SMALL SCALE ENTROPY AND ADIABATIC DENSITY PERTURBATIONS – ANTIMATTER IN THE UNIVERSE

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Abstract. Fluctuations of all scales are equally interesting from the point of view of the characteristics of the singular state and not only those which led to the formation of astronomical objects such as clusters of galaxies, separate galaxies, globular clusters and quasars. In this article estimates are given of the homogeneity of the overall density of hot plasma and the relation between the quantity of baryons and antibaryons at early stages of evolution of the Universe. These estimates are made for small scales, considerably smaller than the scale of the astronomical objects enumerated above. Considerations about the energy balance of hot plasma and distortions of the spectrum of relic radiation due to dissipation of density fluctuations of matter are used for these estimates. The corrected upper limit to early energy injection is given. In our preceding paper (Sunyaev and Zeldovich, 1970) this upper limit was underestimated. Difficulties with a model of the Universe which is symmetric in baryon charge are noted.

In agreement with the hot model of the Universe at an early stage of evolution of the Universe, the state of matter is characterized by a general density which is simply connected with the pressure ($P=\varrho c^2/3=\mathscr{E}/3$) and temperature ($\mathscr{E}=\sigma T^4$). Another characteristic is the density of baryon charge n which (in the presence of antibaryons) is given by $n=N-\tilde{N}$. In the process of expansion, neglecting diffusion and dissipation, the ratio $S_1=s/n$ is conserved, where s is the density of entropy ($s=\sigma T^3$; S is the specific entropy per unit baryon charge). For an explanation of the formation of the astronomical objects enumerated in the abstract, it is necessary to assume some inhomogeneity of the singular state. Fluctuations of $\mathscr{E}$ and fluctuations of velocity and the metric for constant S connected with fluctuations of $\mathscr{E}$, so called adiabatic fluctuations, are possible in principle. An inhomogeneity of n for homogeneous s and $\mathscr{E}$ in the absence of perturbations of velocity and the metric close to the singular state is also possible. In this case it is clear that the specific entropy S is inhomogeneous (due to its denominator) because of which such perturbations are called entropy perturbations. See the comment below about whirl perturbations (Ozernoy and Chernin, 1967).

In principle, it is possible to determine the value (amplitude) of large scale perturbations from the microstructure of the Universe which is presently observable, that is by the presence of stars, star clusters, galaxies and clusters of galaxies. For this purpose, it is necessary to take into account the age of these structures following from observational data on the one hand, and the theory of growth of inhomogeneities on the other. It should be noted that the connection between initial perturbations and present structure is in no way simple and a quantitative explanation of this connection cannot be considered to exist at the present time. It is assumed that separate stars

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arise as the result of thermal instability in already separated and heated regions of increased density, and do not characterize the spectrum of initial perturbations. However, the question of globular clusters already allows two solutions: their origin is secondary (as in the case of stars) or primary (Peebles and Dicke, 1968). It is natural to consider that clusters of galaxies ($M \sim 10^{13}$ to $10^{14}\ M_\odot$) and perhaps separate galaxies reflect the amplitude of perturbations which have corresponding masses. Formation of such objects does not create significant perturbations on still larger scales.

The purpose of this work is to investigate amplitude perturbations of small scale ($M \ll 10^{10}\ M_\odot$ for adiabatic perturbations and $M \ll 10^5\ M_\odot$ for entropy perturbations). These perturbations do not lead to the formation of a macro-structure since, in the first case, they are damped out by dissipative processes and in the second, they are stabilized by pressure. However, for sufficiently large amplitudes small-scale perturbations of both types lead to a considerable energy release and deformation of the spectrum of relic radiation. At the present time there are no substantiated indications of deviations of the spectrum from an equilibrium (black-body) spectrum and, therefore, it is only possible to obtain an upper limit on the amplitude of the extinguished perturbations.

We note that quite large-scale perturbations of the density of matter may, in principle, also be investigated by the angular dependence of the temperature of relic radiation. This method is applicable to perturbations whose amplitudes have not reached unity until the present time, so that they did not appear in the form of objects (super-clusters). The question of spatial fluctuations of relic radiation connected with initial density perturbations is considered in the work of Silk (1967), and Sachs and Wolf (1967), and also in another of our papers (Zeldovich and Sunyaev, 1970).

In this work a comparison is made between perturbations of all scales studied by different methods. It is a curious fact that the simple supposition of a power-law spectrum with constant index is apparently inconsistent with observational data. In this work other types of perturbations – turbulent, magnetic, etc., are not considered, although the methods of this article may give significant limits on the form of the spectrum for them. Moreover, if the data about strong distortions of the short-wavelength part of the spectrum of relic radiation (Shivanandan *et al.*, 1968) are substantiated, then they may require the existence of turbulent (whirl) perturbations for their explanation.

1. Adiabatic Density Perturbations

At the stage of expansion before recombination of the initial plasma adiabatic perturbations on scales less than the Jeans mass (dependent on the pressure of radiation and equal to $M_J \approx 10^{16} \Omega^{-2} M_\odot/(1+z/z_1\Omega)^3$ for $z > z_r = 1500$) are standing sound waves (see Figure 1). In what follows Z denotes the red shift, and $\Omega = \varrho/\varrho_{crit}$ is the dimensionless average density of matter in the Universe and

$$\varrho_{crit} = 3H_0^2/8\pi G = 2 \times 10^{-29}\ \text{g/cm}^3$$

for $H_0 = 100$ km/sec. Existing experimental data does not contradict $1/45 < \Omega < 2$. For $z \sim 4 \times 10^4 \Omega = z_1 \Omega$ the density of matter ϱ_m and radiation* ϱ_r are equal.

We shall characterize wavelengths by the mass of baryons contained in a cube with dimensions of a half-wavelength.

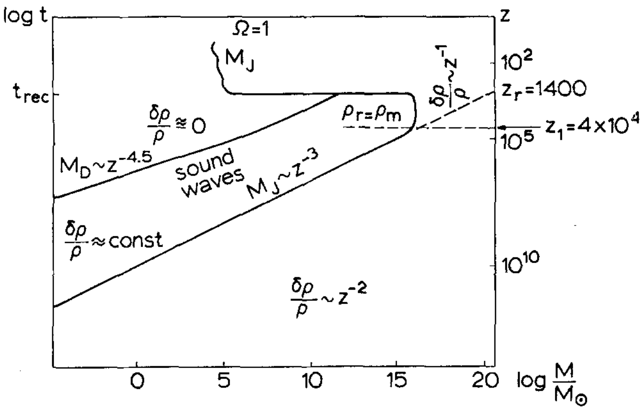

Fig. 1. Diagram of gravitational instability in the hot model of the universe. The region of instability lies to the right of the line $M = M_J(z)$ and the region of stability to the left. The region of damping of sound waves lies to the left of the line $M_D(z)$.

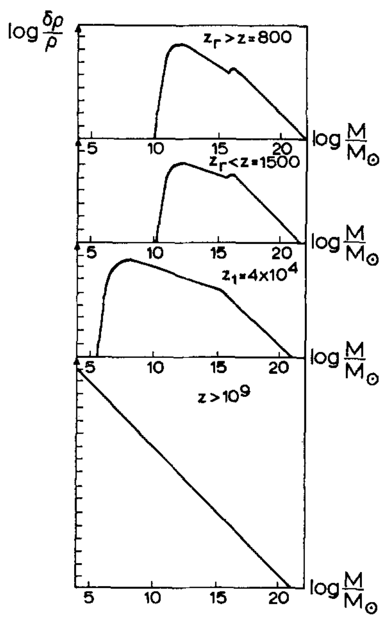

Fig. 2. Evolution of the spectrum of adiabatic density perturbations. It is assumed that $\Omega = 1$ and initially ($z > 10^9$) $\delta\varrho/\varrho \sim M^{-1}$.

* All calculations were made with the assumption that strong distortions of the spectrum of relic radiation (Shivanandan *et. al.*, 1968) do not exist or that the distortion is the result of the sum of the emissions of separate sources superimposed on the equilibrium spectrum with $T_r = 2.7$K.

Silk (1967) showed that due to the interaction of matter and relic radiation oscillations on scales less than

$$M_{\mathrm{D}} = (M_{\tau=1} M_H)^{1/2} \approx 10^7 \left(\frac{z_1}{z}\right)^{4,5} \Omega^{-1/2} M_{\odot} \tag{1}$$

are damped (Figures 1 and 2), where $M_{\tau=1}$ is the mass of a sphere whose optical depth due to Thompson scatteıing equals unity for given times and M_H is the mass of baıyons within the horizon of the Universe. Numerical values in Equation (1) are taken for the case $z > z_1 \Omega$. With the dissipation of the energy of sound waves the energy density of radiation increases and its spectrum is distorted. The question of distortions of the spectrum of relic radiation due to the release of energy at early stages of expansion of the Universe was considered in detail by us previously (Sunyaev and Zeldovich, 1970)*. The absence of significant distortions of the spectrum in the Rayleigh-Jeans regions allows us to establish an upper limit on the energy release as

$$q = \int \frac{Q(t)}{\sigma T_r^4(t)} \, \mathrm{d}t < 2 \times 10^{-2} \Omega^{7/8} \tag{2}$$

for a red shift in the region of integration

$$10^4 \Omega^{-1/2} < z < 5.4 \times 10^4 \Omega^{-6/5}, \tag{3}$$

where $Q(t)$ denotes the rate of energy release; T_r, the temperature of relic radiation at any given moment of time; and $\mathscr{E} = \sigma T_r^4$, the energy density of radiation. Due to the presence of dissipative processes radiation in the interval of red shifts given by Equation (3) damps oscillations corresponding to masses (see Equation (1))

$$5 \times 10^5 \Omega^{4.9} < \frac{M}{M_{\odot}} < 10^9 \Omega^{7/4} \tag{4}$$

if $\Omega < 0.4$ and for $\Omega > 0.4$ when the rate of expansion of the Universe** changes at the lower limit of (3) because ϱ_r becomes less than ϱ_m

$$5 \times 10^5 \Omega^{4.9} < \frac{M}{M_{\odot}} < 6 \times 10^8 \Omega. \tag{5}$$

Limits on the energy release (2) provide an upper limit on the amplitude of adiabatic density perturbations in this mass range. The energy of sound waves

$$E \sim \varrho u^2,$$

where

$$u = \frac{\delta \varrho}{\varrho} a_s \approx \frac{1}{\sqrt{3}} \frac{\delta \varrho}{\varrho} c$$

* As kindly pointed out to us by Dr. Chibisov, the quasi-stationary conditions are not fulfilled when $\sqrt{ak}\, t < 1$. In this case, $\mu = Qt/\sigma T^4$. Hereafter we use this approximation. The results of the paper quoted above, in so far as the q-Equation (27) is concerned, must be altered in accordance with Equation (2) of the present paper. In particular, the estimates of primeval turbulence (Formulae (31) and (32) in the item 3A of the paper quoted above) must be altered.

** In what follows we only obtain formulae for $\Omega < 0.4$. Formulae for $\Omega > 0.4$ are easy to obtain by considering that in this case at the limit $z \sim 10^4$ cosmological time $t \approx (2 \times 10^{17})/(z^{3/2} \Omega^{1/2})$ sec.

is the velocity of matter and a_s is the speed of sound. It is now easy to estimate the energy release for the complete damping of oscillations

$$q \approx \frac{1}{3}\left(\frac{\delta\varrho}{\varrho}\right)^2 . \tag{6}$$

It is interesting that q does not depend on the moment of damping, and therefore on the mass of perturbations if it is located in interval (4) or (5). We note that there is a sufficiently weak dependence of $\delta\varrho/\varrho$ on q. Comparing (2) and (6), we find that in the wide mass range of (4) and (5)

$$\delta\varrho/\varrho < 0.2\Omega^{7/16} . \tag{7}$$

Without the limits following from the observed spectrum of relic radiation, there is only the obvious limit $\delta\varrho/\varrho<1$ for $z \leqslant z_J(M)$; otherwise a significant fraction of

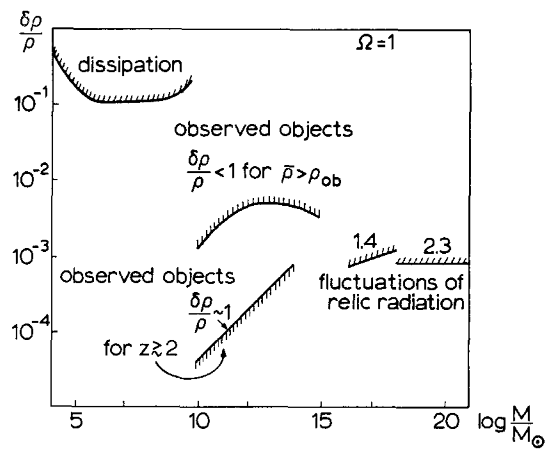

Fig. 3a.

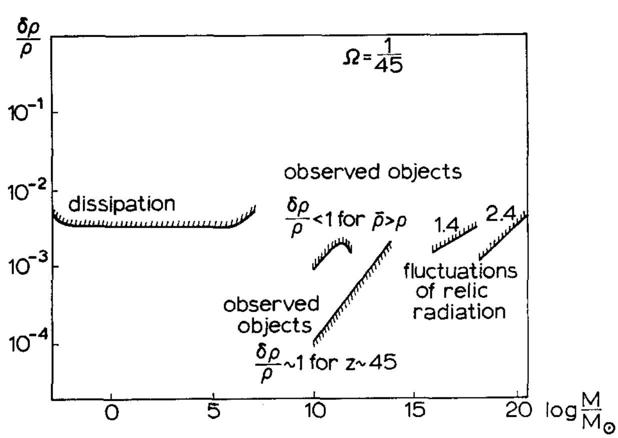

Fig. 3a, b. Limits on the spectrum of adiabatic density perturbations. The numbers indicate applications of experimental results (Conklin and Bracewell, 1967 (1); Stankevich, 1970, (2)) and theoretical formulae (Sachs and Wolf, 1967, (3); Zeldovich and Sunyaev, 1970, (4)).

the matter in the Universe would be in collapsed and super-dense bodies (Novikov and Zeldovich, 1967). The possibility of obtaining upper limits on the amplitudes of perturbations from which observed objects grew is an interesting one. Peebles (1969) showed that nonlinear effects (formation of shock waves) lead to the transfer of energy from long-wavelength oscillations to short wavelength ones whose energy is quickly dissipated, and then distortions of the spectrum of relic radiation occur again. The calculation was done only for the case $\Omega = 1$ for which the present accuracy of measurements of relic radiation does not allow the establishment of interesting limits on the initial $\delta\varrho/\varrho$.

A summary of the limits obtained is given in Figures 3a, b together with the limits found from measurements of the fluctuations of relic radiation and from the condition that density perturbations on the scale of galaxies should be sufficiently large at the moment of recombination for the formation of observed objects. We note that the lower limit of the amplitude of perturbations with $M \sim 10^{10}$ to $10^{14} M_\odot$ is given for the case when the formation of observed objects is due to adiabatic perturbations, in the theory of entropy perturbations and whirl perturbations this limit may be arbitrarily lowered. For the construction of this limit the evolution of density perturbations in the process of expansion of the universe was utilized (see Figure 2): the amplitude of density perturbations in the process of recombination increases

$$\left(\frac{M_J(z_{\text{rec}})}{M}\right)^{1/3}$$

times (Zeldovich and Sunyaev, 1970) and after recombination perturbations grow according to the low $\delta\varrho/\varrho \sim (1+z)^{-1}$ up to $1+z_0 \geqslant 1/\Omega$ if $\Omega \leqslant 1$ since in the open world the growth of perturbations continues only up to $z \sim 1/\Omega$. A rough analysis showed that after this moment they may still grow 2.5 times. An upper limit to the amplitude in the region of observed masses was obtained from the assumption that the density of matter in objects existing at the present time may be less than the average density of matter in the Universe at the time of its formation. Such an estimate for clusters of galaxies (occupying at the present time $\sim 1\%$ of all the area of the sky and apparently occupying a volume at $z \sim 4$ to 5) in the case of small $\Omega < 0.1$ contradicts the slow growth of density perturbations at $z < 1/\Omega$. It would have seemed that a conclusion about $\Omega > 0.1$ could be made from this, but in fact we cannot say hardly anything about the evolution of clusters of galaxies after their separation into independent objects; it is completely possible that they may have expanded several times during simultaneous compressions of their central regions.

Limits are given for each mass at the moment of time when this mass is equal to the Jeans mass

$$z_J = \Omega z_1 \left[\left(\frac{10^{16}\Omega^{-2} M_\odot}{M}\right)^{1/3} - 1\right]$$

i.e. at different moments of time (see Figure 1). For scales exceeding the Jeans mass at the moment of recombination limits are given analogously for the moment of time

$$z_J = \Omega z_1 \left(\frac{10^{16} \Omega^{-2} M_\odot}{M} \right)^{1/3}.$$

They were obtained from observational data on small-scale fluctuations of relic radiation (Conklin and Bracewell, 1967; Stankevich, 1970 ($\Delta T/T < 3 \times 10^{-4}$ on the scale $\theta \gtrsim 1°$); Wilkinson and Partridge, 1967) by use of the equations given in the works of Sachs and Wolf (1967) and Zeldovich and Sunyaev (1970). For calculation according to the equations presented in the latter work it was assumed that initial temperature fluctuations connected with density perturbations are not smoothed out in the process of expansion of the Universe, i.e. the optical depth for Compton scattering is small up to $z \sim 900$.

The amplitude of perturbations on scales $M > M_J(z_{rec})$ were multiplied by the additional factor $z_1 \Omega / z_J$. The point is that in a Universe with pressure $\varrho = \mathscr{E}/3$ (i.e. at $z > z_1 \Omega$) perturbations with $M > M_J$ grow according to the law $\delta\varrho/\varrho \sim t \sim z^{-2}$, while for $p \ll \mathscr{E}/3$ when $z < z_1 \Omega$ the growth of perturbations procedes according to the law $\delta\varrho/\varrho \sim t^{2/3} \sim z^{-1}$ and this correction simplifies the recalculation of Figures 3a, b at any arbitrary early moment of time, which is the same for all masses: it is sufficient to multiply the limits given for $\delta\varrho/\varrho$ by the factor

$$\left(\frac{z_J}{z}\right)^2 = a \left(\frac{M}{M_\odot}\right)^{-2/3} = \frac{8 \times 10^{19} \Omega^{2/3}}{z^2} \left(\frac{M}{M_\odot}\right)^{-2/3},$$

if surely $z > z_J(M)$. For this transformation on a logarithmic scale the function $-\lg a - \frac{2}{3} \lg(M/M_\odot)$ is added.

It is clear from the figure that the spectrum of initial density perturbations which have not yet been dissipated may not quickly increase towards the region of small masses; the limits obtained in the case of small Ω may not be combined with the assumption about a single power-law spectrum of adiabatic density perturbations; a narrow interval of possible spectral slopes $\delta\varrho/\varrho \sim M^{-n}$: $n + \frac{2}{3} \sim 0$ remains only for $\Omega \sim 1$ because of the indeterminacy of estimates.

2. Entropy Fluctuations and Antimatter on Small Scales

The formation of galaxies as a result of entropy fluctuations was considered in Doroshkevich *et al.* (1967) and the formation of globular clusters in Peebles and Dicke (1968). In both cases it is assumed that

$$\delta = \left|\frac{\delta n}{n}\right| \gtrsim \frac{1}{30} \div \frac{1}{300}$$

on a scale $M > 10^5\ M_\odot$ at the moment of recombination of the initial plasma. It is also assumed that the function $\delta(M)$ decreases with increasing scale. Below we consider the question of the value of δ in the region of small scales. By extrapolating the decreasing dependence of $\delta(M)$ to the region $M \ll 10^5 M_\odot$ it can be assumed that $|\delta| \sim 1$ and even $|\delta| > 1$ * is reached for small M.

But in this case in the part of space where $\delta < -1$, $n = \bar{n}(1+\delta) < 0$ which is equivalent to the supposition of the existence of regions with a predominance of antibaryons (details are presented in Appendix II). Keeping in mind that for $kT > 0.05\ m_p c^2$ the number of baryons and antibaryons is generally much larger than the average surplus of baryons in the Universe, the supposition of small regions with surpluses of antibaryons seems natural. One can imagine that the baryon charge, i.e. the difference $n = N - \tilde{N}$, changes not only in value from point to point, but also changes sign and there are regions of antibaryons, $n < 0$. In statistical equilibrium for $\bar{n} = 10^{-8} N$ regions with $n < 0$ contain $(10^{-8})^{-2} = 10^{16}$ baryons on the average and their average baryon charge $|n| \sim 10^8$. Such regions are negligibly small, and annihilation in them due to neighbouring regions occurs quite early, leaving no observable traces. However, in such a picture fluctuations $\delta n/n$ on the scales of observed objects would also be negligible. If entropy fluctuations play an astronomical role, then they are enormously larger than thermodynamic fluctuations. However, in the case one can assume that there must exist volumes for regions with $n < 0$ which are larger than 'thermodynamical' volumes and may be described as regions with $n = \bar{n}(1+\delta) < 0$, $\delta < -1$.

If these regions are sufficiently large, then their annihilation occurs much later than annihilation of the basic mass of equilibrium baryons and antibaryons: the process of annihilation depends on the size of regions, i.e. on the form of the function $\delta(M)$. The release of energy by annihilation at a later stage of expansion (after annihilation of electron-positron pairs) may significantly distort the spectrum of relic radiation. Calculations allow us to present a significant limit on such entropy fluctuations on small scales.

We shall make rough estimates; more precise results are given in Appendix II. The size of the zone in which annihilation effectively occurs is determined by the condition

$$r^2 = 6Dt, \tag{8}$$

where

$$D = \frac{3kT_e c}{2\sigma_T \mathscr{E}} = \frac{10^{32}}{z^3} \frac{\text{cm}^2}{\text{sec}}$$

is the coefficient of ambipolar diffusion of ionized hydrogen plasma (see Appendix I) in a radiation field with temperature $T = 2.7(1+z)\,K$ and $t_{\text{cosm}} \approx 3 \times 10^{19} z^{-2}$ sec is the cosmological time at the stage of expansion of the Universe when the energy density of radiation exceeds the density of matter. Knowing r, we can easily estimate the mass of a region which is able to completely annihilate at a given moment of time as

$$M \approx \frac{4\pi}{3} r^3 \Omega \varrho_{\text{crit}} z^3 \approx \Omega \frac{10^{17}}{z^{4,5}} M_\odot . \tag{9}$$

* We note that this assumption, leading to initial $\delta(M) > 1$ in the region of small masses contradictes nothing since at earlier stages of expansion of the Universe the density is determined by radiation and $\delta\varrho/\varrho$ is small, but at later stages diffusion leads to the smoothing out of density on small scales.

Observations of relic radiation gave information about the energy release in the period defined by Equation (3). In agreement with (9) zones with mass

$$5 \times 10^{-5}\Omega^{6,4} < \frac{M}{M_\odot} < 10^{-1}\Omega^{13/4} \tag{10}$$

are annihilated in this time.

The existence of zones of antimatter and ordinary matter on the scale of (10), i.e. $\delta > 1$ for this scale, contradicts condition (2). Limits calculated on basis of the more precise formulas of Appendix II are given in Figures 4a, b. Since the amplitude of entropy density perturbations is practically constant at the stage before recombination, all the estimates are given for $z \geqslant z_{\text{rec}}$.

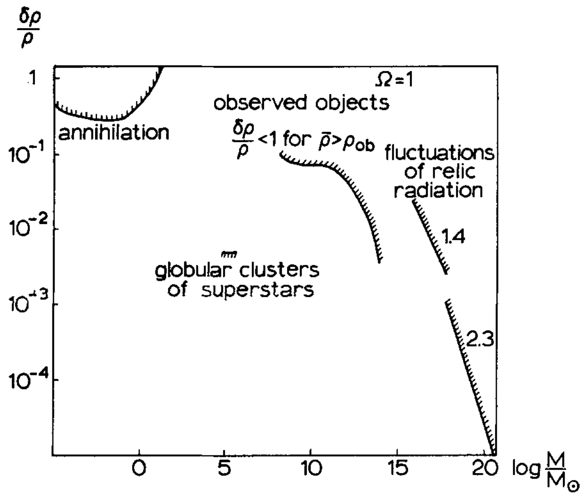

Fig. 4a.

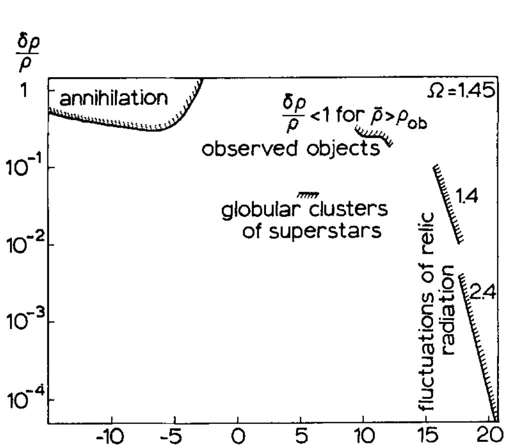

Fig. 4a, b. Limits on the spectrum of entropy density perturbations. The numbers signify the same thing as in Figures 3a, b.

As can be seen from the figures, these limits are difficult to reconcile with the supposition about the value of δ and a decreasing spectrum of entropy density perturbations for $M > 10^5 M_\odot$. In connection with this it is possible to relinquish the hypotheses of entropy perturbations and use adiabatic perturbations with $S = \text{const}$ (true, as was shown in the first part of the article, there are also difficulties with adiabatic perturbations). However, a possible mechanism exists for the excitation of entropy perturbations for which δ may be of order unity, but which never go beyond the limit $\delta \sim -1$. Such a mechanism is obtained, for example, if one imagines that at very early stages when the Universe had still not reached the regime of isotropic expansion (Misner, 1969) it was 100% charge unsymmetric: there were baryons, but no antibaryons and entropy was small. At a nonisotropic stage of expansion an accumulation of entropy occured which was different in different regions, so that the specific entropy S was distributed in a nonequilibrium manner. However, in this case n is automatically positive everywhere. It is obvious that baryon-antibaryon pairs are excited with an increase of temperature, but the initial (until the accumulation of entropy) surplus of baryons in this case is preserved everywhere.

If the existence of significant entropy fluctuations together with the absence of regions with a surplus of antibaryons could be proven, we would obtain important information about very early stages of evolution of the Universe.

In principle, together with fluctuations of the density of baryon charge, fluctuations of other types of charge, leptons and electric charge, should be considered. A particularly large number of questions arises for electric charge and the electromagnetic field connected with it.

3. Antimatter in the Universe on Large Scales

Harrison (1968) considered a charge symmetric model of the Universe with initial fluctuations of the baryon charge from which the spatial distribution of matter and antimatter after annihilation of baryons in the hot model of the Universe, i.e. for $T < 10^{10}$ K. In what follows before the moment of recombination the friction of matter due to radiation impedes the motion of matter so that annihilation is impeded, and regions with matter and antimatter (future objects and anti-objects) exist up to $z \sim 10^3$.

It is true, according to the remark of Bardeen (see the review of Field, 1968), that after recombination of hydrogen regions with one sign of baryon charge would rapidly become separated and objects would be formed with an average density of the order of 10^{-20} g/cm^3 which considerably exceeds the observed density of matter in galaxies and clusters of galaxies ($M > 10^9 M_\odot$).

Observational verification of this hypothesis, as in the case of any other assumption of a significant quantity of antimatter in the Universe, may be carried out in two ways: the first and most obvious is the direct observation of γ-quanta resulting from annihilation; the second is an investigation of the spectrum of relic radiation which should be distorted due to the release of energy in annihilation at early stages of expansion.

The Universe is transparent for γ-quanta resulting from annihilation ($E \sim 100$ MeV) up to $z\sim 70$ if $\Omega\sim 1$ and up to $z\sim 300$ if $\Omega\sim\frac{1}{45}$ (Arons and McCray, 1969). This fact makes any hypothesis about the presence of a significant quantity of antimatter in the Universe generally improbable; for the total energy density in background X-ray and γ-radiation is 3×10^3 times smaller than in relic radiation and constitutes only $(10^{-8}/\Omega)\, z_{\mathrm{ann}}$ fraction of the total energy of matter, where z_{ann} is the red shift when annihilation would have occurred. It follows from this that for $\Omega\sim 1$ in the period $0<z<100$ only one-millionth of the matter could have been annihilated.

It will be shown below that the presence of such objects with arbitrary masses in fact contradicts the limits obtained previously (Zeldovich and Sunyaev, 1969; Sunyaev and Zeldovich, 1970) on the energy release which follow from observations of the relic radiation. The point is that, in spite of the strong connection of matter and radiation, matter will diffuse to the bounds of regions with different baryon charges annihilate, and the energy released in this case will rapidly be converted into radiation, distorting its spectrum. We consider the situation for $z>z_{\mathrm{rec}}$ while for $z<z_{\mathrm{rec}}$ annihilation proceeds rapidly.

The characteristic size of a region with mass M is

$$R\sim\sqrt[3]{\frac{3}{4\pi}\frac{M}{\varrho}}\,.$$

The release of energy in this case for a zone in which annihilation has occurred with size r (See Equation (8)) is

$$q\simeq 3\frac{r}{R}\frac{\varrho_m}{\varrho_r}\,. \tag{11}$$

It was shown previously (Sunyaev and Zeldovich, 1970) that limits (2) on the energy release for $z>10^4$ contradict the division of the Universe into regions of baryons and antibaryons if their masses are smaller than $3\times 10^9\Omega^{5/2}M_{\odot}$. This estimate will be made more precise below. It follows from (1.3) that the rate of diffusion quickly grows with decreasing z which increases the energy release and before recombination ($z>1500$) and the characteristic cooling time of the plasma

$$t_{\mathrm{cool}}\approx\frac{m_e c}{\sigma_T\mathscr{E}}=\frac{m_e c}{\sigma_T\mathscr{E}_0 z^4} \tag{12}$$

becomes of the order of the characteristic heating time in the case of annihilation

$$t_{\mathrm{heat}}\approx\frac{n_e k T_e}{q\mathscr{E}}\,t_{\mathrm{cosm}}\approx A\,\frac{T_e(z)}{z^{5/2}} \tag{13}$$

only for the condition $T_e\gg T_r$. Here, as in (2),

$$q=\int\frac{Q(t)}{\mathscr{E}}\,\mathrm{d}t\,,$$

is the rate of energy release, and t_{cosm} is the cosmological time at that stage of expansion of the Universe. Distortions of the spectrum of relic radiation due to the exchange of energy between the plasma and radiation in the case $T_e \gg T_r$ were considered in Zeldovich and Sunyaev (1969). It was shown that within one standard deviation existing observations limit the parameter*

$$y = \int \frac{kT_e}{m_e c^2}\, d\tau < 0.15 \tag{14}$$

(τ – optical depth for Thompson scattering) and the total energy release

$$q < e^{4y} - 1 \sim 0.8\,. \tag{15}$$

It is easy to find an upper limit $\bar{T}_e < 4 \times 10^6 \Omega^{-1/2}$ K at $z \sim 1500$ from Equation (14). Then on the basis of (11) and (15) it can be shown that division of the Universe into regions of matter and antimatter is forbidden by observations if the masses of these regions are less than $3 \times 10^{17} \Omega^{11/2} M_\odot$. If it is taken into account in (14) that $r < R$, then upper limits on the temperature of the plasma in the boundary layer and at the same time on the maximum mass of objects from antimatter increase still more. This result presents great difficulties for Harrison's model which are all the more severe before recombination when there still are no galaxies and a magnetic field is probably absent.

Acknowledgement

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Appendix I. Coefficient of Ambipolar Diffusion in a Radiation Field

It is easy to find the coefficient of diffusion of electrons in the radiation field $D = kT_e b$ where $b = v/f$ is the mobility if we know the friction force acting on electrons in a radiation field with energy density $\mathscr{E}$

$$f = \tfrac{4}{3} v \frac{\sigma_T \mathscr{E}}{c}\,. \tag{I.1}$$

Considering that the presence of protons weakly connected with radiation increases the mobility by a factor of 2, we obtain finally

$$D = \frac{3}{2} \frac{kT_e c}{\sigma_T \mathscr{E}}\,. \tag{I.2}$$

* We introduce a note here which is most directly connected with our previous article (Sunyaev and Zeldovich, 1970). If the situation $T_e \gg T_r$ occurred for a long time in the process of evolution of the universe, then for $y < 0.5$ this should have practically not been evident in the earlier distortions of the spectrum of relic radiation in the Rayleigh-Jeans region and the limits (2) are trustworthy.

In a Universe with $T_r = 2.7(1+z)$ K the coefficient of diffusion is numerically equal to

$$D = \frac{3 \times 10^{31} T_e(z)}{z^4} \frac{\text{cm}^2}{\text{sec}}; \tag{I.3}$$

and with $T_e \approx T_r$

$$D = \frac{10^{32}}{z^3} \frac{\text{cm}^2}{\text{sec}}. \tag{I.4}$$

Appendix II. Calculation of the Annihilation of a Random Distribution

Let the density of baryons per unit of co-moving volume equal

$$\left.\begin{aligned} N(\mathbf{x}, t) &= \bar{n}(1 + \delta(\mathbf{x}, t)), \\ \delta(\mathbf{x}, t) &= \int \delta_k(t)\, e^{i\mathbf{k}\mathbf{x}} d^3 k, \end{aligned}\right\} \tag{II.1}$$

where $\mathbf{x}$ is the co-moving coordinate with $\mathbf{x}$ and the volume normalized so that they coincide with the physical coordinates and volume at the present time. It is natural to assume that

$$\bar{\delta}^2(x) = \int |\delta_k|^2\, d^3 k; \tag{II.2}$$

and $\delta(x)$ has a normal (Gaussian) distribution

$$P(\delta) = \frac{1}{\sqrt{2\pi\bar{\delta}^2}}\, e^{-\delta^2/2\bar{\delta}^2}. \tag{II.3}$$

In this case the probability that $\delta < -1$ and the fractional volume with a predominance of antibaryons which is equal to this probability is found from the formula

$$w = \frac{1}{2\sqrt{2}} \operatorname{erf}\left(\frac{1}{\bar{\delta}}\right). \tag{II.4}$$

The density of antibaryons averaged over volume is

$$\tilde{N} = \bar{n} \sqrt{\frac{\bar{\delta}^2}{2\pi}}\, e^{-1/2\bar{\delta}^2} \tag{II.5}$$

Correspondingly, the average density of baryons is $N = \bar{n} + \tilde{N}$.

We derive the equation for the Fourier component δ_k considering the diffusion of baryons and antibaryons in the linear approximation, i.e. without taking into account the indirect influence of annihilation on the local temperature and pressure of the medium. Annihilation clearly does not have a direct influence on the equation for δ_k; in fact, we derive two equations of diffusion and annihilation for N and $\tilde{N}$: namely,

$$\begin{aligned} \partial N/\partial t &= D\Delta N - AN\tilde{N}, \\ \partial \tilde{N}/\partial t &= D\Delta \tilde{N} - AN\tilde{N}, \end{aligned} \tag{II.6}$$

and subtract one from the other in which case the annihilation term disappears, leading to

$$\frac{\partial(N-\tilde{N})}{\partial t}=\frac{\partial n}{\partial t}=\bar{n}\frac{\partial\delta}{\partial t}=D\bar{n}\Delta\delta\,, \tag{II.7}$$

The equation for δ_k has the form

$$\frac{\mathrm{d}\delta_k}{\mathrm{d}t}=-\,Dk^2z^2\delta_k\,, \tag{II.8}$$

where z^2 appears due to the transformation from co-moving to rest coordinates; D is the diffusion coefficient which was shown in Appendix I to be given by $D=D_0z^{-3}$ during the period of expansion of the universe which is of interest here. Finally, the time $t_{\mathrm{cosm}}=3\times10^{19}z^{-2}$ sec $=t_0/z^2$ during the period when radiation is dominant. Hence,

$$\left.\begin{aligned}&\frac{\mathrm{d}\ln\delta_k}{\mathrm{d}z}=2D_0k^2t_0z^{-4}\,,\\&\delta_k(z)=\delta_k(\infty)\exp\left(-\frac{2D_0k^2t_0}{3z^3}\right).\end{aligned}\right\} \tag{II.9}$$

For a power-law dependence on the wave vector of the initial amplitude $\delta_k(\infty)$

$$\delta_k(z=\infty)=Bk^n$$

we obtain a simple expression for the mean square value

$$\bar{\delta}^2=B^24\pi\int_0^\infty k^{2n}\exp\left(-\frac{2}{3}\frac{D_0k^2t_0}{z^3}\right)k^2\mathrm{d}k=B^2\Gamma\left(n+\frac{1}{2}\right)\left(\frac{3}{2}\frac{z^2}{D_0t_0}\right)^{n+3/2}. \tag{II.10}$$

The equations presented in principle answer the question about the quantity of antibaryons remaining at a given moment z. It was noted above that the equation of diffusion for the difference $N-\tilde{N}$ does not depend on the rate of annihilation. In fact, annihilation proceeds quickly and, therefore, in regions where $N-\tilde{N}>0$, $\tilde{N}=0$; but in the region where $N-\tilde{N}<0$, $N=0$ which was taken into account in the expression for $\tilde{N}$ given above. Knowing, the way $\tilde{N}$ changes, we can find the energy released in annihilation and then the distortion of the spectrum of relic radiation by the equations derived previously (Sunyaev and Zeldovich, 1970). It is clear that the precise calculations give a result which coincides with the estimates of Section 2 of the main paper.

References

Doroshkevich, A. G., Zeldovich, Ya. B., and Novikov, I. D.: 1967, *Astron. Zh.* **44**, 295.

Conklin, E. K. and Bracewell, R. N.: 1967, *Nature* **216**, 777.

Field, G. B.: 1968, in *Stars and Stellar Systems*, vol. IX (ed. by A. M. Sandage), Univ. of Chicago Press.

Harrison, E. R.: 1967, *Phys. Rev. Letters*, **18**, 1011.

Misner, C. W.: 1969, *Phys. Rev. Letters* **22**, 1071.
Ozernoy, L. M. and Chernin, A. D.: 1967, *Astron. Zh.* **44**, 1131.
Peebles, P. J.: 1969, preprint of Palmer Physical Laboratory, Princeton University.
Peebles, P. J. E. and Dicke, R. H.: 1968, *Astrophys. J.* **154**, 898.
Sachs, R. K. and Wolf, A. M.: 1967, *Astrophys. J.* **147**, 73.
Shivanandan, K., Houck, T. R., and Harwit, M. O.: 1968, *Phys. Rev. Letters* **21**, 1460.
Stankevich, K. S.: 1970, *Australian J. Phys.*, in press.
Sunyaev, R. A. and Zeldovich, Ya. B.: 1970, *Astrophys. Space Sci.* **7**, 20.
Wilkinson, D. T. and Partridge, R. B.: 1967, *Nature* **215**, 719.
Silk, J.: 1967, *Astrophys. J.* **151**, 459.
Zeldovich, Ya. B. and Novikov, I. D.: 1967, *Relativistic Astrophysics*, Nauka, Moscow (in Russian).
Zeldovich, Ya. B. and Sunyaev, R. A.: 1969, *Astrophys. Space Sci.* **4**, 302.
Zeldovich, Ya. B. and Sunyaev, R. A.: 1970, *Astrophys. Space Sci.* **7**, 201.