EFFECTS OF MASS TRANSFER, FREE-CONVECTION CURRENTS AND HEAT SOURCES ON THE STOKES' PROBLEM FOR AN INFINITE VERTICAL PLATE

A. A. RAPTIS and G. J. TZIVANIDIS

Department of Mechanics, University of Ioannina, Greece

(Received 7 January, 1981)

Abstract. An exact analysis of the effects of heat sources, mass transfer and free-convection currents on the flow past an impulsively started vertical plate is investigated. Closed-form solutions to the velocity and temperature field have been derived by using Laplace transform and expressions are given for the Skin-friction and the rate of heat transfer. Variations of the above quantities are presented graphically, and the paper is concluded with a quantitative discussion.

1. Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate in its own plane, was studied first by Stokes (1851). Recently Georgantopoulos *et al.* (1979) have investigated the effects of mass transfer and free-convection currents on the flow past an impulsively-started vertical plate. But in these papers the effects of temperature-dependent sources have not been taken into account. Such a situation exists in many industrial or technological applications, solar energy problems, or in problems of space sciences. Hence, the object of the present paper is to study the effects of temperature-dependent sources as an extension of the problem which has been solved by Georgantopoulos *et al.* (1979).

2. Mathematical Analysis

Using the geometry, assumptions, and physical variables of Georgantopoulos *et al.* (1979), the problem is governed by the equations

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2}, \tag{1}$$

$$\rho c_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + Q', \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}, \tag{3}$$

where $Q'/\rho c_p$ is the volumetric rate of heat generation (absorption). We consider in the

Astrophysics and Space Science **78** (1981) 351–357. 0004–640X/81/0782–0351$01.05.

present analysis the heat generation (absorption) of the type

$$Q' = Q(T'_\infty - T'). \tag{4}$$

Under that condition, the non-dimensional equations, initial and boundary conditions of the problem are

$$\frac{\partial u}{\partial t} = G\Theta + GcC + \frac{\partial^2 u}{\partial y^2}, \tag{5}$$

$$P\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial y^2} - S\Theta, \tag{6}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}, \tag{7}$$

where

$$u = 0, \Theta = 0, C = 0, \quad \text{for all} \quad y \geqslant 0, x' > 0, t \leqslant 0,$$

$$\left.\begin{array}{lll} u = 1, \Theta = 1, C = 1 & \text{at} & y = 0 \\ u = 0, \Theta = 0, C = 0 & \text{as} & y \to \infty \end{array}\right\} t > 0. \tag{8}$$

The non-dimensional quantities are defined by Georgantopoulos *et al.* (1979) and $\acute{S} = Q\nu^2/kV_0^2$ (heat source parameter).

Equations (5)–(7), subject to the boundary conditions (8), can be obtained by Laplace-transform technique, and the solutions are given by the expressions (9) and (10), i.e.,

$$\Theta = \tfrac{1}{2}\exp(y\sqrt{S})\left[\exp(-2y\sqrt{S})\operatorname{erfc}\left(\sqrt{\frac{P}{t}}\frac{y}{2} - \sqrt{\frac{St}{P}}\right) + \right.$$
$$\left. + \operatorname{erfc}\left(\sqrt{\frac{P}{t}}\frac{y}{2} + \sqrt{\frac{St}{P}}\right)\right],$$

$$u = \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \frac{G}{2S}\left[\exp\left(\frac{St}{1-P} + \sqrt{\frac{S}{1-P}}y\right) \times \right.$$
$$\times \left[\exp\left(-2\sqrt{\frac{S}{1-P}}y\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\frac{St}{1-P}}\right) + \right.$$
$$+ \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\frac{St}{1-P}}\right) - \exp\left(-2\sqrt{\frac{S}{1-P}}y\right)\operatorname{erfc}\left(\tfrac{1}{2}\sqrt{\frac{P}{t}}y - \right.$$
$$\left. - \sqrt{\frac{St}{P(1-P)}}\right) + \operatorname{erfc}\left(\tfrac{1}{2}\sqrt{\frac{P}{t}}y + \sqrt{\frac{St}{P(1-P)}}\right)\Bigg] +$$
$$+ \exp(\sqrt{S}\,y)\left[\exp(-2\sqrt{S}\,y)\operatorname{erfc}\left(\tfrac{1}{2}\sqrt{\frac{P}{t}}y - \sqrt{\frac{St}{P}}\right) + \right.$$
$$\left.\left. + \operatorname{erfc}\left(\tfrac{1}{2}\sqrt{\frac{P}{t}}y + \sqrt{\frac{St}{P}}\right)\right] + 2\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right)\right] -$$

$$
-\frac{Gc}{(1-Sc)}\left[\operatorname{terfc}\left(\frac{y}{2\sqrt{t}}\right) - y\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right) + \right.
$$

$$
+\frac{y^2}{2}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \operatorname{terfc}\left(\tfrac{1}{2}\sqrt{\frac{Sc}{t}}y\right) +
$$

$$
\left. + \sqrt{\frac{Sct}{\pi}}\,y\exp\left(-\frac{Scy^2}{4t}\right) - \frac{Scy^2}{2}\operatorname{erfc}\left(\tfrac{1}{2}\sqrt{\frac{Sc}{t}}y\right)\right]. \tag{10}
$$

By use of Equtions (9) and (10), the expressions for the non-dimensional skin friction τ, at the plate and for non-dimensional rate of heat transfer Nu, are given by (11) and (12)

$$
\tau = -\frac{1}{\sqrt{\pi t}} - G\left[\frac{1}{\sqrt{S(1-P)}}\exp\left(\frac{St}{(1-P)}\right) - \exp\left(\frac{St}{(1-P)}\right)\Big/ S \times\right.
$$

$$
\times\left[\sqrt{\frac{S}{1-P}}\left(2\text{-erfc}\left(\sqrt{\frac{St}{1-P}}\right)\right) + \frac{1}{\sqrt{\pi t}}\exp\left(\frac{St}{(1-P)}\right)\right] +
$$

$$
\left. + \frac{1}{S}\frac{1}{\sqrt{\pi t}}\right] + \frac{2Gc}{(1-Sc)}\left(\sqrt{\frac{t}{\pi}} - \sqrt{\frac{Sct}{\pi}}\right) + \frac{G}{S}\exp\left(-\frac{St}{P}\right) \times
$$

$$
\times\left[\sqrt{\frac{S}{1-P}} + \left(\sqrt{\frac{S}{1-P}}\right)\left(\operatorname{erfc}\left(\sqrt{\frac{St}{P(1-P)}}\right) - 2\right) - \right.
$$

$$
\left. - \sqrt{\frac{P}{t\pi}}\exp\left(-\frac{St}{P(1-P)}\right)\right] - \frac{G}{\sqrt{S}} - \frac{G}{S} \times
$$

$$
\times\left[\sqrt{S}\left(\operatorname{erfc}\left(\sqrt{\frac{St}{P}}\right) - 2\right) - \sqrt{\frac{P}{t\pi}}\exp\left(-\frac{St}{(1-P)}\right)\right]. \tag{11}
$$

$$
Nu = \sqrt{\frac{P}{\pi t}}\exp\left(-\frac{S}{P}t\right) + \sqrt{S}\left(1 - \operatorname{erfc}\left(\sqrt{\frac{St}{P}}\right)\right). \tag{12}
$$

3. Discussion

In order to investigate the effects, of the heat sources, when the plate moves with constant velocity on its own plane, numerical calculations are carried out for different values of G (Grashof number), Sc (Schmidt number), Gc (modified Grashof number), S (heat source parameter), when the rate of Prandtl number P is equal to 0.71, which corresponds, to the air. The Grashof number, G, represents here the effects of the free convection currents, and receives positive, zero or negative values. The case $G < 0$ corresponds physically to an externally-heated plate as the free convection currents are moving towards the plate. The case $G > 0$ corresponds to an externally cooled plate and the case $G = 0$ corresponds to absence of the free convection currents. The results

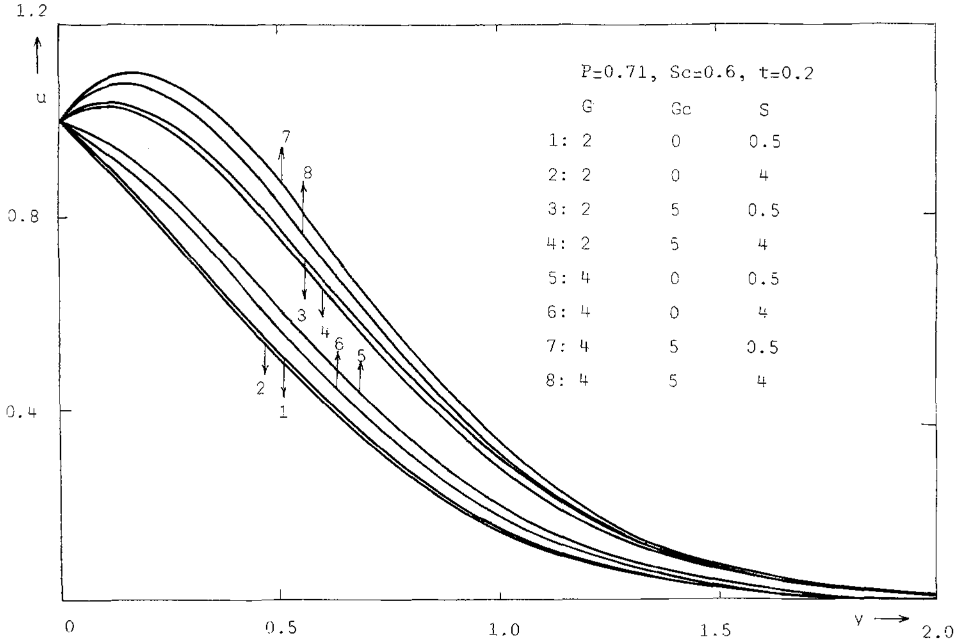

Fig. 1. Velocity profile.

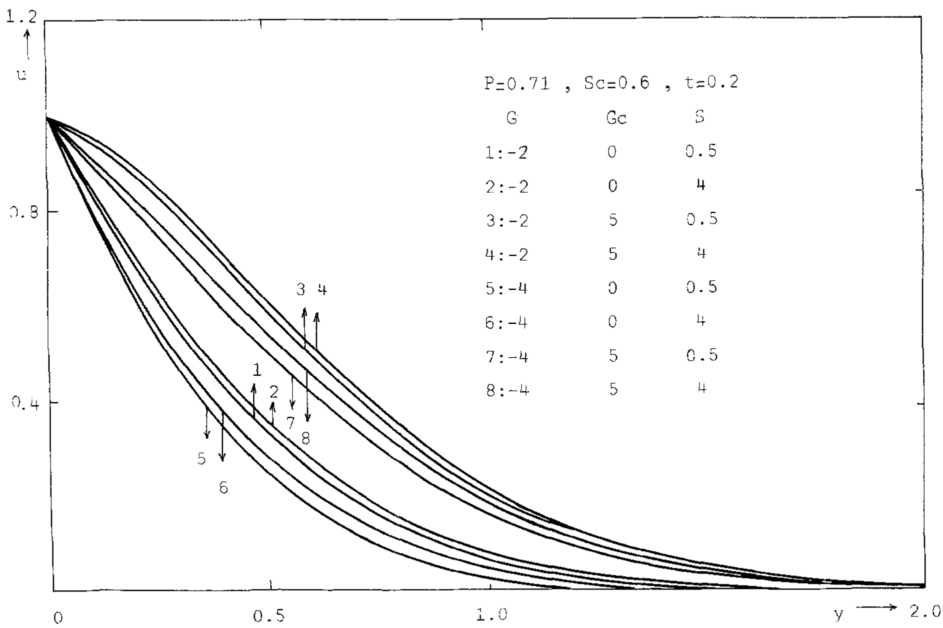

Fig. 2. Velocity profile.

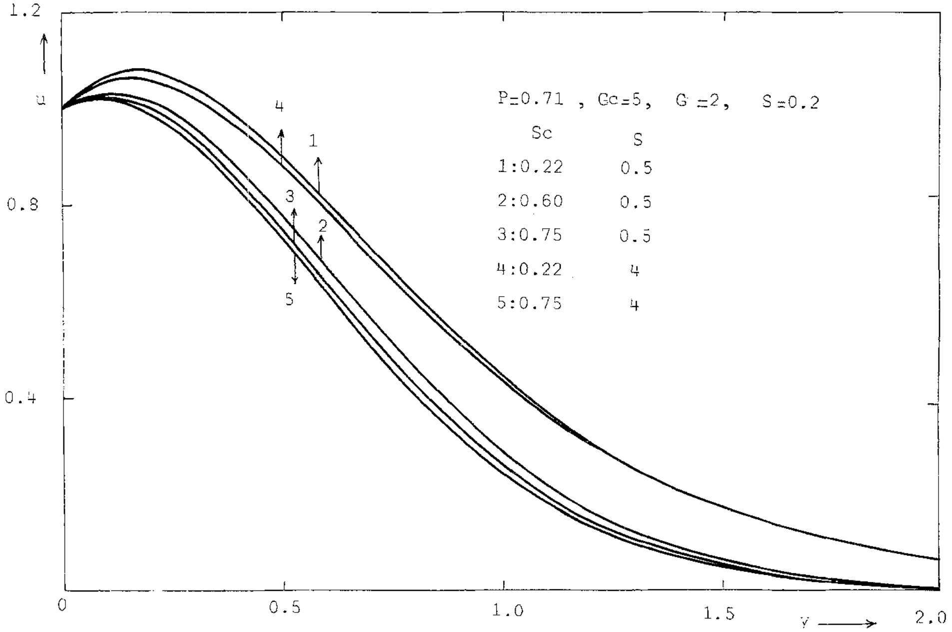

Fig. 3. Velocity profile.

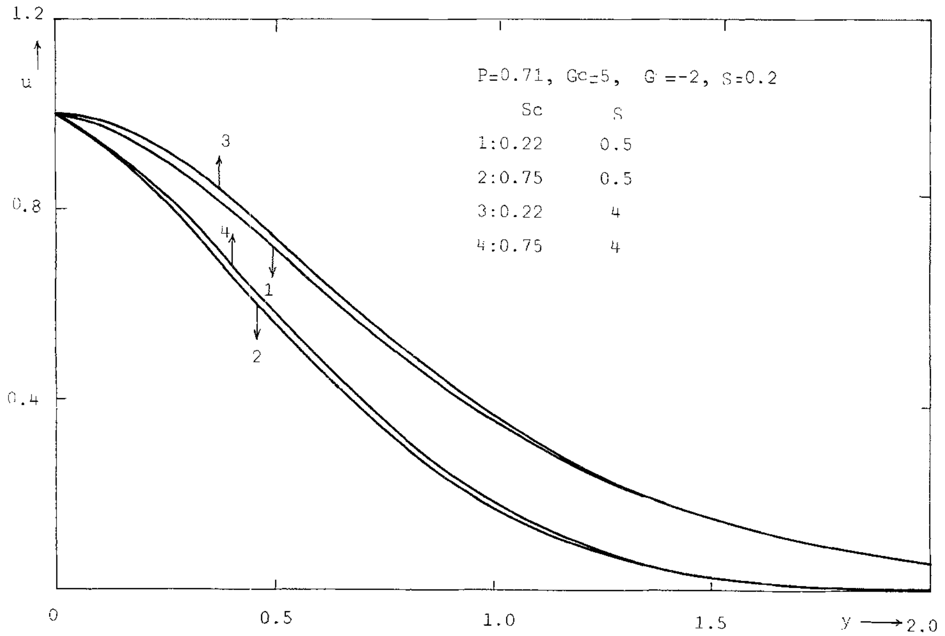

Fig. 4. Velocity profile.

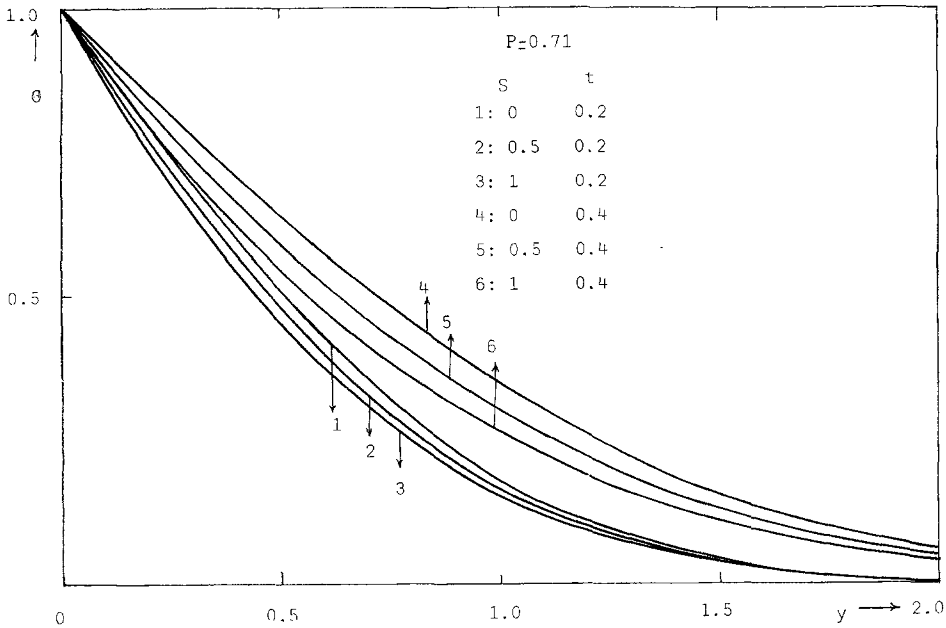

Fig. 5. Temperature profile.

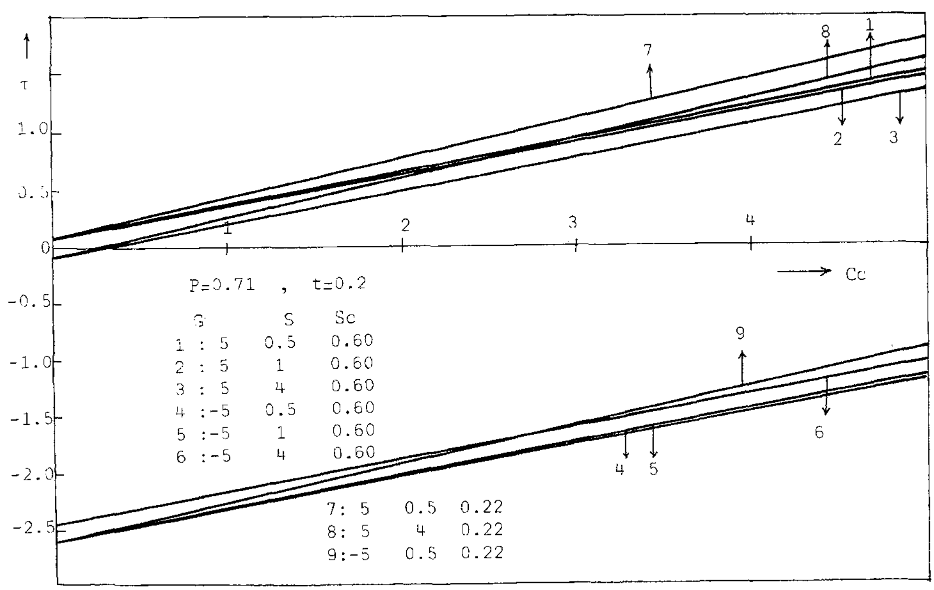

Fig. 6. Skin-friction.

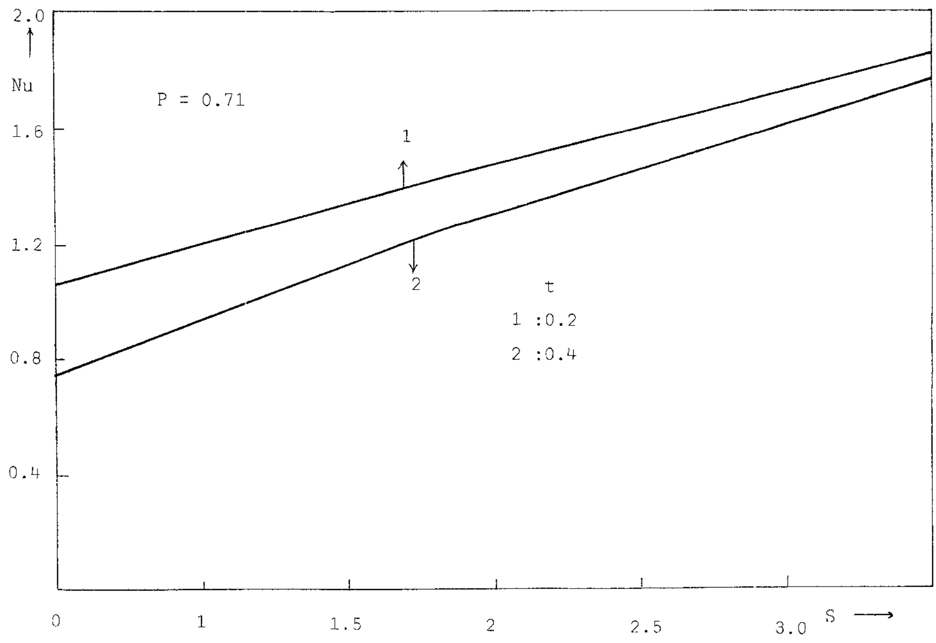

Fig. 7. Rate of heat transfer.

obtained in non-dimensional formes are displayed on Fifures 1–7. The non-dimensional velocity profiles are shown on Figures 1–4. From Figures 1 and 3 we observe that, when $G > 0$, an increase in heat source parameter leads to a fall in the velocity. The opposite phenomenon is presended (Figures 2 and 4) when $G < 0$. The variations of non-dimensional temperature and rate of heat transfer for different values of S and time t are shown in Figures 5 and 7, respectively. We observe that, when S increases the temperature decreases while the rate of heat transfer increases. Finally, from Figure 6, we observe that, when $G > 0$, an increase in S leads to a fall in the skin-friction and a rise in the skin-friction when $G < 0$.

References

Georgantopoulos, G..A., Nanousis, N. D. and Goudas, C.L.: 1979, *Astrophys. Space Sci.* **66**, 13.
Stokes, G. C.: 1851 *Camb. Phil, Trans*, **9**, 8.