FREE-CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE

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Abstract. The free-convection flow of an incompressible and viscous fluid past an exponentially accelerated infinite vertical plate is analysed. The Laplace transform method is used to obtain the expressions for velocity and skin-friction. The effect of various parameters, occuring into the problem, is discussed with the help of graphs and table.

1. Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite plate was first studied by Stokes (1851). This is also known as Rayleigh's problem in the literature. Due to the importance of this problem in technology, it has been considered by number of researchers for bodies of different shapes such as cylinder, sphere, etc. (cf. Illingworth, 1950; Stewartson, 1951; Sakiadis, 1961; Hall, 1969; or Elliott, 1969).

Free-convection flow encountered in cooling of nuclear reactor or in the study of structures of stars and planets. From this point of view, Soundalgekar (1977) studied the free-convection flow past an impulsively started infinite vertical plate, when it is cooled or heated by the free-convection currents. The object of the present paper is to study the free-convection flow due to an exponentially accelerated infinite vertical plate. The solution of the problem has been obtained with the aid of the Laplace transform technique.

2. Mathematical Analysis

Here, an unsteady motion of an incompressible and viscous fluid past an infinite vertical plate has been considered. The x'-axis is chosen along the infinite plate in the upward direction and the y'-axis normal to the plate. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u' = Ue^{a't'}$ in its own plane and its temperature instantaneously rises or falls to T'_w, which is thereafter maintained constant. Then under usual Boussinesq's approximation the flow can be shown to be governed by system of non-dimensional equations of the form

$$\frac{\partial u}{\partial t} = G\theta + \frac{\partial^2 u}{\partial y^2}\,, \tag{1}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P}\,\frac{\partial^2 \theta}{\partial y^2}\,, \tag{2}$$

subject to the initial and boundary conditions

$$t \leq 0: \quad u(y,t) = 0\,, \quad \theta(y,t) = 0\,,$$

$$t > 0 \begin{cases} u(0,t) = e^{at}\,, & \theta(0,t) = 1\,, \\ u(\infty,t) = 0\,, & \theta(\infty,t) = 0\,. \end{cases} \tag{3}$$

The non-dimensional quantities introduced in the above equations are:

$$\begin{aligned} &y = y'\,U/\nu\,, \quad t = t'\,U^2/\nu\,, \quad u = u'/U\,, \\ &a = a'\,\nu/U^2\,, \quad G = \nu g\beta'(T'_w - T'_\infty)/U^3\,, \\ &\theta = (T' - T'_\infty)/(T'_w - T'_\infty)\,, \quad P = \mu c_p/k\,, \end{aligned} \tag{4}$$

where all the physical quantities have their usual meanings.

The solution of Equation (2) with the boundary conditions (3) has been obtained by Soundalgekar (1977). We shall now solve Equation (1) with the help of the Laplace transform technique. By doing so we obtain, in closed form, the expression for velocity field

$$\begin{aligned} u = {} & \tfrac{1}{2}e^{at}[e^{-y\sqrt{a}}\,\mathrm{erfc}(\eta - \sqrt{at}) + e^{y\sqrt{a}}\,\mathrm{erfc}(\eta + \sqrt{at})] + \\ & + \frac{Gt}{P-1}\left[\left\{(1+2\eta^2)\,\mathrm{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}}\,e^{-\eta^2}\right\} - \right. \\ & \left. - \left\{(1+2\eta^2 P)\,\mathrm{erfc}(\eta\sqrt{P}) - \frac{2\eta\sqrt{P}}{\sqrt{\pi}}\,e^{-\eta^2 P}\right\}\right], \end{aligned} \tag{5}$$

where $\eta = y/2\sqrt{t}$.

If we use the expression (5), the skin-friction in non-dimensional form is given by

$$\begin{aligned} \tau &= -\left.\frac{\partial u}{\partial y}\right|_{y=0}, \\ &= \frac{1}{\sqrt{\pi t}} + e^{at}\,\mathrm{erf}(\sqrt{at}) - \frac{2G\sqrt{t}}{\sqrt{\pi}\,(\sqrt{P}+1)}\,. \end{aligned} \tag{6}$$

3. Discussions and Results

In order to have physical point of view of the problem, numerical calculations are carried out for different values of G, t, P, and a. The values of the Prandtl number P are taken as 0.71 and 7.0 for air and water, respectively. We have plotted velocity profiles of air and water in Figures 1 and 2, respectively. It is seen from these figures that the effect of the parameter a is to increase the velocity of air and water in both cases, cooling and heating of the plate. In the case of cooling of the plate, the velocity of air and water

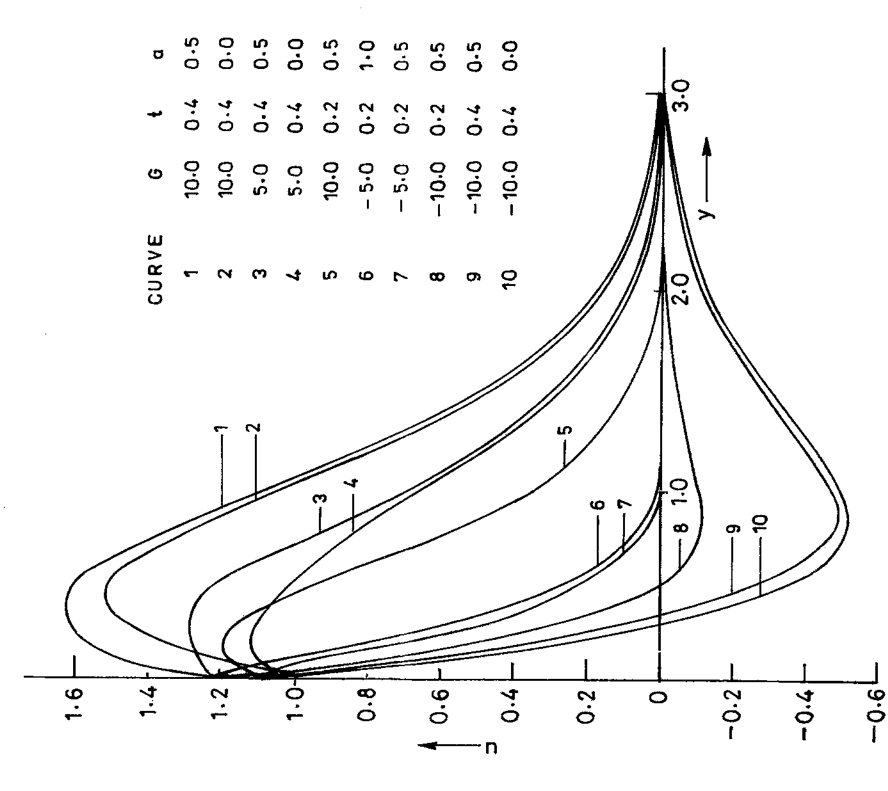

Fig. 1. Velocity profiles of air ($P = 0.71$).

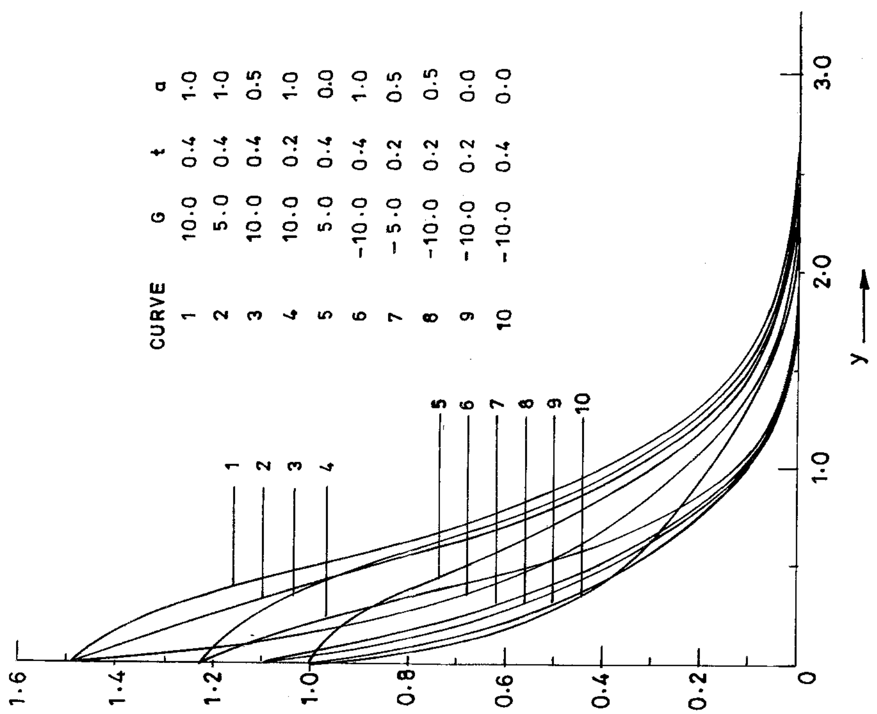

Fig. 2. Velocity profiles of water ($P = 7.0$).

TABLE I

Values of skin-friction

G	t	P	τ for following values of a		
			$a = 0.0$	$a = 0.5$	$a = 1.0$
−10.0	0.2	0.71	4.0002	4.3818	4.5778
		7.0	2.6457	3.0273	3.2233
	0.4	0.71	4.7651	5.3427	5.7033
		7.0	2.8495	3.4271	3.7877
10.0	0.2	0.71	−1.4770	−1.0954	−0.8994
		7.0	−1.2258	0.2590	0.4550
	0.4	0.71	−2.9809	−2.4033	−2.0427
		7.0	−1.0654	−0.4878	−0.1272

increases when the plate is being cooled owing to greater cooling of the plate due to free-convection currents. Also, we observe that in the case of heating of plate (Figure 1) the velocity of air takes negative values and this means there is reverse type of air motion.

Table I shows the numerical values of skin-friction and from which we conclude that the skin-friction of both, air and water increases as the parameter a increases.

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