C. L. HAMBLIN

# QUANDARIES AND THE LOGIC OF RULES

By a *quandary* I shall mean a circumstance in which someone cannot act without breaking one of a set of rules; and by a *rule* I mean any formulation of a prescription, permission or prohibition of one or more deeds, or a combination of these. A quandary, therefore, occurs in a *circumstance* for some *person* (or perhaps group of persons), relative to a set of *rules*. Doing nothing counts as doing something, so that a quandary cannot be evaded by doing nothing.

A parking lot contains two signs

(1A)    Keys to be left in dash

and

(1B)    Remove your ignition key.

For someone committed to parking (and who cares about the signs), these create a quandary: he cannot obey both at once. Generally, quandaries reflect an inconsistency in a set of rules. On the other hand, not every inconsistency generates a quandary. I am issued with a piece of paper which says at the top

(2A)    The holder is entitled to proceed without hindrance in all parts of the precincts

and at the bottom

(2B)    Admission is not permitted to security areas.

If security areas are part of the precincts, this is inconsistent; but there is no quandary. If I enter security areas I contravene (2B), but if I do not, I contravene nothing. The inconsistency between (2A) and (2B) is not of such a kind that I am unable, as in the case of (1A) and (1B), to conform with both.

Rules include statutes, orders by superior officers, rules of the road; rules of games; etiquette; recipes and instructions-for-use; advice to

parents; rules of deduction; and rules of language and of linguistic interchange. They are sometimes represented as having four basic logical forms, picturable in a square of opposition of traditional properties:
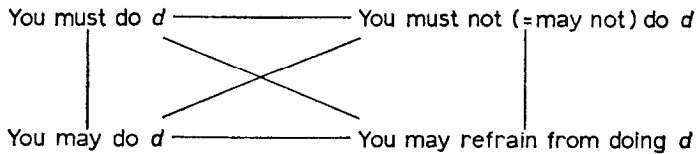
You must do $d$ —————————— You must not ($=$may not) do $d$

You may do $d$ —————————— You may refrain from doing $d$

Fig. 1.

Variants of formulation, irrelevant for our purpose, include: in place of 'You must do $d$', '$d$ is obligatory', 'Do $d$', 'You should do $d$', 'You are to do $d$', 'You ought to do $d$', 'Let $x$ do $d$' and so on; in place of 'You may do $d$', '$d$ is permitted', 'Do $d$ if you wish', and so on; and similarly for the others.

The four forms, perhaps, are interdefinable using internal and external negations. But they do not enter equally into the generation of quandaries. The conjunction of the two top-row forms, 'You must do $d$' and 'You must not do $d$', is quandarian, and these are the forms (to a sufficient approximation) of (1A) and (1B); but forms at the extremes of diagonals, as are (2A) and (2B), generate no quandaries, but at most some uncertainty as to what might be in the mind of the rule-maker.

It needs only a moment's extra thought to see that rules of bottom-row forms cannot take part in the generation of quandaries at all. To say that I *may* do something, or to say that I *may refrain* from doing it, gives me no instructions capable of being incompatible with other instructions. If we set out to determine the capacity of a given set of rules to generate quandaries we may as well start by crossing out all the permissive ones, and consider only those that are prescriptive or prohibitional. At least, this is a first approximation to the truth; though we shall find later that we need to discuss whether permissive rules may sometimes play a subsidiary role.

Now let us consider how a quandary may arise in the course of a sequence of actions. Let $P_0$ and $P_1$ be two people (or players, or participants) and let $D_0$, $D_1$, $D_2$, ... be deeds that they may do. An *event*

consists of the doing of a deed by a person, and is thus an ordered pair $\langle P_i, D_j \rangle$. I shall suppose two events take place in order:

$$E_0: \langle P_0, D_0 \rangle$$
$$E_1: \langle P_1, D_1 \rangle$$

and that the question now arises of whether $P_0$ shall or shall not do deed $D_2$,

$$E_2: \langle P_0, D_2 \rangle?$$

And I shall suppose that there is a set of rules $\rho$ that include or entail, firstly, a rule $r_0$ that in these circumstances – that is, after $E_0$ and $E_1$ have occurred – $P_0$ *must* do $D_2$, and secondly, a rule $r_1$ that in the same circumstances he *must not*. For example, let $E_0$ be $P_0$'s promise to his girl-friend to meet her at six, and $E_1$ the recall of $P_0$ to his unit by his commanding officer $P_1$, by five. The rule that we should keep promises, especially to girl-friends, entails $r_0$, that he must meet her as planned; the rule that he should obey lawful commands entails $r_1$, that he must not. Let us suppose he does $D_2$, meets her.

From what we have said, in doing $D_2$, $P_0$ has broken the derived rule $r_1$; that is, looking at the matter synoptically, he has failed to conform to the rules $\rho$. But has he? There is at least one argument to the effect that he has not, or, at least, must not be held responsible for so doing.

### I. ARGUMENT THAT THE ILLEGALITY IS IN $E_1$

Let us simplify our problem by assuming that events before, after and in between the three we are considering have no effect on the application of the rules $\rho$ to them. The overall effect of a set of rules is to divide possible sequences of events into *legal sequences* and *illegal sequences*. In the present case the rules $\rho$ class any sequence of the form

(S)        ..., $E_0$, ..., $E_1$, ..., $E_2$, ...

that is, any sequence of events in which $E_0$, $E_1$ and $E_2$ occur, in that order, independently of what is before, after or between, as illegal. And they also class as illegal any sequence of events of the form

(S')        ..., $E_0$, ..., $E_1$, ..., $E_2'$, ...

where $E_2'$ is any event $\langle P_0, D_2 \rangle$ of $P_0$'s not doing $D_2$ but doing in place of it some other deed $D_2'$.

But if all sequences of forms (S) or (S') are illegal, then all sequences of the form

(S'')    ..., $E_0$, ..., $E_1$, ...

are illegal. In fact the set of sequences (S'') is the join of the sets (S) and (S').

If follows that we must look for the source of the illegality not in $E_2$ or $E_2'$ – the doing of $D_2$ or $D_2'$ by $P_0$ – but in $E_0$ and $E_1$. In doing $D_2$, $P_0$ did not break the rules: they were already broken when he was put, *by the occurrence of $E_1$*, in the position of having no legal continuation.

In the case of our example this does not seem a very plausible conclusion, since $P_0$'s orders of recall would normally be regarded as overruling his promise. But that is because we weight the rules differently. If we were to balance them by treating the claims of true love as equal to those of military duty, we could find ourselves reasoning that the C.O. had no right to demand that $P_0$ break his promise at short notice.

## II. ARGUMENT THAT THE ILLEGALITY IS IN $E_0$

But this is not all. $P_0$ is court-martialled and pleads as suggested. The court dismisses the plea and reasons: "Knowing that you might be recalled to your unit $(E_1)$, you had no right to make the promise $(E_0)$, and thereby put yourself in an impossible position." How do we assess this reasoning?

Sometimes, perhaps, people are placed in quandaries through no fault of their own. By hypothesis, this is not the case here: not only is $P_0$ able to do something other than $D_0$, but he is also in a position to predict the possibility that $P_1$ will do $D_1$ and that he, $P_0$, will then be in a quandary in connection with $D_2$. On the other hand (we suppose), if he does not do $D_0$, no such situation can arise. It can reasonably be argued that he should not to $D_0$. At least if he does so and subsequently $P_1$ does do $D_1$, he must take the consequences.

And what of $P_1$? "$P_0$'s quandary", $P_1$ can say, "is no concern of mine. It is not my business to save him from it."

In effect it is now argued that any sequence

..., $E_0$, ...

however continued, is illegal. Even if $E_0$ is not followed by $E_1$, the possibility that it could be so followed -- with consequent quandary for $P_0$ in respect of $D_2$ – is sufficient to demonstrate illegality in the sequence in respect of $E_0$.

### III. THE EFFECT OF PERMISSIVES

This argument would be very much strengthened if, besides the top-row rules, explicit or entailed, $r_0$ and $r_1$, there were a bottom-row rule $r_2$ of the form (say)

$P_1$ may do $D_1$.

That is, if the C.O. is *entitled* to issue the recall order, $P_0$ is *not entitled* to make his promise. The permissive rule $r_2$, though it cannot contribute to the logical generation of a quandary, is relevant to deciding the legal status of deeds which causally generate it. In effect, it contributes to the disjunctive syllogism:

Either $P_1$ must not do $D_1$ or $P_0$ must not do $D_0$.
But $P_1$ may do $D_1$.
Therefore, $P_0$ must not do $D_0$.

Whether the first premiss of this inference really follows from the fact that $E_0$ and $E_1$ together lead to a quandary for $P_0$ over $D_2$, we have not yet decided; but if it does, the bottom-row $r_2$ can serve as second premiss to produce the top-row conclusion.

It is, of course, possible that there should exist another rule or entailed rule $r_3$ of the form

$P_0$ may do $D_0$.

Rules $r_2$ and $r_3$ together would seem to defeat any attempt to reflect blame for the quandary back on to $E_0$ or $E_1$. Under these circumstances we may be uneasy about the consistency of the set of rules. It is important to emphasise, however, that rules $r_0$–$r_3$ are not inconsistent in the sense of making a quandary unavoidable. $P_0$ does not break these rules if he does not do $D_0$, nor $P_1$ if he does not do $D_1$, and in either case no quandary arises. Moreover, the rules do not produce a simple incon-sistency; there is no deed that is both permitted and forbidden, or both

obligatory and omissible. If these rules are inconsistent they are inconsistent in a new sense we have not yet adequately characterised.

## IV. FOUR KINDS OF QUANDARY-FREEDOM

Even if we confine ourselves to top-row rules, there are several kinds of consistency or inconsistency; that is, in this case, of quandary-freedom or quandary-incurrence. I shall describe four kinds of quandary-freedom: they are not exhaustive.

### (1) *Absolute quandary-freedom*

It is possible for a set $\rho$ of rules to be such that it could under no circumstances produce any quandaries at all. The most obvious example is the empty set; but we need not be so extreme. The single rule 'We should keep our promises' is, to be sure, not absolutely quandary-free, since one may get into a quandary by making incompatible promises; but if we were to replace it by the rule 'We should keep any promise that is not incompatible with previous promises', no quandary would be possible except on the improbable supposition that it is possible to make two promises at the same time. In fact quite complicated sets of rules may be absolutely quandary-free: they may be so formulated that none of their top-corner rules can possibly conflict.

### (2) *Legislative quandary-freedom*

An absolutely quandary-free set of rules has the property that one cannot get into a quandary even if the rules are broken many times over, either by oneself or others or both. But there are many practical sets of rules in connection with which we settle for less than this. We often reason that it is sufficient that quandaries should be impossible *provided everyone obeys the rules*. Then if a quandary does occur it will be possible to put the blame on a previous infraction. I shall call this *legislative quandary-freedom*: unlike absolute quandary-freedom it can be achieved by *adding* rules to the quandary-infected system, namely, rules which will prevent the quandary from legally arising. The rule 'We should keep our promises' will not legally lead to quandaries if there is added to it the rule 'One should not make a promise inconsistent with previous promises', and the system consisting of just these two rules is legislatively quandary-free.

Most board and card games are quandary-free in at most this weakened sense. To take a rather artificial example, suppose that in Bridge I play two cards on one trick: then I both must and cannot play a card on the last trick of the hand, since, first, the rules say that I must play a card on each trick and, second, I cannot do so without somehow illegally getting an extra card. Thus the system of rules of Bridge is not absolutely quandary-free: but it may be (and presumably is) legislatively so, since this situation could not have arisen unless a rule had been broken in the first place.

### (3) Strategic quandary-freedom

Even legislative quandary-freedom may not be demanded in practical sets of rules, provided potential quandaries are predictable so that steps may be taken to avoid them. But in fairness to individuals subject to the rules it would often be reasonable to stipulate that it should not be possible for someone to be put in a quandary by the actions of others, without a chance to avoid it by actions of his own.

What is demanded is that for each person there should be a possible *strategy* which, if he follows it, will keep him out of quandaries independently of what is done by others. It will follow that no one can be forced into a quandary by the actions of others and, of course, that it is possible that no one will be put in a quandary at all. If we add, as seems reasonable, the condition that each person's actions in accordance with his strategy are everywhere legal, it is clear that the criterion is weaker than that of legislative quandary-freedom: I shall call such a set of rules *strategically quandary-free*.

There are other interesting kinds of quandary-freedom intermediate between, or parallel with, those considered. For example, there might be a strategy that avoided quandaries for some person $p$ but involved $p$ in breaking a rule along the way; or there might be a case in which a set of rules is strategically quandary-free for $p$, but only if $p$'s strategy involves taking some action which creates a quandary for someone else. I shall not try to sum up these possibilities.

### (4) Minimal quandary-freedom

Finally, let us weaken our requirement yet further: a set of rules is quandary-free in a minimal sense if it is possible for everyone so to act

that no quandaries arise. Again, we may add that they may do so legally so that the requirement is equivalent to: 'It is not impossible that everyone's actions be always legal'. This is as weak a consistency requirement as we could demand of any set of rules. It is compatible with the requirement that it should require the co-operative efforts of different people to keep everyone out of quandaries.

## V. SYMBOLIC FORMULATION

Now let us sharpen up these definitions by stating them in a precise context. Let $P$ be a set of people and $D$ a set of deeds, exhaustive and mutually exclusive. Let it be assumed that each $p \in P$ does just one $d \in D$ at each time. Some member of $D$, $D_0$ say, may be nominated as a 'null deed' to allow for occasional inactivity. Times are represented by integers $n \in N$.

I do not intend to construct a complete model of people doing deeds. Some of the limitations of the present model are essential, some inessential. The limitation that there is a first integer and thus a beginning of time is relatively inessential, and could be removed at some slight cost in complexity. The fact that time is regarded as discrete is not important so long as elementary time-intervals can be regarded as very short. On the other hand, deeds, as we ordinarily understand them, are perhaps not uniformly subdivisible into elementary deeds. For simplicity, I shall make no provision for variation of logical or physical possibility of deeds with context.

$H = D^P$, the set of functions from $P$ to $D$, is called the set of *happenings*: a happening consists of everybody's doing just one thing, and there is a happening at each time. $W = H^N$, the set of functions from $N$ to $H$, is called the set of *worlds*. In any given world, each person does just one deed at each time.

For $n \in N$, $H^n$ is the set of functions from times up to but not including $n$, to happenings. (Each integer $n$ is defined as the set $\{0, 1, ..., n-1\}$ of the preceding ones.) We can call $H^n$ the set of *histories at n*, since each of its members is an allocation of a deed to each person at each time preceding $n$. In general the set of *histories* is $J = \bigcup_n H^n$.

Within such a model, in which everyone does something at each time, we may represent all top-row rules as prohibitions; since to prescribe a

given deed or one of a set of deeds (of a person at a time) is to prohibit all others. An *elementary rule* is the prohibition of a particular deed of a particular person at some time, given a history at that time. Thus the set of elementary rules is

$$R = J \times P \times D.$$

If $r \in R$ is an elementary rule of the form $\langle j, p, d \rangle$, $j \in H^n$, it specifies that, given history $j$, $d$ is prohibited for $p$ at $n$.

There is a *quandary* for $p$ at $n$, relative to a set $\rho \subset R$ of rules, given $j \in H^n$, if $\langle j, p, d \rangle \in \rho$ for all $d \in D$.

A possible artificiality in the model consists in the fact that it allows that rules may be conditional on any event of past history. In fact, information is imperfect and flows at different rates, and reaction cannot always be expected to be immediate. Another limitation is that no allowance is made for rules that prescribe or prohibit joint simultaneous actions of different people. If such rules are allowed they introduce the complicating possibility of quandaries dependent on them; where, for example, there is a rule that at least one of $P_0$, $P_1$ must do $D_1$ but each is forbidden.

I assume that any set of top-row rules – in whatever rule-language one may devise – denotes or is representable as a subset $\rho \subset R$ of rules of elementary prohibitional form. We can now define our four kinds of quandary-freedom.

*Absolute quandary-freedom* obtains if there is no circumstance in which all deeds are prohibited; thus

$$- (\exists n \in N, p \in P, j \in H^n) \, (\forall d \in D) \, (\langle j, p, d \rangle \in \rho).$$

To define *legislative quandary freedom* we need the concept of a legal history. This is a history such that no deed done in it was against the rules $\rho$. Let $j \in H^n$ be a history, and let $k \in H^m$, $m < n$, be a subhistory of it, $k \subset j$. If $j$ is to be a legal history we require that no deed $d$ of any $p$ at $m$ be prohibited. Thus for the set of legal histories at $n$, relative to rules $\rho$, we have

$$L_{\rho,n} = \{ j \in H^n : \, - (\exists k \subset j, \, m \in n, \, p \in P, \, d \in D, \, h \in H) \, (k \in H^m \cdot \\ \cdot \langle m, h \rangle \in j \cdot \langle p, d \rangle \in h \cdot \langle k, p, d \rangle \in \rho) \}.$$

A set $\rho$ of rules is *legislatively quandary-free* if

$$-(\exists n \in N, p \in P, j \in L_{\rho,n}) (\forall d \in D) (\langle j, p, d \rangle \in \rho).$$

A *strategy* is a function from histories to deeds, $S = D^J$; and we say that a person *follows* a given strategy if, for any $j \in J$, he does the corresponding $d \in D$.* Now for the set of histories within which $p$ always follows strategy $s$, we have

$$T_{\rho,s} = \{ j \in J: (\forall d \in D, h \in H, n \in N, m \in n, k \subset j) ((j \in H^n \cdot$$
$$\cdot \langle p, d \rangle \in h \cdot \langle m, h \rangle \in k) \supset \langle k, d \rangle \in s) \}.$$

A set $\rho$ of rules is *strategically quandary-free* if everyone has a strategy such that, if he follows it, given any legal history he always has a legal deed,

$$(\forall p \in P) (\exists s \in S) (\forall n \in N, j \in L_{\rho,n} \cap T_{p,s}) (\exists d \in D) (\langle j, p, d \rangle \notin \rho).$$

Finally, *minimal quandary-freedom* relative to $\rho$ consists of the existence of at least one infinite legal history,

$$L_{\rho,N} \neq 0.$$

## VI. MIXED KINDS OF INCONSISTENCY

To import permissives into the model as well as rules that prescribe or forbid is to invite yet further multiple distinctions of kinds of consistency and inconsistency. But I do not think the presence of permissive rules necessarily much alters what we say about quandary-freedom. That is, although permissives may influence the allocation of the blame for any illegality associated with a quandary, they do not play any part in determining where a quandary occurs.

Thus it might be thought that there is an inconsistency in any case in which the doing of permitted deeds results in a quandary; and that the earlier deeds themselves could be regarded as raising an associated quandary for someone who saw the true quandary coming. But this is a questionable principle, as we may demonstrate.

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* More accurately, a strategy is an incomplete function, which specifies a deed for each circumstance that can arise *if the strategy is uniformly followed* (but independently of what others may do). But this concept is much more difficult to consign to symbols, and the added complication would not achieve any worth-while result here.

Let us suppose that a remote community in which bigamy of both sexes is permitted achieves an overlay of Christianity. $X$'s wife $Y$ contracts a second marriage with his neighbour $Z$. $X$ is bound to love, honour and cherish his wife and not to covet his neighbour's wife, but they happen to be the same wife, $Y$. This is a "permitted" state of affairs. Are we to say that the social mores are inconsistent? And, if so, how are we to categorise the inconsistency?

If the community, in adopting Christianity, had adopted a prohibition of bigamy, there would have been a clear simple inconsistency between this prohibition and the older permission. But let us suppose that such an explicit prohibition is not in force. Since no one *has* to marry anyone, the rules are strategically quandary-free; but not legislatively so, since the marriages of $X$ to $Y$ and $Y$ to $Z$ were not themselves illegal. The kind of inconsistency the rules incur in permitting bigamy is not to be regarded as of the same kind as if bigamy were simultaneously permitted and prohibited; nor is the decision whether or not to contract a bigamous marriage itself the subject of a quandary of the same kind as the one that ensues. A new kind of inconsistency is generated by the interaction of permission and prohibition, but it is additional to, and does not modify, the quandary that would be there independently of the permission.

That there is no general principle to the effect that one is prohibited from taking actions that result in quandaries may be seen by considering a case into which probabilities enter. Let us suppose that $X$ in marrying $Y$ knows he is at some risk of quandary should $Y$ subsequently marry $Z$ but thinks the risk small. If $Y$ does remarry, $X$ may reason, he will take the awful consequences; but this is no reason for not marrying $Y$ in the first place.

## VII. POTENTIAL UNITY OF RULE-SYSTEMS

In interpreting rules one is tempted to introduce irregularities and exceptions, and to appeal to dimly-understood meta-rules to justify modifications of the originals. This may be practical politics, but it is not necessary to good logic. Often it reveals that the rules have been badly stated or understood in the first place.

Take, for example, the case of 'higher' and 'lower' systems of rules, the first taking precedence over the second. This is a well-understood

concept; but we do not need to regard it as raising special or distinctive logical issues. Careful statement of any rule may require the incorporation of provisos, of forms such as 'unless there is conflict with a rule of type so-and-so'. But once these are written in, they express all that needs to be conveyed by an assertion that there are higher rules, and the logic goes through without this gloss.

Again, it could easily be (and sometimes has been) asserted that the concept of culpability is not closely related to the concept of rule-breaking, and requires extensive separate consideration. I shall not assert that this is not so; but it is clear that some, at least, of the considerations that are said to divide the two concepts are the result of sloppy logic. If there is a rule that I must give way to vehicles approaching from the right and I have a collision with such a vehicle in special circumstances of poor visibility, it is possible that I shall be held blameless. But all that this shows is that the rule, as it stands, is not fully stated and should really be understood to say something such as "A driver must *when possible, under normal circumstances,* give way ...". There are various good reasons, in practice, for leaving out qualifications; but their literal absence should not be allowed to make the logician's job harder than it need otherwise be.

*University of New South Wales*