HYPERONIC EQUATION OF STATE

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(Received 4 August, 1969)

Abstract. An equation of state for cold matter at neutron star densities, $\varrho > 10^{14}$ gm/cm^3, is evaluated. The gas is considered to be a degenerate mixture of neutrons, protons, leptons, hyperons and massive baryons. We derive the equilibrium equations including the effects of nuclear interactions among all the hadrons.

1. Introduction

The purpose of this work is to calculate an equation of state for the interior of stars in which the densities exceed that of nuclear matter. This is essential to the calculation of the maximum mass neutron star that may be formed after a supernova explosion. In a previous paper (Langer *et al.*, 1969) an equation of state was developed for densities up to 1.0×10^{14} gm/cm^3. This was accomplished by including nuclear interactions and minimizing the relevant thermodynamic potentials. When one extends the method of calculating the equation of state beyond this point, certain complexities enter the problem. Due to the combination of nuclear interactions plus fermi level of the baryon, the effective mass of the neutrons increases with increasing density. A point is reached, therefore, where the chemical potential of the neutron exceeds the rest mass of the sigma minus hyperon – a mass of 1197 MeV (Table I). From this point on, hyperons (and other massive baryons) enter as components of the equation of state. Each hyperon in turn interacts with all other baryons and must satisfy its own set of equilibrium conditions, including, of course, conservation of charge and baryon number. One is hurt by two principal problems: on one side is the necessity to use a proper nuclear interaction and on the other is the increasing numerical difficulty which goes almost exponentially with the addition of new hyperons as the density increases. The latter problem enters because of the requirement of solving a set of coupled equations for all components in the equation of state. These are in reality integral equations since the binding per particle must be found by integrating the contribution of nuclear interactions due to all other particles. Tsuruta and Cameron (1966) developed an approximate treatment for a hyperon equation of state by including nuclear interactions only after the equilibrium number densities of non-

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Astrophysics and Space Science 6 (1970) 217–227;

TABLE I

Particle	Mass MeV	Charge q	Spin s	Threshold density $\times 10^{14}$ gm/cm^3 Noninteracting	V_α	V_γ
e^-	0.511	-1	$\frac{1}{2}$			
μ^-	105.7	-1	$\frac{1}{2}$	7.75	2.30	2.35
n	939.6	0	$\frac{1}{2}$			
p	938.3	$+1$	$\frac{1}{2}$			
Σ^-	1197.0	-1	$\frac{1}{2}$	11.2	2.60	2.83
Λ^0	1115.0	0	$\frac{1}{2}$	19.1	4.02	4.17
Δ^-	1236.0	-1	$\frac{3}{2}$	19.1	4.45	4.49
Σ^0	1197.0	0	$\frac{1}{2}$	70.6	9.28	8.93
Δ^0	1236.0	0	$\frac{3}{2}$	112.0	12.1	11.6

interacting particles in the gas had been found. In this paper, the full set of coupled equations is solved, using of course a good fast computer.

The nuclear interaction used in this work is that developed by Weiss and Cameron (1969) based upon the Levinger-Simmons V_α and V_γ potentials. Levinger and Simmons (1961) originally developed these potentials from neutron scattering data between 20 and 340 MeV. A word of caution must therefore be entered. The equation of state extends in the upper region beyond this energy for some of the constituents. The maximum energy of any of the constituents never tends to more than 410 MeV, however, since the equation of state is cut off at the point at which the pressure equals the energy density. This limit represents the requirement that the speed of sound shall never exceed the speed of light (Zeldovich, 1962).

2. Equation of State

We have to find the equation of state for a mixture of leptons and baryons where nucleon–nucleon interactions are included (electrostatic interactions are neglected because their energies are orders of magnitude smaller than the other interaction energies). The equation of state will follow from the conditions which determine the equilibrium mixture of the gas. These conditions will follow from minimizing the thermodynamic potential

$$\Phi = \Phi(P, T, n_i), \tag{1}$$

where T is the temperature, P the pressure and n_i the number densities for particle i. Φ will be minimum for fixed T ($=0$ K) and P when $\mathrm{d}\Phi = 0$, or

$$\frac{\partial \Phi}{\partial n_j} + \sum_i \frac{\partial \Phi}{\partial n_i} \frac{\partial n_i}{\partial n_j} = 0 \tag{2}$$

for all j.

The chemical potential is defined as

$$\mu_i = \left(\frac{\partial \Phi}{\partial n_i}\right)_{T,P}. \tag{3}$$

For neutron star matter the degenerate gas mixture must be minimized subject to the two constraints of charge neutrality and baryon conservation. The new Φ' with the Lagangian constraints α and β is,

$$\Phi' = \Phi + \alpha \sum_i q_i n_i + \beta \sum_k n_k, \tag{4}$$

where q_i is the charge on i, i is summed over all particles and k over all baryons. Writing out Φ'

$$\Phi' = \sum_i \int E_i(n_i)\,\mathrm{d}n_i + \tfrac{1}{2}\sum_k \int\int b_k(n_k, n_k)\,\mathrm{d}n_k\,\mathrm{d}n_k + \sum_{k\neq j} \int\int b_k(n_k, n_j)\,\mathrm{d}n_k\,\mathrm{d}n_j + \alpha \sum_i q_i n_i + \beta \sum_k n_k. \tag{5}$$

The first term is the free energy of the gas where $E_i(n_i) = (p^2(n_i)\,c^2 + m_i^2 c^4)^{\frac{1}{2}}$ is a function of number density through the relationship of momentum to number density for a degenerate fermi gas. The second term is the interaction term for like particles, and the third integral is the interaction energy for unlike particles. $b_k(n_k, n_j)$ is the interaction energy as a function of number density or momentum.

In terms of the average binding energy for an interaction $\langle b(i,j)\rangle$, we have

$$\Phi' = \sum_i \int E_i(n)\,\mathrm{d}n_i + \tfrac{1}{2}\sum_k n_k^2 \langle b(k,k)\rangle + \sum_{k\neq j} n_k n_j \langle b(k,j)\rangle + \alpha \sum_i q_i n_i + \beta \sum_k n_k. \tag{6}$$

Before describing the derivation of the $b(i,j)$, let us solve

$$\frac{\partial \Phi'}{\partial n_i} = 0. \tag{7}$$

Taking the derivatives with respect to n_e and n_n we find that

$$\alpha = \mu(e^-) = E_f(e^-), \tag{8}$$

$$\beta = -\mu(n) = -\left(E_f(n) + n_n \langle b(n,n)\rangle + \tfrac{1}{2} n_n^2 \frac{\partial \langle b(n,n)\rangle}{\partial n_n} + \sum_{k\neq n_n}\left\{ n_k \langle b(n_n,k)\rangle + n_k n_n \frac{\partial \langle b(n_n,k)\rangle}{\partial n_n}\right\}\right). \tag{9}$$

Equation (9) shows the form for the chemical potential μ in which E_f is the free energy at the fermi surface. The remaining solutions of Equation (7), expressed in terms of the chemical potentials are

(a) for leptons, l,

$$\mu(l) = \mu(e^-); \tag{10}$$

(b) for baryons with $q=0$,

$$\mu(q=0) = \mu(n) = \mu(p) + \mu(e^-); \tag{11}$$

(c) for baryons with $q=-1$

$$\mu(q=-1) = \mu(n) + \mu(e^-); \tag{12}$$

(d) for baryons with $q=+1$

$$\mu(q=+1) = \mu(n) - \mu(e^-). \tag{13}$$

The relationship between number density and fermi momentum p_f in a completely degenerate gas is

$$n(i) = \frac{(2s_i+1)\,a_0}{m_e^3}\,p_f^3(i), \tag{14}$$

where

$$a_0 = 8\pi\left(\frac{m_e c^2}{hc}\right)^3 = 1.76\times10^{30}\ \mathrm{cm}^{-3}$$

and $(2s_i+1)$ is the spin multiplicity of particle i.

In the thermodynamic potential (5) we have included the effects of nucleon-nucleon interactions between all pairs of particles. The average binding values $\langle b(i,j)\rangle$ for a two body interaction potential have been derived by Weiss and Cameron (1969) (see also Langer *et al.* (1969)). We briefly outline their results below and refer the interested reader to their paper.

Weiss and Cameron (1969) determined the binding energies for a degenerate gas of interacting fermions, where only two body interactions are considered, based on the V_α and V_γ nucleon-nucleon potentials of Levinger and Simmons (1961).

These velocity dependent potentials have the following form

$$V_\alpha = -V_0 J_1(r) - \frac{\lambda}{M}\,\mathbf{P}\cdot J_2(r)\,\mathbf{P},$$

$$J_1(r) = J_2(r) = \begin{cases} 1 & r<b, \\ \frac{1}{2} & r=b, \\ 0 & r>b; \end{cases} \tag{15}$$

$\mathbf{P} = i\hbar\nabla$, $V_0 = 16.9$ MeV, $\lambda = -0.21$ and $b = 2.4$ fermis.

$$V_\gamma = -V_0 J_1(r) + \frac{1}{M}\left[P^2\omega(r) + \omega(r)\,P^2\right],$$

$$V_0 J_1(r) = \left[1 + 2\omega(r)\right]\left\{112\exp(-1.4r) - \frac{\hbar^2}{M}\frac{(\omega'(r))^2}{(1+2\omega(r))}\right\}, \tag{16}$$

$$\omega(r) = 5\exp(-3.6r).$$

These potentials provide a fair to good fit for ^1S and ^1D phase shifts in low energy scattering. They are assumed to have the same form in each of the four possible spin-parity states, singlet-even, singlet-odd, triplet-even, triplet-odd. All four states are present when unlike particles interact, but only the singlet-even and triplet-odd appear for like particles.

Weiss and Cameron found it necessary to adjust the various interaction strengths to agree with nuclear matter results; in particular they reproduced the saturation density and the volume and symmetry energy coefficients in mass formulae for nuclei.

The average binding is found by first calculating the interaction part of the ground state in lowest order (Hartree-Fock terms)

$$\langle V \rangle = \tfrac{1}{2} \sum_{i,j} \{\langle ij| V |ij\rangle - \langle ij| V |ji\rangle\}, \tag{17}$$

where V is the two-nucleon potential energy operator and the sum is over all the single

TABLE II

V_α potential

ϱ gm/cm³	P dynes/cm²	n_e 10^{30} cm⁻³	n_{μ^-} 10^{30} cm⁻³	n_n 10^{30} cm⁻³	n_p 10^{30} cm⁻³
1.00×10^{14}	4.08×10^{32}	9.81×10^{5}	0.0	5.93×10^{7}	9.81×10^{5}
1.51×10^{14}	1.32×10^{33}	2.40×10^{6}		8.75×10^{7}	2.40×10^{6}
2.00×10^{14}	2.95×10^{33}	3.47×10^{6}		1.14×10^{8}	4.37×10^{6}
2.30×10^{14}	4.50×10^{33}	5.98×10^{6}	1.47×10^{5}	1.31×10^{8}	6.13×10^{6}
2.60×10^{14}	6.90×10^{33}	7.51×10^{6}	7.43×10^{5}	1.46×10^{8}	8.38×10^{6}
2.66×10^{14}	7.41×10^{33}	7.61×10^{6}	7.88×10^{5}	1.48×10^{8}	9.22×10^{6}
2.81×10^{14}	8.48×10^{33}	7.65×10^{6}	8.08×10^{5}	1.52×10^{8}	1.16×10^{7}
3.00×10^{14}	9.84×10^{33}	7.57×10^{6}	7.73×10^{5}	1.57×10^{8}	1.48×10^{7}
3.51×10^{14}	1.42×10^{34}	7.17×10^{6}	5.90×10^{5}	1.67×10^{8}	2.48×10^{7}
4.02×10^{14}	1.95×10^{34}	6.75×10^{6}	4.13×10^{5}	1.75×10^{8}	3.52×10^{7}
4.20×10^{14}	2.18×10^{34}	6.62×10^{6}	3.65×10^{5}	1.79×10^{8}	3.84×10^{7}
4.45×10^{14}	2.55×10^{34}	6.45×10^{6}	3.01×10^{5}	1.83×10^{8}	4.31×10^{7}
5.06×10^{14}	3.52×10^{34}	5.95×10^{6}	1.41×10^{5}	1.90×10^{8}	5.35×10^{7}
5.51×10^{14}	4.29×10^{34}	5.60×10^{6}	5.03×10^{4}	1.94×10^{8}	6.08×10^{7}
6.00×10^{14}	5.23×10^{34}	5.20×10^{6}	0.0	1.99×10^{8}	6.87×10^{7}
6.52×10^{14}	6.36×10^{34}	4.79×10^{6}		2.04×10^{8}	7.73×10^{7}
7.08×10^{14}	7.74×10^{34}	4.36×10^{6}		2.09×10^{8}	8.65×10^{7}
7.55×10^{14}	9.04×10^{34}	4.00×10^{6}		2.13×10^{8}	9.45×10^{7}
8.05×10^{14}	1.05×10^{35}	3.65×10^{6}		2.17×10^{8}	1.03×10^{8}
8.57×10^{14}	1.23×10^{35}	3.29×10^{6}		2.22×10^{8}	1.12×10^{8}
9.28×10^{14}	1.50×10^{35}	2.86×10^{6}		2.28×10^{8}	1.24×10^{8}
9.60×10^{14}	1.64×10^{35}	2.68×10^{6}		2.30×10^{8}	1.28×10^{8}
1.00×10^{15}	1.86×10^{35}	2.42×10^{6}		2.34×10^{8}	1.36×10^{8}
1.21×10^{15}	2.85×10^{35}	1.62×10^{6}		2.47×10^{8}	1.64×10^{8}
1.62×10^{15}	5.68×10^{35}	6.46×10^{5}		2.71×10^{8}	2.12×10^{8}
2.00×10^{15}	9.20×10^{35}	2.05×10^{5}		2.91×10^{8}	2.53×10^{8}
2.76×10^{15}	1.92×10^{36}	$< 10^{2}$		3.32×10^{8}	3.30×10^{8}
3.01×10^{15}	2.31×10^{36}	$< 10^{2}$		3.51×10^{8}	3.50×10^{8}
3.51×10^{15}	3.17×10^{36}	$< 10^{2}$		3.86×10^{8}	3.86×10^{8}

TABLE III

V_α potential

ϱ gm/cm^3	n_{Σ^-} 10^{30}cm^{-3}	n_{Λ^0} 10^{30}cm^{-3}	n_{Δ^-} 10^{30}cm^{-3}	n_{Σ^0} 10^{30}cm^{-3}	n_{Δ^0} 10^{30}cm^{-3}	Γ
1.00×10^{14}	0.0	0.0	0.0	0.0	0.0	2.99
1.51×10^{14}						2.98
2.00×10^{14}						2.96
2.30×10^{14}						2.95
2.60×10^{14}	1.26×10^5					2.94
2.66×10^{14}	8.24×10^5					2.90
2.81×10^{14}	3.10×10^6					2.50
3.00×10^{14}	6.49×10^6					2.36
3.51×10^{14}	1.70×10^7					2.40
4.02×10^{14}	2.80×10^7	2.59				2.53
4.20×10^{14}	3.15×10^7	4.69×10^5				2.82
4.45×10^{14}	3.63×10^7	2.42×10^6	8.59×10^4			2.93
5.06×10^{14}	4.46×10^7	9.64×10^6	2.82×10^6			2.58
5.51×10^{14}	4.94×10^7	1.60×10^7	5.70×10^6			2.54
6.00×10^{14}	5.42×10^7	2.35×10^7	9.32×10^6			2.56
6.52×10^{14}	5.89×10^7	3.17×10^7	1.36×10^7			2.61
7.08×10^{14}	6.36×10^7	4.06×10^7	1.86×10^7			2.67
7.55×10^{14}	6.73×10^7	4.80×10^7	2.31×10^7			2.73
8.05×10^{14}	7.11×10^7	5.55×10^7	2.80×10^7			2.78
8.87×10^{14}	7.50×10^7	6.32×10^7	3.34×10^7			2.84
9.28×10^{14}	7.99×10^7	7.29×10^7	4.08×10^7	8.28×10^5		3.36
9.60×10^{14}	8.19×10^7	7.68×10^7	4.39×10^7	2.38×10^6		3.00
1.00×10^{15}	8.49×10^7	8.27×10^7	4.88×10^7	5.98×10^6		2.93
1.21×10^{15}	9.51×10^7	1.02×10^8	6.71×10^7	2.63×10^7	1.17×10^6	2.98
1.62×10^{15}	1.11×10^8	1.32×10^8	9.98×10^7	6.68×10^7	3.69×10^7	3.18
2.00×10^{15}	1.24×10^8	1.54×10^8	1.28×10^8	9.75×10^7	8.38×10^7	3.45
2.76×10^{15}	1.49×10^8	1.92×10^8	1.82×10^8	1.47×10^8	1.79×10^8	3.87
3.01×10^{15}	1.35×10^8	2.05×10^8	2.15×10^8	1.63×10^8	2.12×10^8	3.89
3.51×10^{15}	1.13×10^8	2.38×10^8	2.71×10^8	1.90×10^8	2.69×10^8	3.90

particle quantum numbers. Then by averaging Equation (17) over the states available in a degenerate gas, we have an average value for the binding terms.

We used the analytic results of Weiss and Cameron for the V_α and V_γ potential averaged over a degenerate gas in our calculations of the average binding energy per particle. In Equations (6, 9, 11–13) these interactions appear as all the possible pair wise interactions between like and unlike particles.

We solved the set of μ_i relationships for the following particles: e^-, μ^- (muon), p, n, Σ^-, Λ^0, Δ^- (also called $n^{*-}_{3/2}$), Σ^0, $\Delta^0(n^*_{3/2})$, whose properties are listed in Table I. The μ_i are solved in terms of a parameter, the number density of any particle, including charge neutrality. As all the baryon μ's are coupled to each other through the two body interactions the problem must be solved on a computer.

It should be emphasized that we approximate the potential interaction for the hyperons. We use the V_α and V_γ forms with the same interaction constants as for the neutrons and protons for the hyperon-hyperon and hyperon-nucleon interaction.

TABLE IV

V_γ potential

ϱ gm/cm³	P dynes/cm²	n_{e^-} 10^{30} cm^{-3}	n_{μ^-} 10^{30} cm^{-3}	n_n 10^{30} cm^{-3}	n_p 10^{30} cm^{-3}
1.00×10^{14}	3.54×10^{32}	8.45×10^{5}	0.0	5.93×10^{7}	8.45×10^{5}
1.50×10^{14}	1.22×10^{33}	2.07×10^{6}		8.75×10^{7}	2.07×10^{6}
2.00×10^{14}	3.01×10^{33}	4.00×10^{6}		1.15×10^{8}	4.00×10^{6}
2.35×10^{14}	4.90×10^{33}	5.46×10^{6}	2.46×10^{4}	1.31×10^{8}	5.48×10^{6}
2.83×10^{14}	8.66×10^{33}	7.72×10^{6}	8.39×10^{5}	1.55×10^{8}	1.06×10^{7}
3.00×10^{14}	9.92×10^{33}	7.67×10^{6}	8.17×10^{5}	1.59×10^{8}	1.37×10^{7}
3.51×10^{14}	1.42×10^{34}	7.29×10^{6}	6.40×10^{5}	1.69×10^{8}	2.37×10^{7}
4.01×10^{14}	2.28×10^{34}	6.80×10^{6}	4.34×10^{5}	1.76×10^{8}	3.52×10^{7}
4.17×10^{14}	2.49×10^{34}	6.66×10^{6}	3.81×10^{5}	1.79×10^{8}	3.84×10^{7}
4.49×10^{14}	3.00×10^{34}	6.45×10^{6}	3.01×10^{5}	1.83×10^{8}	4.43×10^{7}
5.04×10^{14}	3.97×10^{34}	6.03×10^{6}	1.62×10^{5}	1.90×10^{8}	5.35×10^{7}
5.67×10^{14}	5.19×10^{34}	5.48×10^{6}	2.76×10^{4}	1.96×10^{8}	6.39×10^{7}
6.17×10^{14}	6.29×10^{34}	5.06×10^{6}	0.0	2.00×10^{8}	7.20×10^{7}
6.49×10^{14}	7.08×10^{34}	4.78×10^{6}		2.02×10^{8}	7.73×10^{7}
7.05×10^{14}	8.64×10^{34}	4.32×10^{6}		2.07×10^{8}	8.65×10^{7}
7.64×10^{14}	1.04×10^{35}	3.82×10^{6}		2.11×10^{8}	9.65×10^{7}
8.25×10^{14}	1.26×10^{35}	3.32×10^{6}		2.15×10^{8}	1.07×10^{8}
8.93×10^{14}	1.53×10^{35}	2.84×10^{6}		2.19×10^{8}	1.19×10^{8}
9.70×10^{14}	1.89×10^{35}	2.36×10^{6}		2.24×10^{8}	1.31×10^{8}
1.06×10^{15}	2.33×10^{35}	1.90×10^{6}		2.28×10^{8}	1.44×10^{8}
1.16×10^{15}	2.90×10^{35}	1.47×10^{6}		2.32×10^{8}	1.58×10^{8}
1.57×10^{15}	5.81×10^{35}	4.65×10^{5}		2.46×10^{8}	2.05×10^{8}
1.73×10^{15}	7.29×10^{35}	2.56×10^{5}		2.51×10^{8}	2.23×10^{8}
2.10×10^{15}	1.15×10^{36}	$< 10^{2}$		2.61×10^{8}	2.61×10^{8}
2.75×10^{15}	2.01×10^{36}	$< 10^{2}$		3.03×10^{8}	3.03×10^{8}
3.12×10^{15}	2.62×10^{36}	$< 10^{2}$		3.26×10^{8}	3.26×10^{8}
3.34×10^{15}	3.05×10^{36}	$< 10^{2}$		3.40×10^{8}	3.40×10^{8}

3. Conclusion

The solution of the coupled set of equations as described in the previous section yields the number densities of all particles described in Table I as a function of the energy density, ϱ, in gm/cm^3, of the system. The equation of state is then found from

$$P = -\left(\frac{\partial U}{\partial V}\right)_s = n^2\left(\frac{\partial V(n)}{\partial n}\right)_s, \tag{18}$$

where U is the total energy density, n is the baryon number density and $V(n)$ is the energy per particle of the medium.

The values of the number densities of the particles, the pressure and Γ, defined as $(P+U/P)\,\partial P/\partial U$ are given in Tables II and III, and IV and V for the V_α and V_γ potentials respectively. It may be noted that the massive baryons enter the equation of state for both potentials in the following order: Σ^-, Λ^0, Δ^-, Σ^0 and Δ^0 last. The equation of state for the hyperonic gas is plotted in Figure 1 for both the V_α and the

TABLE V

V_γ potential

ϱ gm/cm^3	n_{Σ^-} 10^{30} cm^{-3}	n_{Λ^0} 10^{30} cm^{-3}	n_{Δ^-} 10^{30} cm^{-3}	n_{Σ^0} 10^{30} cm^{-3}	n_{Δ^0} 10^{30} cm^{-3}	Γ
1.00×10^{14}	0.0	0.0	0.0	0.0	0.0	3.19
1.50×10^{14}						3.22
2.00×10^{14}						3.18
2.35×10^{14}						3.14
2.83×10^{14}	2.02×10^{6}					2.66
3.00×10^{14}	5.19×10^{6}					2.40
3.51×10^{14}	1.56×10^{7}					2.44
4.01×10^{14}	2.79×10^{7}					2.47
4.17×10^{14}	3.14×10^{7}	9.62×10^{4}				2.62
4.49×10^{14}	3.75×10^{7}	2.45×10^{6}	3.19×10^{4}			2.68
5.04×10^{14}	4.47×10^{7}	9.26×10^{6}	2.60×10^{6}			2.54
5.67×10^{14}	5.13×10^{7}	1.89×10^{7}	7.02×10^{6}			2.48
6.17×10^{14}	5.59×10^{7}	2.69×10^{7}	1.11×10^{7}			2.57
6.49×10^{14}	5.85×10^{7}	3.22×10^{7}	1.40×10^{7}			2.63
7.05×10^{14}	6.27×10^{7}	4.15×10^{7}	1.95×10^{7}			2.72
7.64×10^{14}	6.74×10^{7}	5.07×10^{7}	2.53×10^{7}			2.71
8.25×10^{14}	7.20×10^{7}	5.98×10^{7}	3.20×10^{7}			2.86
8.93×10^{14}	7.62×10^{7}	7.03×10^{7}	3.96×10^{7}	3.64×10^{5}		2.92
9.70×10^{14}	8.08×10^{7}	7.96×10^{7}	4.78×10^{7}	5.37×10^{6}		2.96
1.06×10^{15}	8.54×10^{7}	8.92×10^{7}	5.67×10^{7}	1.47×10^{7}		2.98
1.16×10^{15}	9.01×10^{7}	9.88×10^{7}	6.64×10^{7}	2.68×10^{7}	1.34×10^{6}	3.07
1.57×10^{15}	1.05×10^{8}	1.28×10^{8}	9.94×10^{7}	6.91×10^{7}	4.12×10^{7}	3.31
1.73×10^{15}	1.11×10^{8}	1.37×10^{8}	1.12×10^{8}	8.27×10^{7}	6.35×10^{7}	3.36
2.10×10^{15}	1.44×10^{8}	1.57×10^{8}	1.17×10^{8}	1.10×10^{8}	1.16×10^{8}	3.14
2.75×10^{15}	9.18×10^{7}	1.92×10^{8}	2.11×10^{8}	1.54×10^{8}	2.10×10^{8}	3.80
3.12×10^{15}	6.88×10^{7}	2.10×10^{8}	2.57×10^{8}	1.74×10^{8}	2.56×10^{8}	4.05
3.34×10^{15}	5.59×10^{7}	2.21×10^{8}	2.84×10^{8}	1.86×10^{8}	2.83×10^{8}	4.11

V_γ potentials. Near nuclear densities, it is seen that the pressures for both cases are the same, a conclusion one would expect since the Levinger-Simmons potentials are both fitted to agree in this region. Below this region, the V_γ is softer than the V_α by a few percent. At higher densities, however, the V_γ is more repulsive and the equation of state for this potential becomes stiffer than that for the V_α potential (the pressure for the V_γ is typically 15% greater than that for the V_α). The equation of state for both cases is terminated at that density at which the pressure becomes equal to the energy density; these are the last points listed in Tables II and IV.

The thresholds for the appearance of the particles are listed in Table I for both the V_α and V_γ potential, as well as for the noninteracting case. Langer and Cameron (1969) have shown that neutron stars containing hyperons have their vibrational energy damped rapidly on an astronomical time scale. Our results would indicate that neutron stars with central densities $>2.60 \times 10^{14}$, for V_α, and $>2.83 \times 10^{14}$ for V_γ will not sustain vibrations.

The number densities of the electrons, muons, neutrons, protons, sigma minuses, sigma zeros, delta minuses and lambda zeros as a function of density are illustrated

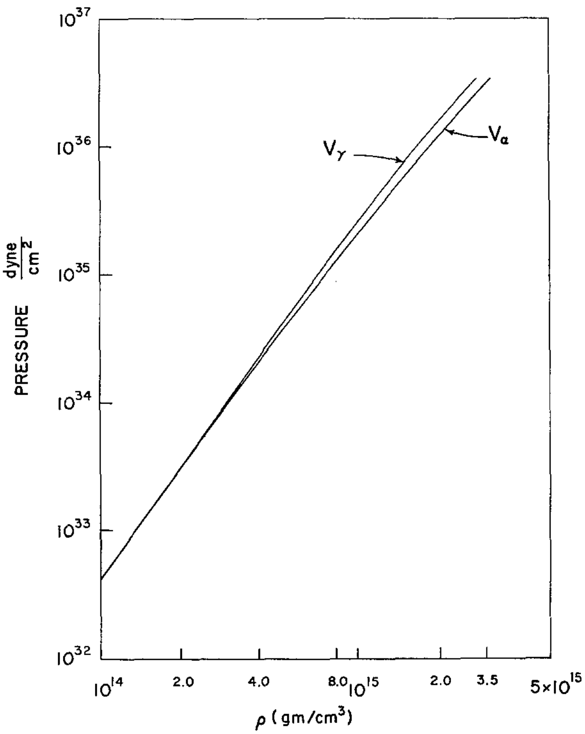

Fig. 1. Pressure versus density for the V_α and V_γ potentials.

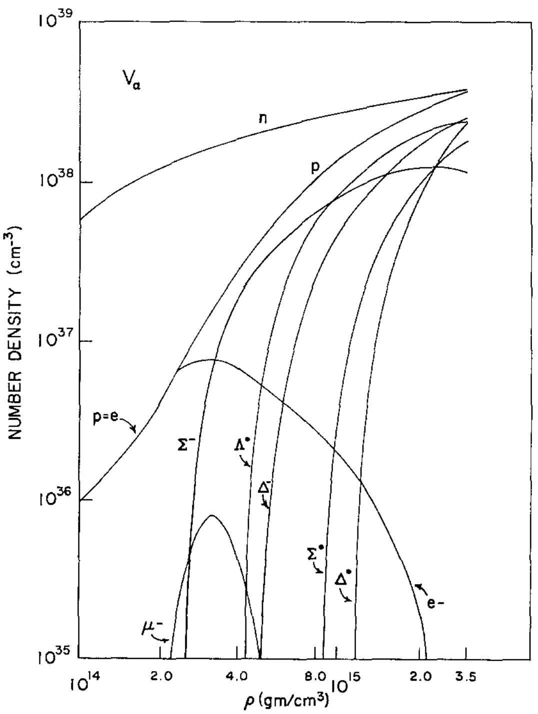

Fig. 2. Number densities of all the constituents of the hyperonic gas for the V_α potential as a function of density ϱ.

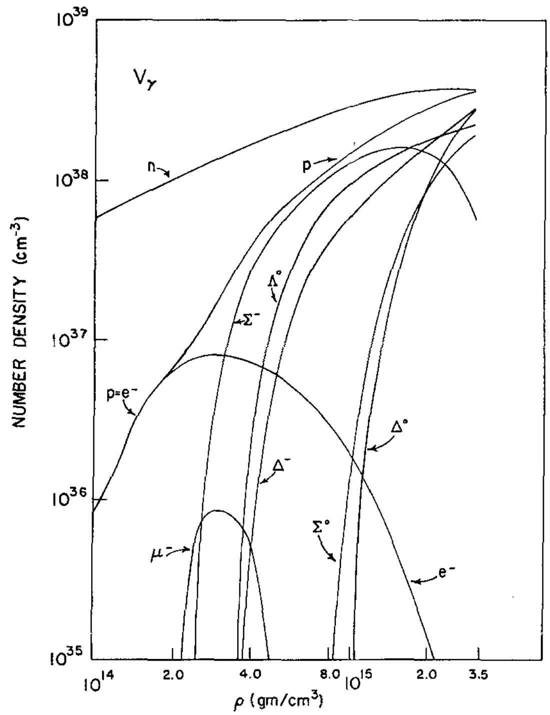

Fig. 3. Same as Figure 2 except that V_γ is the potential.

in Figures 2 and 3 for the V_α and V_γ potentials respectively. From these figures we can note the decrease in the electron number density and the disappearance of the muons. By Equation (10) we see that the presence of the muon is connected to the electron and when the electron fermi energy falls below the muon rest mass, the muons are no longer present.

The effect of the nuclear potentials is to increase the number densities of the other baryons compared to the neutrons. The decrease of the electrons is related to the fast rise of the protons and even faster rise of the sigma minus particles. Once the Σ^- appear they can replace the electrons in maintaining charge neutrality. As the density increases, the potential effect of the baryons becomes more important than the kinetic energy of the electrons.

We also note that negative pions do not appear in our density range as $\mu(e^-) \ll m_\pi$. As long as the potentials, V, are long range attractive and short range repulsive, pions are excluded.

Γ, which is a measure of the stiffness of the equation of state, softens at the onset of the hyperons. Its general behavior is to decrease to a minimum at $\varrho \sim 3.0 \times 10^{14}$ gm/cm^3 and increase until the equation of state is no longer evaluated. The local maxima and minima at a few points are results of small fluctuations of the equation of state as calculated on the computer. Since Γ is a measure of the slope of P versus U, it is sensitive to these small changes in the equation of state.

Acknowledgements

We wish to thank Dr. A. G. W. Cameron for his encouragement and many helpful discussions.

This research has been supported in part by a grant from the National Science Foundation. We wish to thank Dr. Robert Jastrow for his hospitality at the Goddard Institute for Space Studies. One of us, W. L., wishes to acknowledge the support of an NRC-NAS research associateship.

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