (F_{2800}) and of the relative sunspot numbers R on a daily basis.

The aim of the paper is to work out an optimum method of radio-flux prediction (daily, monthly, and yearly values of the Zürich numbers which are currently predicted by means of the various methods presented so far (Xanthakis, 1966, 1967b; Vassilyev *et al.*, 1975; Vitinsky, 1973, 1976; etc.).

2. Data and Technique

Our analysis was based on 9300 pairs of daily values of the radio-flux at 2800 MC/S as well as of the Zürich numbers which are available from the quarterly *Bulletin of Solar Activity* covering the period 1957–1976.

As a first step we calculated the correlation coefficients between the observed daily, mean monthly and mean yearly values of the Wolf numbers R and the corresponding values of the radio-emission index F_{2800} by means of the normal mathematical process (Kendal, 1958).

The relationship between the two components on a daily basis has been expressed by the following equation

$$F_{2800} = C + KR\,, \tag{1}$$

where F_{2800} is the total flux from the whole disk and C and K are constants calculated with the aid of the least-squares method.

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A comparison of the differences, $F_{2800}^{\mathrm{ob}} - F_{2800}^{\mathrm{com}}$ shows that Equation (1) is very satisfactory. The accuracy of Equation (1) was computed with the help of the formula $[1 - (\sigma/\bar{F}_{2800})] \times 100\%$ where σ is the standard deviation and $\bar{F}_{2800}$ the mean observed value of the index F_{2800} during the investigated period. On the average the accuracy was found equal 94, 91, and 89% for the mean annual, mean monthly, and daily values of the radio-emission index F_{2800}, respectively. Details for the numerical results are given in Table I.

The trend shown in Figure 1 is the result of the linear correlation between F_{2800} (emission from the undisturbed solar disk plus emission from radio-regions with associated and nonassociated sunspots) and the Zürich numbers R. It appears from this

TABLE I

Relation between the Zürich numbers and the radio-fluxes in the frequency 2800 MC/S

		Individual years		
		Daily values		
Years	Correlation $r_{R,F}$	C	K	Accuracy (%)
Solar cycle No. 19				
1957	0.83	106.52	0.66	90
1958	0.85	116.30	0.61	92
1959	0.90	95.43	0.71	93
1960	0.89	90.84	0.63	92
1961	0.81	78.70	0.49	91
1962	0.86	71.33	0.49	92
1963	0.82	72.01	0.32	95
1964	0.63	69.70	0.25	96
1965	0.83	71.81	0.28	97
1966	0.86	74.45	0.59	92
1967	0.85	88.58	0.59	91
1968	0.71	94.60	0.52	89
1969	0.82	90.97	0.57	90
1970	0.72	100.04	0.52	90
1971	0.80	76.83	0.62	90
Solar cycle No. 20				
1972	0.88	77.36	0.63	92
1973	0.90	75.16	0.48	94
1974	0.88	71.28	0.45	93
1975	0.85	69.51	0.43	94
1976	0.85	69.46	0.31	96
	0.74			

Relation: $F_{2800} = C + KR$.

Accuracy: $\left[1 - \frac{\sigma}{\bar{F}_{2800}}\right] \times 100\%$.

Table I (continued)

Grouped years					
Daily values					
Years	Correlation $r_{R,F}$	C	K	Accuracy (%)	Max. S.C. No. 19
1957–59	0.87	103.88	0.67	92	Epoch of max. S.C. No. 19
1967 – 69	0.80	90.89	0.56	91	Epoch of max. S.C. No. 20
1962–65	0.87	68.85	0.56	96	Epoch of max. S.C. No. 19
1973–76	0.90	69.63	0.51	95	Epoch of max. S.C. No. 20
1957–59	0.91	76.31	0.78	91	
1967–69					
1962–65	0.90			94	
1973–76					
1957–65	0.83	87.60	0.52	93	Solar cycle No. 19
1966–76	0.82	80.00	0.50		Solar cycle No. 20
1957–76	0.90	88.15	0.65		Solar cycle Nos. 19, 20
1957–76					
for R 50	0.74	69.36	0.62	94	
1957–76					
for R 50	0.90	61.56	0.85	91	

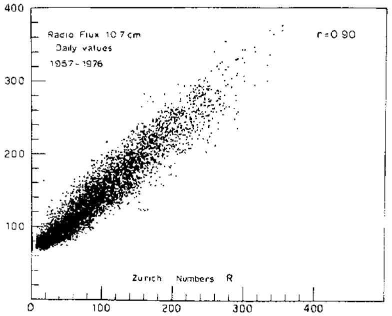

Fig. 1. Plot of diurnal radio-flux density values (2800 MC/S) against Zürich numbers for the period 1957–1976.

figure that the correlation between these two components is very high. The correlation coefficient is marked on the upper right part of the figure.

A second degree curve is fitted to the points as well but the improvement in correlation is not statistically significant and the alteration to the trend was found very small. Extrapolating to zero sunspot numbers the line of the most probable relation between F_{2800} and R, we obtain the basic component or the quiet Sun emission. The emission above the base level, which is variable, is related to the presence of sunspots and it is the so-called slowly varying components (SVC).

It is assumed that the relation (1) applies to all years of a given solar cycle. Nevertheless, linear correlations between the daily values of radio-flux and the corresponding values of the Wolf numbers for the ascending and the descending branch of the solar cycle No. 20 have shown that there is in fact an improvement in the correlation coefficients and the trend alters a little bit from what it is seen in Figure 1. Moreover, the differences between the observed and computed values of solar flux have been reduced and the accuracy of the calculation became 2–4% higher.

This result led us to examine the behaviour of the daily solar radio-flux values when correlated with the corresponding low or high values of the Wolf numbers R in some detail. To this end the data used in the previous analysis were separated into groups as it follows:

Group 1: Daily values of R and F_{2800} for the period: 1957–1959 (epoch of maximum of S.C. No. 19).

Group 2: Daily values of R and F_{2800} for the period: 1967–1969 (epoch of maximum of S.C. No. 20).

Group 3: Daily values of R and F_{2800} for the period: 1962–1965 (epoch of minimum of S.C. No. 19).

Group 4: Daily values of R and F_{2800} for the period: 1973–1976 (epoch of minimum of S.C. No. 20).

Group 5: Daily values of R and F_{2800} for the period: 1957–1959 and 1967–1969 (epoch of maximum of both S.C. Nos. 19 and 20).

Group 6: Daily values of R and F_{2800} for the period: 1962–1965 and 1973–1976 (epoch of minimum of both S.C. Nos. 19 and 20).

Group 7: Daily values of R and F_{2800} for the period: 1957–1965 (solar cycle No. 19).

Group 8: Daily values of R and F_{2800} for the period: 1966–1976 (solar cycle No. 20).

Thus, for each group new correlation coefficients were determined and new relations representing the observed values of F_{2800} were established. The numerical results are given in Table I. In order to obtain the relation corresponding to each group one should replace the constants C and K of the relation $F = C + KR$, with the corresponding values tabulated in Table I. The accuracy was calculated with the help of the formula $(1 - (\sigma/F_i)) \times 100\%$.

It must be noted here that groups Nos. 5 and 6 were formed in order to show our assumption that the behaviour of the diurnal values of the parameters R and F should be analogous during the maximum or minimum of solar activity.

Figures 2a–h show the plot of the daily Zürich numbers against daily flux values at 2800 MC/C for the different groups of years cited above. Corresponding correlation coefficients are marked on the figures. From these figures we see that during the epoch of solar maximum and the solar minimum the relationship between R and F_{2800} is linear. However, in Figures 2a, b, and e we notice that the points are rather scattered. Especially these corresponding to high values of R. This is because the fluctuation of the daily radio-flux index gets its maximum power during the period of maximum of solar activity.

In Figures 2g and h a second-degree curve may be fitted to the points as well, but our determination has proved that the improvement in the correlation coefficient is negligible and the alterations to the trend are very small.

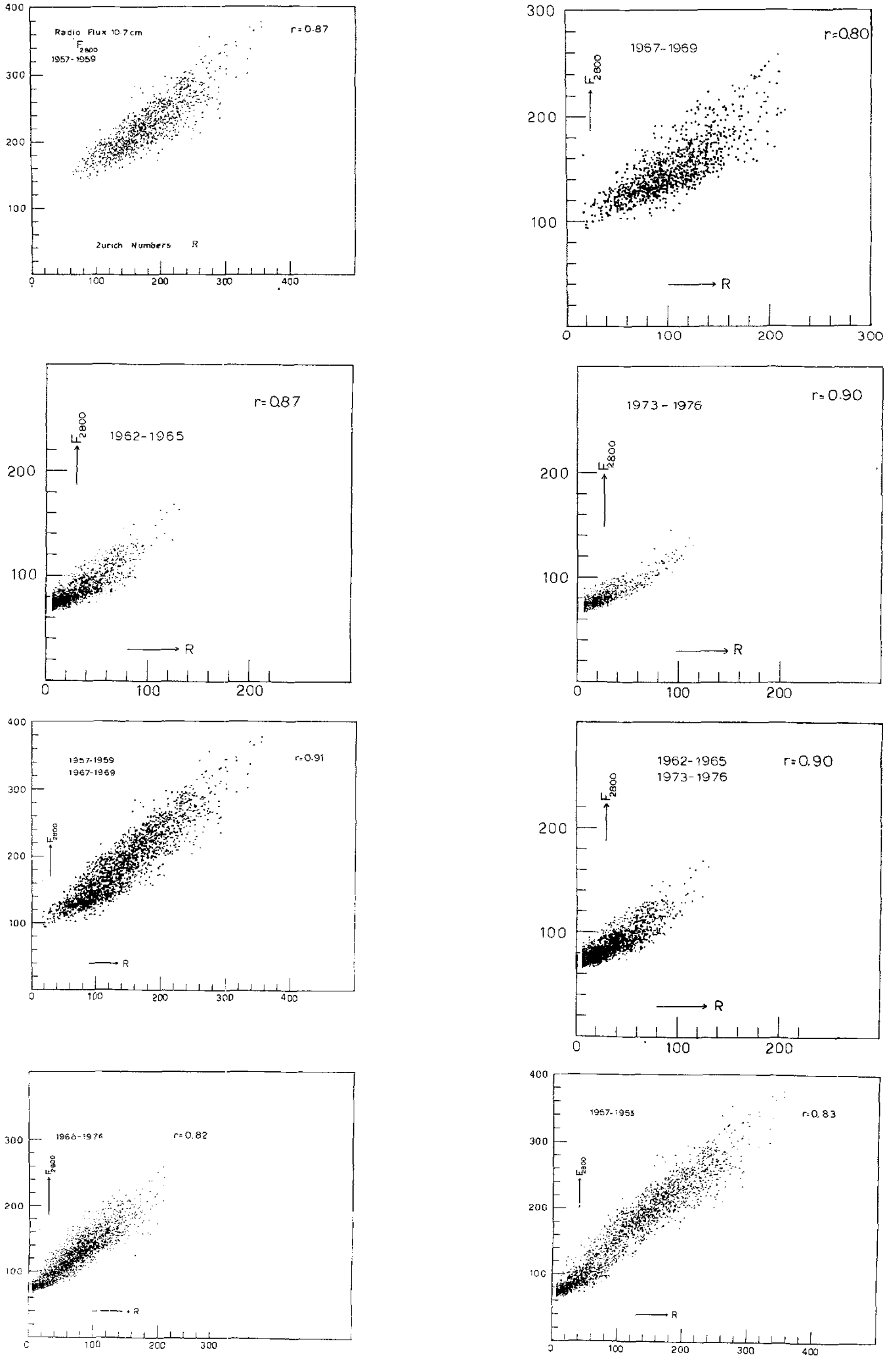

Fig. 2a–h. Daily values of the radio flux at 10.7 cm against Zürich numbers for the different groups of years. On the upper right part of each figure the correlating coefficient is marked.

An additional striking feature exhibited by the trend in Figures 2a–h is that if we extrapolate the trend to zero Zürich number it cuts the solar flux index axis with a positive intercept which implies that the solar flux is not zero but positive though small even when the Sun should be considered completely quiet. The intercept involves variation of the solar radio-flux of the order of 70–100 flux units. This conclusion is in accordance with the conclusions drawn previously by Covington and Harvey (1960) who used sunspot areas instead of Wolf numbers and of Das Gupta and Basu (1964) who found variations of the order of 10–15 flux units. It must be noted, however, that the cited investigators have examined the problem from a different consideration and their data were rather limited.

As it may be seen in Figures 2a–h all the points fit the population trend quite well with the exception of Figures 2g and h. In these figures the plotted points could be selected into two groups, i.e., points corresponding to values of $R < 50$ and points corresponding to values of $R \geq 50$. It appears then that the two lines of the most probable relation between the two parameters for the cited groups of points intersect and by extrapolating them to zero Zürich numbers cut the flux axis with a positive intercept of the order of 70–90 flux units.

It is worth mentioning here that Waldmeier (1971) has found that the mean annual values of the Zürich numbers are bi-linearly correlated to the means of radio-flux – i.e., the slope of the linear relationship is dependent upon the state of solar activity.

However, our results shown in Figures 2g and h have been extracted, not from annual values, but based on 9300 diurnal values and agree with Waldmeier's conclusion. It was only to verify this extraordinary behaviour of radio-flux values that we splitted our data into two groups.

The first group consists of points having $R < 50$ and the second one of points having $R \geq 50$. This is shown in Figure 3 which is a composite graph.

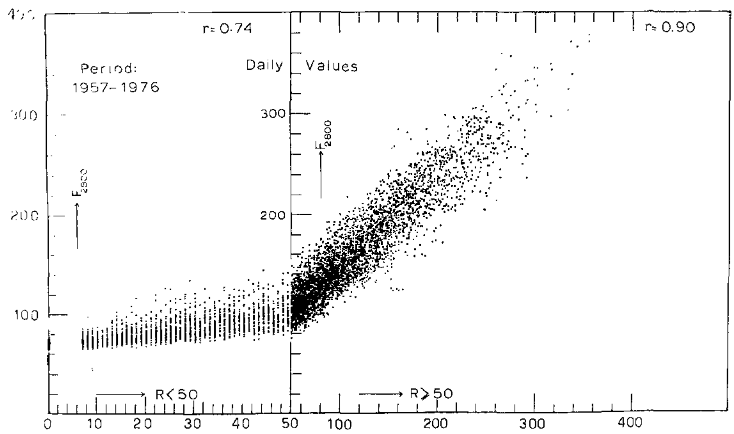

Fig. 3. A composite of daily values of the radio-frequency (2800 MC/S) against Zürich numbers R. The plot is splitted into two groups of points having $R < 50$ and $R \geq 50$, respectively. Due to the different scale used for the two groups of points the behaviour of the low and high values of the flux index F_{2800} is demonstrated.

With the help of this figure we can have a first estimation of the quiet-Sun radio-flux at 2800 MC/S by extrapolating the best fitting linear approximation to the flux axis. The numerical results are given in Table I which summarizes correlation coefficients constants, accuracy, etc., for grouped and for individual years.

Finally, it must be noted that by evaluating our linear relations using the diurnal values published in the *Quarterly Bulletin of Solar Activity* for the current cycle we found that Equation (1) is also valid for predicting the mean monthly, quarterly and yearly values of the total radio-flux density at 2800 MC/S. The standard deviation was found equal to $\sigma = \pm 27.8$ units, $\sigma = \pm 32.2$ units, and $\sigma = \pm 36.3$ units, respectively.

3. Prediction on the Radio-Flux Density at 10.7 cm

What follows from our previous analysis in 2 is that one with the help of the relation $F_{2800} = C + KR$ (where $C = 68.15$ and $K = 0.65$) is able to determine the total solar flux density at 10.7 cm from one day to years ahead provided that the corresponding predicted values of the Zürich numbers R are available.

An evaluation of the reliability of the relation is given in Figure 4a–c which contains three curves illustrating the time variation of the frequency 2800 MC/S for the current solar cycle. Figure 4a is based on predicted mean monthly values of the Zürich numbers while Figure 4b is based on observed values of R. The last curve has been plotted directly using the observed mean monthly values of F_{2800} for comparison. Both the relative sunspot numbers and the total radio-flux values have been taken from the relevant publications of the SGD (Coffey, 1976–1984).

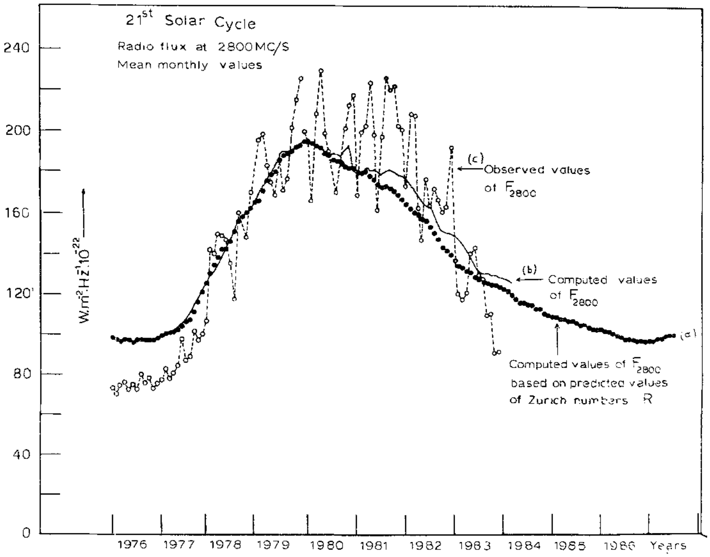

Fig. 4. Observed and computed mean monthly values of radio-flux at 10.7 cm for the twenty-first solar cycle. The standard deviation is equal to $\sigma = \pm 46$ units.

The accuracy between the observed and computed values of the radio-flux was found equal to 70%. The standard deviation constitutes ±46 units.

The studies by Xanthakis (1966, 1967a) have shown that the mean monthly and the mean annual values of the relative sunspot numbers can be satisfactorily represented as function of the time of rise T_R and certain supplementary periodic terms. According to Xanthakis, the variation of the mean annual values of R within the 22nd sunspot cycle can be represented by the relations

$$R_m = C + 2T_0(T_0 - T_R)^2 \quad \text{(year of maximum)}, \tag{2}$$

$$R = R_m \cos^2 \frac{\pi}{2T_R} t + 2T_0 \sin \frac{\pi}{4} t \cos N\pi \quad \text{(ascending branch)}, \tag{3}$$

$$R = R_m \cos^2 \frac{\pi}{2(11 - T_R)} t - 2T_0 \sin \frac{\pi}{4} \quad \text{(descending branch)}; \tag{4}$$

where $C = 84$, $T_0 = 5.76$, $N = 22$ (number of the cycle), and $T_R = 4.5$ yr; and $t = 1, 2, 3, 4$, etc. The value $T_R = 4.5$ has been derived modifying slightly Xanthakis relation giving the time of rise for the forthcoming sunspot cycles up to 35th solar cycle (Xanthakis, 1967b).

Figure 5 illustrates our prediction for the next solar cycle. The next maximum is expected in 1990.5 having a probable value of $R = 102$ units and, therefore, $F_{2800} = 155$ flux units. Bars indicate the uncertainties of the predicted values of F_{2800}.

As it is seen in this figure the last half of the cycle is a little bit lower than the first half. This contradicts with the general opinion that only odd cycles present this phenomenon.

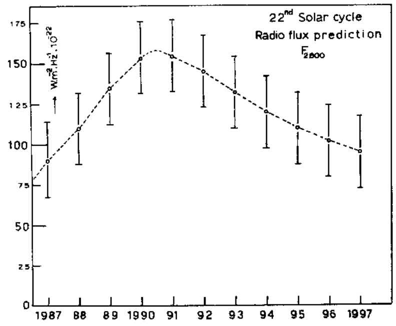

Fig. 5. Prediction of the radio-flux density 2800 MC/S for the twenty-second solar cycle. The prediction was based on predicted Zürich relative sunspot numbers R. The maximum is expected in 1990.5.

4. The Relationship Between Mean Distance of the Photospheric Granules and the Solar Flux

As we have already noted there is a constant radio-emission at 2800 MC/S from the Sun even when the Zürich numbers are practically zero. Attempting to explain this intersection of the flux axis by the most probable line of the correlation we thought it worthwhile to examine the relationship between radio-flux density and the distance of the photospheric granules pointed out recently by Macris and Rösch (1983).

According to Macris and Rösch there is a relationship between F_{2800} and d^{11} (distance in arc sec). Their studies were based on photographic plates of the Sun taken during the period 1966–1978 and on corresponding mean yearly values of F_{2800}. On the whole they have used 10 points corresponding to the 10 sporadic dates during which the pictures of the Sun have been taken. As a result they have found a correlation coefficient between d^{11} and F_{2800} equal to $r = 0.977$.

In fact we found that if we multiply the reverse of the distance d^{11} by a factor of 150, then we obtain the value which results after the intersection of the flux axis by the expolation of the most probable line fitted to the points in Figures 2a–h and which might give an explanation of the base level radio-emission emitted from the sum.

However, we considered it more consistent by correlating d^{11} and F_{2800} to use the daily values of the radio-flux instead of the mean yearly values since the lifetime of the photospheric granules is a matter of ours and not of years (Macris and Rösch, 1983).

Both mean distances of the granules have a good correlation with the radio-flux giving a correlation coefficient equal to $r = 0.77$ which is about 20% smaller than the correlation given by Macris and Rösch (1983) who used mean yearly values of F_{2800}. The linear relation between d^{11} and F_{2800} is given by means of the equation

$$d^{11} = 2.19 + 0.024R\,. \tag{5}$$

Figure 6 shows the various measures of d^{11} which are linearly correlated with the corresponding daily values of F_{2800}.

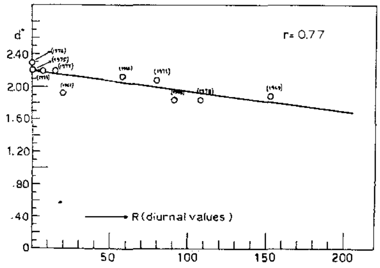

Fig. 6. Correlation between mean distance of the photospheric granules (after Macris and Rösch, 1983) and daily values of the radio-frequency 2800 MC/S. The correlation coefficient is marked on the upper right part of the figure.

However, we are of the opinion that if one wants to demonstrate the statistical significance of a given conclusion, an attempt must be made to include in the analysis a sufficient number of cases; even weak correlations are considered significant if based on a large body of observational material. Conclusions concerning relationships must be based on a large number of observations so that not completely or change phase or direct relationships being reversed and vice versa, and so on, when sufficient number of observations are used afterwards.

Seeing the matter from this point of view and not having at present adequate data to verify the relationship between d^{11} and F_{2800} on a daily basis we have been restrained to apply the aforesaid idea which, however, it seems to be very promising.

5. Conclusions

We have presented a detailed statistical analysis of the daily values of the total radio-flux at 2800 MC/S. We have provided correlations between the relative sunspot numbers and the radio-flux density based on diurnal values for each year and for a group of years during the period 1957–1976.

Simple linear relations between R and F_{2800} have been established. The different behaviour of the ascending and the descending branch of solar cycles is verified. Additional evidence in favor of the Waldmeier's remark that small and high values of R behave differently has been proved by using diurnal values for the period 1957–1976.

A prediction for the next solar cycle is given, although prediction of future sunspot cycles are notoriously inaccurate to some degree even though all forecast are based upon observed patterns of solar activity that have been mainfaired through previous cycles. So, all the predictions should be used with due caution. Finally, we want to emphasize that the parameters R and F_{2800} despite of their close correlation are physically and statistically different from each other and, therefore, in no case we must substitute the one by the other. But to use R for predicting the radio-flux density it appears to be simple and promising.

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