ON THE THEORY OF CYCLOTRON LINES IN THE SPECTRA OF MAGNETIC WHITE DWARFS

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Abstract. The radiation transfer at the gyrofrequency in the coronae of magnetic white dwarfs is considered. The electron distribution over Landau levels, taking both radiative and collisional transitions into account, is obtained. The emissivity and absorption coefficients of extraordinary radiation at the gyrofrequency are calculated. The ranges of parameters where cyclotron lines are observed in emission or absorption are found. The upper limit on coronal plasma density (2×10^{11} cm^{-3}) for isolated magnetic white dwarfs with absorption lines in the spectrum is specified.

1. Introduction

The interpretation of absorption lines and bands, which are observed in the optical and UV spectra of magnetic white dwarfs (Angel, 1978; Green and Liebert, 1981; Greenstein and Oke, 1982), as cyclotron features, has given impetus to the study of the interaction between hot, nonrelativistic plasma and cyclotron radiation. This interaction strongly depends on the parameter $\nu_{\text{eff}} \tau_c$, where ν_{eff} is the collision frequency and τ_c is the cyclotron transition time. In a collisional limit, $\nu_{\text{eff}} \tau_c \gg 1$, the electron distribution function is kept isotropic by collisions and does not depend on specific intensity. Relevant calculations are given by Pavlov *et al.* (1980a) and Meggitt and Wickramasinghe (1982). In a collisionless limit, $\nu_{\text{eff}} \tau_c \ll 1$, the distribution function is anisotropic, i.e., the electron distribution over Landau levels is close to the Boltzmann distribution and is determined by the specific intensity of the extraordinary mode emission at the cyclotron fundamental (Zheleznyakov, 1983). Under such conditions, the radiation transfer can be described in terms of the cyclotron scattering.

In Zheleznyakov's 1983 paper, the plasma self-radiation was assumed to be mainly bremsstrahlung. In this paper, again in a limit $\nu_{\text{eff}} \tau_c \ll 1$, we shall search for the deviation of the electron distribution function from the Boltzmann distribution due to collisional transitions between Landau levels and shall consider the influence of this deviation on the extraordinary radiation transfer at the cyclotron fundamental.

We shall show that the collisional excitation of Landau levels followed by cyclotron radiation makes a larger contribution to the hot plasma radiation at gyrofrequency than does bremsstrahlung. Correspondingly, cyclotron absorption (radiative excitation of Landau levels) followed by collisional de-excitation makes a larger contribution to the cyclotron absorption than bremsstrahlung absorption.

This was pointed out by Bogovalov (1984) for the hot spot plasma of X-ray pulsars. However, taking into account the collisional excitation on the X-ray pulsars changes the estimate of the hot spot cyclotron radiation intensity only by a factor of $\sqrt[3]{10}$. We shall

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show that the inclusion of the collisional transitions between Landau levels in the plasma on magnetic white dwarfs changes appreciably the quantitative estimate of the specific intensity.

2. The Transfer Equation of Extraordinary Radiation at Gyrofrequency

The difference between the plasma on X-ray pulsars and the plasma on magnetic white dwarfs consists in the different relationship between the parameters $\hbar\omega_B$ and $\varkappa T_\parallel$ (ω_B is the electron gyrofrequency, $\varkappa$ is the Boltzmann constant, and $T_\parallel$ is the longitudinal temperature). The condition $\hbar\omega_B \gg \varkappa T_\parallel$ is satisfied for X-ray pulsars. In this case, particle collisions will at most excite the lower Landau levels and cause a limited portion of the electrons to transit from the ground state to the first level, while the brightness temperature T_b of the atmospheric radiation of X-ray pulsars does not exceed the value $T_\parallel$ ($\varkappa T_b \ll \hbar w_B$). Under such conditions, the Landau-level populations decrease sharply with increasing level number (the plasma becomes quantized, and the bulk of interactions with radiation occur when transitions take place between two lower levels), and the number of photons in the mode at the cyclotron fundamental is far less than unity, i.e., the induced radiation is insignificant. Relevant transfer equations are given, for example, by Mihalas (1978).

In the coronae of magnetic white dwarfs the inverse inequality, $\hbar\omega_B \ll \varkappa T_\parallel$, is satisfied. The high energy of the longitudinal motion of electrons allows transitions to higher Landau levels due to collisions. The coronal specific intensity is characterized by the brightness temperature T_b, which ranges from $\varkappa T_b < \hbar\omega_B$ to $\varkappa T_b = \varkappa T_\parallel \gg \hbar\omega_B$. When $\varkappa T_b \gtrsim \hbar\omega_B$, the number of photons in the mode exceeds unity, and the induced radiation becomes essential. Thus, when analyzing the interaction of the radiation with the coronal hot plasma on magnetic white dwarfs, the plasma should be considered as a multilevel system where not only the spontaneous radiation and real absorption but also the contribution of induced processes to the radiative transitions should be taken into account. Such a consideration is carried out in the present paper.

In this section, we shall derive the transfer equation for the extraordinary mode emission at the gyrofrequency (since transitions associated with this mode are more probable than those associated with the ordinary mode at the cyclotron fundamental or either ordinary or extraordinary modes at higher harmonics; see Zheleznyakov, 1983). It is assumed that the longitudinal temperature is constant, for example, because of the longitudinal acceleration (heating) of the electrons.

We shall describe the variation in the Landau level populations caused by the electron collisional transitions in the relaxation time approximation, assuming that

$$\left(\frac{\partial N_j}{\partial t}\right)_{\text{st}} = -\nu_{\text{eff}}(N_j - N_j^{(0)})\,; \tag{2.1}$$

where N_j is the population of the jth Landau level and $N_j^{(0)}$ is the population corresponding to the equilibrium distribution with the transverse temperature $T_\perp = T_\parallel$, such that

$$N_j^{(0)} = N \frac{\hbar \omega_B}{\varkappa T_\parallel} e^{-(\hbar \omega_B/\varkappa T_\parallel)j} . \tag{2.2}$$

The collision frequency can be found from the formula (Ginzburg, 1970)

$$\nu_{\text{eff}} = \frac{5.5N}{T_\parallel^{3/2}} K , \tag{2.3}$$

where the Coulomb logarithm is given by

$$K = \ln (10^4 T_\parallel^{3/2} N^{-1/3}) . \tag{2.4}$$

In a plasma with $T_\parallel \sim 10^7$ K and $N \sim 10^9$–10^{13} cm^{-3} the logarithm $K \simeq 10$. It is assumed that the longitudinal momentum distribution is the same for all levels and is described by the function

$$f(p_\parallel) = \frac{1}{(2\pi m \varkappa T_\parallel)^{1/2}} e^{-p_\parallel^2/2m\varkappa T_\parallel} . \tag{2.5}$$

We now consider the change in the population of the jth Landau level due to transitions between $(j + 1)$th and jth levels and between jth and $(j - 1)$th levels because of photon emission or absorption, as well as the change due to collisions. Let $N_{\mathbf{k}}$ be the number of photons in the extraordinary mode at the cyclotron fundamental. Then, the balance equation has the form

$$\begin{aligned} \frac{\partial N_j}{\partial t} = N_{j+1} \int A_1^0 (j + 1)(1 + N_{\mathbf{k}})\, \mathrm{d}\omega\, \mathrm{d}\Omega \, - \\ - N_j \int A_1^0 (j + 1) N_{\mathbf{k}}\, \mathrm{d}\omega\, \mathrm{d}\Omega - N_j \int A_1^0 j (1 + N_{\mathbf{k}})\, \mathrm{d}\omega\, \mathrm{d}\Omega \, + \\ + N_{j-1} \int A_1^0 j N_{\mathbf{k}}\, \mathrm{d}\omega\, \mathrm{d}\Omega - \nu_{\text{eff}} (N_j - N_j^{(0)}) , \end{aligned} \tag{2.6}$$

bearing in mind that the probability of spontaneous radiation to a unit solid angle in a unit frequency region is given by

$$A_j^{j-1} = \frac{e^2 \omega \omega_B}{4\pi m c^3} j (1 + \cos^2 \alpha)\, \delta \left\{ \omega \left(1 - \frac{p_\parallel}{mc} \cos \alpha \right) - \omega_B \right\} \equiv A_1^0 j , \tag{2.7}$$

where α is the angle between the photon wave vector and magnetic field B (Melrose and Zheleznyakov, 1981; Zheleznyakov, 1984). Then, due to δ-function, we have

$$\int A_1^0 (1 + N_{\mathbf{k}})\, \mathrm{d}\omega\, \mathrm{d}\Omega = A \left(1 + \int_0^\pi \psi(\alpha) N_{\mathbf{k}} \sin \alpha\, \mathrm{d}\alpha \right) , \tag{2.8}$$

where

$$A = \frac{4}{3}\frac{e^2\omega\omega_B}{mc}, \qquad \psi(\alpha) = \tfrac{3}{8}(1 + \cos^2\alpha). \tag{2.9}$$

Here, A stands for the spontaneous cyclotron transition probability and the function $\psi(\alpha)$ characterizes the antenna pattern of cyclotron radiation.

It follows from (2.6) that under steady-state conditions

$$N_j - \frac{q}{q+1} N_{j-1} - \frac{j-1}{j}\left(N_{j-1} - \frac{q}{q+1} N_{j-2}\right) = \frac{\varepsilon}{q+1}\frac{1}{j}(N_{j-1} - N_{j-1}^{(0)}), \tag{2.10}$$

where

$$\varepsilon = \nu_{\text{eff}}/A, \qquad q = \int_0^\pi \psi(\alpha) N_{\mathbf{k}} \sin\alpha \, \mathrm{d}\alpha. \tag{2.11}$$

In a quasi-classical limit, $q \gg 1$, the electron distribution over Landau levels is wide, and the difference in the populations of the neighboring levels is small. Equation (2.10) is readily solved by substituting the differences by derivatives. However, in the coronae of the magnetic white dwarfs, the parameter q (the number of photons in the extra-ordinary mode at the cyclotron fundamental averaged over the pattern $\psi(\alpha)$; see (2.11)) can be either more and less than unity. Hence, for the correct analysis of the radiation transfer in magnetic white dwarf coronae it is necessary to obtain a solution to (2.10) without substituting the differences by derivatives.

The solution in finite differences to (2.10) with the small parameter $\varepsilon \ll 1$ will be obtained by the method of successive approximations. In a zeroth approximation, $\varepsilon = 0$, we have ($j \geq 2$)

$$\tilde{N}_j = \frac{q}{q+1}\tilde{N}_{j-1} + \frac{j-1}{j}\left(\tilde{N}_{j-1} - \frac{q}{q+1}\tilde{N}_{j-2}\right). \tag{2.12}$$

It follows from (2.12) that disregarding the collisions

$$\tilde{N}_j = \tilde{N}_1\left(\frac{q}{q+1}\right)^{j-1} + \left(\tilde{N}_1 - \frac{q}{q+1}\tilde{N}_0\right)\sum_{k=2}^{j}\frac{1}{k}\left(\frac{q}{q+1}\right)^{j-k}, \tag{2.13}$$

where $\tilde{N}_0$ and $\tilde{N}_1$ are the populations of the zeroth and first Landau levels, which are constants to be determined. The first term in (2.13) describes the Boltzmann distribution of electrons with a temperature $T_\perp$ given by

$$e^{-\hbar\omega_B/\varkappa T_\perp} = \frac{q}{q+1}. \tag{2.14}$$

In a quasi-classical limit, $q \gg 1$, this relation can be reduced to $\varkappa T_\perp \simeq \hbar\omega_B q$ (Zheleznyakov, 1983).

To elucidate the meaning of the second term in (2.13), we determine the particle flux from the $(j+1)$th to the jth level from the formula

$$Q_j = A(j+1)(1+q)\tilde{N}_{j+1} - A(j+1)q\tilde{N}_j . \tag{2.15}$$

Substituting (2.13) into (2.15) we find that

$$Q_j = A(1+q)\left(\tilde{N}_1 - \frac{q}{q+1}\tilde{N}_0\right). \tag{2.16}$$

It is easy to see that the second term in (2.13) describes the nonzero particle flux from the upper to the lower Landau levels and that this particle flux does not depend on j. Due to this last circumstance the difference $(Q_j - Q_{j-1})$ that determines the source power at the jth level is equal to zero for any finite j. Hence, the second term in (2.13) describes a steady-state flux of particles from a source located at an infinitely high Landau level.

The absence of a source of particles with infinitely high energy means that* $\tilde{N}_1 = (q/q+1)\tilde{N}_0$ (see (2.16)). Using this equality and determining $\tilde{N}_0$ from the normalization condition $\Sigma_{j=0}^{\infty}\tilde{N}_j = N$, we finally rewrite (2.13) in the form

$$\tilde{N}_j = \frac{N}{1+q}\left(\frac{q}{q+1}\right)^j . \tag{2.17}$$

A substitution of (2.17) into the right-hand side of (2.10) and solving for N_j yields

$$N_j = N_1\left(\frac{q}{q+1}\right)^{j-1} + \left(N_1 - \frac{q}{q+1}N_0\right)\sum_{k=2}^{j}\frac{1}{k}\left(\frac{q}{q+1}\right)^{j-k} + $$
$$+ \frac{\varepsilon}{q+1}\sum_{i=1}^{j}(\tilde{N}_{i-1} - N_{i-1}^{(0)})\sum_{k=i}^{j}\frac{1}{k}\left(\frac{q}{q+1}\right)^{j-k} . \tag{2.18}$$

The first two terms are analogous to those in (2.13), and the third term describes the flux of particles which are brought into the upper Landau levels by collisions and relax to the quasi-equilibrium distribution (2.17) due to cyclotron losses. The absence of particle sources at infinitely high Landau levels leads to the relation $N_1 = (q/q+1)N_0$. Then N_0 can be determined from the normalization condition $\Sigma_{j=0}^{\infty}N_j = N$. Finally, the electron distribution over Landau levels takes the form

$$N_j = \frac{N}{1+q}\left\{1 - \varepsilon\ln\frac{1+\varkappa T_{\parallel}/\hbar\omega_B}{1+q}\right\}\left(\frac{q}{q+1}\right)^j +$$
$$+ \frac{\varepsilon}{q+1}\sum_{k=1}^{j}\left(\frac{q}{q+1}\right)^{j-k}\frac{1}{k}\left[e^{-(\hbar\omega_B\varkappa T_{\parallel})k} - \left(\frac{q}{q+1}\right)^k\right] . \tag{2.19}$$

* The relation $\tilde{N}_1 = (q/q+1)\tilde{N}_0$ can also be obtained from the sum finiteness condition $\Sigma_{j=0}^{\infty}\tilde{N}_j$, where $\tilde{N}_j$ is determined by (2.13).

In the transition from (2.18) to (2.19), the order of summation was changed, and summation in index i was performed in the last term.

We now calculate the coefficients of radiation transfer in a plasma with the non-equilibrium distribution of electrons over Landau levels (2.19). The plasma emissivity in the extraordinary mode at the cyclotron fundamental can be found from the formula (Zheleznyakov, 1977)

$$a_{11} = \hbar\omega \sum_{(q)\to(m)} A_q^m N_q = \hbar\omega \sum_{j=1}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \, A_j^{j-1} f(p_\parallel) N_j \,. \tag{2.20}$$

On substituting (2.5) and (2.6) into (2.20) we have

$$a_{11} = \frac{\hbar\omega^3}{(2\pi)^3 c^2} \, F \sum_{j=1}^{\infty} jN_j \,, \tag{2.21}$$

$$\begin{aligned} F &= \frac{\sqrt{2}\,\pi^{3/2} e^2}{(m\varkappa T_\parallel)^{1/2}\,\omega_B} \, \frac{\omega_B^2}{\omega^2} \, \frac{1+\cos^2\alpha}{|\cos\alpha|} \, e^{-z_1^2} = \\ &= \sqrt{\frac{\pi}{8}} \, \frac{\omega_L^2}{\omega_B c} \, \frac{\omega_B^2}{\omega^2} \, \beta_{T_\parallel}^{-1} \, \frac{1+\cos^2\alpha}{|\cos\alpha|} \, e^{-z_1^2} \, \frac{1}{N} \,, \end{aligned} \tag{2.22}$$

$$z_1 = \frac{\omega - \omega_B}{\sqrt{2}\,\beta_{T_\parallel}\,\omega\,|\cos\alpha|} \,;$$

where $\omega_L^2 = 4\pi e^2 N/m$ is the Langmuir frequency and $\beta_{T_\parallel}^2 = \varkappa T_\parallel/mc^2$. Substitution of (2.19) into (2.21) yields

$$a_{11} = \frac{\hbar\omega^3}{(2\pi)^3 c^2} \, FN \left[q + \varepsilon \left(\frac{\varkappa T_\parallel}{\hbar\omega_B} - q \right) \right]. \tag{2.23}$$

Unlike the known formulae (see, for example, Zheleznyakov, 1983), this expression has a term which accounts for collisions. Besides, this relation is valid at an arbitrary q, including both a quasiclassical limit, $q \gg 1$, and a quantum limit, $q \ll 1$.

The cyclotron absorption coefficient can be determined from the formula

$$\mu_{11} = \frac{(2\pi)^3 c^2}{\omega^2} \sum_{(q)\rightleftarrows(m)} A_q^m (N_m - N_q) = F \sum_{j=1}^{\infty} j(N_{j-1} - N_j) = FN \,. \tag{2.24}$$

To derive this relation, Equations (2.5), (2.6), and (2.22) were used. It is easy to see from (2.24) that μ_{11} does not depend on the form of the transverse distribution (see, for example, Roulands *et al.*, 1966). Therefore, in particular, the absorption coefficient of the extraordinary mode at the cyclotron fundamental has the same form in a classical plasma (Zheleznyakov, 1970, 1983) and in a quantized plasma (Pavlov *et al.*, 1980b).

Equations (2.23) and (2.24) determine the coefficients in the transfer equation

$$\cos\alpha \, \frac{\mathrm{d}I_\omega}{\mathrm{d}z} = (a_{11} + a_b) - (\mu_{11} + \mu_b) I_\omega \,. \tag{2.25}$$

The terms a_b and μ_b denote the bremsstrahlung emissivity and bremsstrahlung absorption coefficients. According to Zheleznyakov (1983), we can make the rough assumption that within the line

$$\mu_b = \frac{\omega_\mathrm{L}^2 \nu_\mathrm{eff}}{\omega_B^2 c} \, \beta_{T_\parallel}^{-2} \simeq 10^{-4} \, \frac{N^2}{B^2 T_\parallel^{5/2}} \,, \qquad a_b = \frac{\omega^2 \varkappa T_\parallel}{(2\pi)^3 c^2} \, \mu_b \,. \tag{2.26}$$

In the following we shall neglect the angular dependence of emissivity (2.23) and absorption coefficient (2.24). We put $\cos\alpha = 1/\sqrt{2}$ in all expressions for F (2.22) and approximate the number q (2.11) by the expression

$$q = \int_0^\pi \frac{1}{2} \, N_\mathrm{k} \sin\alpha \, \mathrm{d}\alpha \equiv \frac{(2\pi)^3 c^2}{\hbar \omega^3} \, \frac{1}{2} \int_0^\pi I_\omega \sin\alpha \, \mathrm{d}\alpha \,. \tag{2.27}$$

In the Schwarzschild–Schuster approximation, Equation (2.25) can be reduced to

$$\frac{1}{2} \, \frac{\mathrm{d}I_+}{\mathrm{d}z} = -(\mu_{11} + \mu_b) I_- \,, \qquad \frac{1}{2} \, \frac{\mathrm{d}I_-}{\mathrm{d}z} = 2(a_{11} + a_b) - (\mu_{11} + \mu_b) I_+ \,, \tag{2.28}$$

where

$$I_+ = \int_0^\pi I_\omega \sin\alpha \, \mathrm{d}\alpha \,, \qquad I_- = \int_0^{\pi/2} I_\omega \sin\alpha \, \mathrm{d}\alpha - \int_{\pi/2}^\pi I_\omega \sin\alpha \, \mathrm{d}\alpha \,. \tag{2.29}$$

We then substitute

$$x = I_+ / B_\omega^{(0)} \,, \qquad y = I_- / B_\omega^{(0)} \,, \tag{2.30}$$

where $B_\omega^{(0)} = \omega^2 \varkappa T_\parallel / (2\pi)^3 c^2$ use a new independent variable

$$\tau = \int_z^\infty \mu_{11} \, \mathrm{d}l \,. \tag{2.31}$$

In view of (2.23) and (2.24) we finally arrive at

$$\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}\tau} = (1 + p)y\,, \tag{2.32a}$$

$$\frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}\tau} = p(x - 2) + \varepsilon(x - 2)\,. \tag{2.32b}$$

Here, p denotes the ratio μ_B/μ_{11}, and ε, the ratio ν_{eff}/A (see (2.9)). Under the conditions of magnetic white dwarfs $p \ll 1$. Note that both p and ε are present in Equation (2.32b), i.e., ultimately, the bremsstrahlung processes and collisional transitions between Landau levels exert the same influence on the ratio flux $y/2$. Therefore, all the formulae from Zheleznyakov's (1983) paper hold, but with p substituted by $p + \varepsilon$.

We now compare the parameters p and ε. Taking Equations (2.3), (2.9), (2.22), (2.24), and (2.26) into account, we find that at the center of the line

$$\frac{\varepsilon}{p} = \frac{9\sqrt{\pi}}{16}\frac{mc^3}{e^2\omega_B}\beta_{T_\parallel} \simeq 7 \times 10^{10}\,\frac{T_\parallel^{1/2}}{B}\,. \tag{2.33}$$

At $B \simeq 10^8$ G, $T_\parallel \simeq 10^7$ K the ratio ε/p is about 2×10^6. Therefore, the basic mechanism responsible for the change in the radiation flux in the cyclotron line is collisional transition (which, however, gives a small correction to the electron distribution over Landau levels). In the theory of stellar atmospheres such a situation is classed as a case where the line is 'fixed by collisions' (Thomas, 1957; Mihalas, 1978).

The high value of the ratio ε/p permits one to neglect the first term in the right-hand side of Equation (2.32b), and describes the transfer of extraordinary radiation at the cyclotron fundamental by the equations

$$\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}\tau} = y\,, \qquad \frac{1}{2}\frac{\mathrm{d}y}{\mathrm{d}\tau} = \varepsilon(x - 2)\,. \tag{2.34}$$

The value ε is determined by formula (2.11). A detailed analysis of the solutions of Equations (2.34) for a homogeneous layer is made in Zheleznyakov's (1983) paper.

3. Analysis of a Relative Part Played by the Photosphere and Coronal Plasma in the Formation of the Radiation Spectral of White Dwarfs

Comparison of the coronal parameters with each other and with the photospheric radiation temperature enables one to distinguish a number of characteristic regions within the parameters N, B, and $T_\parallel$ (assuming that the photospheric temperature is given). Let us fix, as usual, one of the parameters and consider the characteristic regions

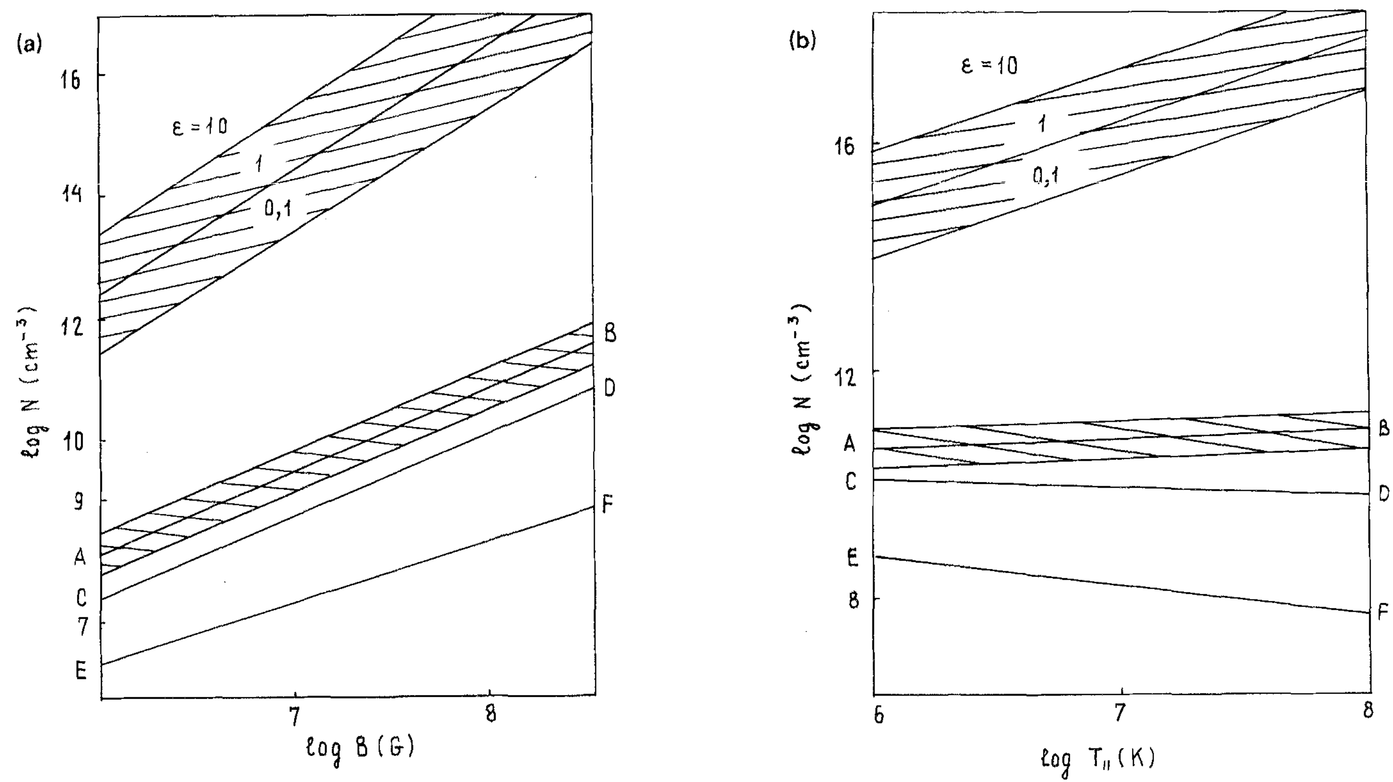

Fig. 1. The hot plasma parameters: (a) in the 'electron density-magnetic field' diagram ($T_{\parallel} = 10^7$ K); and (b) in the 'electron density-longitudinal temperatures' diagram ($B = 10^8$ G).

in the plane of two remaining ones. We fixed $T_\parallel$ and considered the plane of parameters $N - B$ (Figure 1a). Sometimes it is more convenient to consider the characteristic regions in the plane $N - T_\parallel$, with fixed B (Figure 1b; see also Tikhomirov, 1984).

First of all, note that Equation (2.34) and its solutions refer to a collisionless limit where the parameter

$$\varepsilon = \nu_{\text{eff}} \tau_c \equiv \nu_{\text{eff}}/A = 1.25 \times 10^{10} NB^{-2} T_\parallel^{-3/2} \ll 1 . \tag{3.1}$$

The lines ε = const. for $T_\parallel \simeq 10^7$ K are plotted in Figure 1a. The dashed strip corresponds to the parameter ε that ranges from 0.1 to 10. Our consideration relates to the region below this strip.

On the other hand, the cyclotron processes in a coronal plasma are essential when, due to cyclotron scattering, the plasma's optical depth exceeds unity – i.e., the inequality

$$\mu_{11} H > 1 \tag{3.2}$$

is satisfied, where H denotes the reduced height of the corona

$$H = \frac{2\varkappa T_\parallel R_*^2}{GM_* m_p} \simeq 3 \times 10^6 \text{ cm} , \tag{3.3}$$

($T_\parallel \simeq 10^7$ K, $R_* \simeq 5 \times 10^8$ cm, $M_* \simeq M_\odot$). The condition (3.2) is met above the line EF in Figure 1a.

Inequalities (3.1) and (3.2) determine the same ranges of parameters as those in Zheleznyakov's (1983) paper. However, substitution of the parameter p in the transfer equation in Zheleznyakov's (1983) paper by the parameter ε in the present paper significantly changes the boundaries of characteristic regions determines by the lines AB and CD in Figure 1a.

Depending on the relation between the coronal parameters, two cases are possible: a 'thin' corona

$$2 \sqrt{\varepsilon}\, \mu_{11} H \ll 1 , \tag{3.4}$$

and a 'thick' corona, where the inverse inequality is satisfied. All the photons emitted in the thin corona have enough time to escape before they are absorbed because of collisional transitions. But in the thick corona, photons escape only from the surface layer of thickness $(2 \sqrt{\varepsilon}\, \mu_{11})^{-1}$, and thermal equilibrium with a temperature of radiation and plasma equal to $T_\parallel$ is established in the corona (see Figure 3a in Zheleznyakov's 1983 paper). In Figure 1a the line AB is determined by the equation

$$\log N = \tfrac{4}{3} \log B + 0.1 \tag{3.5}$$

and separates the cases of thick and thin coronae (in the dashed strip $2 \sqrt{\varepsilon}\, \mu_{11} H$ ranges from 0.3 to 3). A thin corona is located below this line. Compared to the analogous line in Zheleznyakov's (1983) paper, the line (3.4) has a slower slope and corresponds to an electron density two orders lower at $B = 10^8$ G.

We now consider the influence of incident radiation. Since the brightness temperature of the photosphere T_{ph} is less than $T_\parallel$, the contribution of the intrinsic radiation of the corona to the extraordinary radiation at the fundamental exceeds that of the photospheric radiation passing through the corona, even in the range of parameters corresponding to a thin corona. The criterion of a significant contribution of the photosphere can be the inequality

$$\frac{T_{\mathrm{ph}}}{1 + \mu_{11}H} > 2\varepsilon\mu_{11}HT_{\parallel} \,. \tag{3.6}$$

For $T_{\mathrm{ph}}/T_{\parallel} = 3 \times 10^{-3}$ the condition (3.6) is satisfied below the line CD in Figure 1a. The relative positions of the lines AB and CD in this paper are the same as those of the corresponding lines in Zheleznyakov's (1983) paper.

Finally, we compare the brightness temperature of extraordinary radiation in the cyclotron line at the fundamental with the temperature of the surrounding continuum. This last temperature is determined by the photospheric radiation and is equal to T_{ph}. The temperature in the line depends on the coronal parameters. Analysis shows that at $T_{\mathrm{ph}}/T_{\parallel} = 3 \times 10^{-3}$ and $B \sim 10^6$–3×10^8 G the boundary where the brightness temperature of radiation in the line coincides with the radiation temperature in the continuum lies in the dashed strip along the line AB. In a weaker magnetic field with $B < 10^6$ G this boundary shifts to a thin corona, while in a stronger field with $B > 3 \times 10^8$ G it shifts to a thick corona. Therefore, in Figure 1a, where only fields with $B = 10^6$–3×10^8 G are indicated, the cyclotron line in the extraordinary radiation at the fundamental is observed in emission above the line AB and in absorption if the parameters of coronal plasma are below this line.

The transfer of radiation at higher harmonics is qualitatively the same as in Zheleznyakov's (1983) paper. According to this paper, in the region above the line CD, cyclotron lines at higher harmonics and the line in ordinary radiation at the fundamental should be observed in emission. Thus, we infer that the cyclotron spectrum is a succession of emission lines in the range of parameters corresponding to a thick corona (i.e., above the line AB), between AB and CD the line in the extraordinary radiation at the fundamental is in absorption, all the remaining lines are emission features and below CD all the lines are absorption features.

All known observations show that the cyclotron lines and bands in the spectra of isolated magnetic white dwarfs are absorption features. Hence, for the theory to agree with the observations, the hot plasma parameters should correspond to a thin corona and be located below the line AB* in Figure 1a, i.e., the coronal plasma density should satisfy the condition $N < 1.3B^{4/3}$ (for $T_{\parallel} \simeq 10^7$ K). At $B \leq 2 \times 10^8$ G this condition requires that $N \leq 2 \times 10^{11}$ cm^{-3}. We recall that the lower boundary is defined by condition (3.2), which is satisfied above the line EF in Figure 1a (at N below EF the absorption lines are weak).

Thus, according to our investigation, the mere presence of absorption features in the spectra of isolated magnetic white dwarfs implies that the coronal plasma density of these stars lies within rather narrow limits. Specifically, N should lie between the lines

* If absorption lines are observed at higher harmonics, then the plasma parameters should lie still lower, below the line CD.

AB and *EF*, in the band shown in Figure 1a – i.e., it should satisfy the inequality

$$1.8B < N < 1.3B^{4/3} . \qquad (3.7)$$

The width of this band depends weakly on the magnetic field, and at $B \leq 2 \times 10^8$ G it does not exceed 2.5 orders of magnitude in electron density.

We assumed $T_{\parallel}$ to be fixed and equal to 10^7 K. The temperature dependence is investigated in the same manner as the dependence on magnetic field (in the plane of parameters $N - T_{\parallel}$ at fixed B, Figure 1b); an analogous plot is given in Tikhomirov's (1984) paper. The analysis shows that the lower boundary of inequality (3.7), which determines the condition of effective interaction of coronal plasma with radiation, changes only as $T_{\parallel}^{-1/2}$. The upper boundary, where the photospheric temperature ($T_{\text{ph}} = 3 \times 10^4$ K) and the brightness temperature of extraordinary radiation at the fundamental T_b are equal, changes by not more than 15%, as $T_{\parallel}$ varies in the range 10^6–10^8 K. At $T_{\parallel} < 10^6$ K, this boundary lies in the thick corona, at $T_{\parallel} > 10^8$ K, in the thin corona, where it has a constant level of 8×10^{11} cm^{-3} (for $B = 10^8$ G). Therefore, the upper limit on density given in this paper ($N \leq 2 \times 10^{11}$ cm^{-3} for $B \leq 2 \times 10^8$ G) does not, in fact, depend on the coronal temperature.

In conclusion it should be noted that in this paper we neglect the angular dependence of the transfer coefficients and the quasilinear intensity exchange of the cyclotron harmonics. These approximations are acceptable in the analysis of the characteristic ranges of parameters *N*, *B*, and $T_{\parallel}$. However, for correct calculation of relative line depths in the region below the line AB, it is necessary to study in more detail the transfer processes taking the above factors into account. This investigation will be made elsewhere.

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