CORE-ENVELOPE MODELS FOR MASSIVE FLUID SPHERES

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(Received 18 July, 1988)

Abstract. The solution of equation of state corresponding to equality $\Gamma = \Gamma_3$ gives non-terminating solutions for isothermal neutron star cores. Hence, for this equality, core-envelope models have been developed by taking another equation of state, corresponding to the condition Γ_3 = constant, in the envelope. Various static, pulsational, and rotational parameters pertaining to neutron star models are calculated. These models are gravitationally bound and stable for radial perturbations and slow rotations.

1. Introduction

A single equation of state or mass distribution is unlikely to describe a physical structure completely. A relativistic massive sphere-like neutron star can be considered as a configuration made of two regions, core and envelope. Hence, there is need to consider a physically-reasonable model with two or more density distributions. In this arrangement there is more flexibility in adjusting the various parameters of the model in a multidensity distribution.

In general relativity the core-envelope model was first developed by Bondi (1964) for cores with $3P = E$. Das and Narlikar (1975) have discussed core-envelope models with

$$P = KE\,, \qquad K = \text{constant}$$

in the core, and

$$\mathrm{d}P/\mathrm{d}E = K'\,, \qquad K' = \text{constant}$$

in the envelope.

Durgapal and Gehlot (1969, 1971) have discussed two density models with constant density in the core. Pandey *et al.* (1983) have solved equation of state $P = KE$ in the core for finite central densities and polytropic equation of state in the envelope.

The models of neutron star based upon an isothermal equation of state in core with a finite central density is rather simple in the sense that a massive sphere with an isothermal equation of state has an infinite size and has to be terminated abruptly at some place. The models with an isothermal equation of state must have an envelope of some other kind. The solution within the core and the envelope should match at the core-envelope boundary. The models with constant density core give discontinuity in the value of density at the core-envelope boundary. In the Das and Narlikar (1975) model, $\mathrm{d}P/\mathrm{d}E$ is discontinuous at the core-envelope boundary.

For an isothermal core and a suitable envelope, the continuity of density and $\mathrm{d}P/\mathrm{d}E$ can be ascertained at the core-envelope boundary. By considering $\mathrm{d}P/\mathrm{d}E$ = constant for the core, Durgapal *et al.* (1980a, b) established continuity of $\mathrm{d}P/\mathrm{d}E$ at the core-envelope boundary.

Astrophysics and Space Science **151** (1989) 255–264.

In our earlier paper (Fuloria *et al.*, 1988) we have given the parameters of isothermal neutron star cores for the condition $\Gamma = \Gamma_3$. For this equality the equation of state gives a non-terminating solution (i.e., as $P \to 0$, $r \to \infty$). Hence, in this paper we have presented core-envelope models for this equation of state in the core. In the envelope we have taken the equation of state corresponding to the condition Γ_3 = constant. With these density distributions we have established continuity of dP/dE along with that of ν, λ, P, and E at the core-envelope boundary. For these models we have calculated static (mass, size, gravitational binding, central and surface red-shifts), pulsational and rotational parameters. All these models are gravitationally bound and stable for radial perturbation and rotation.

2. Equations of Hydrostatic Equilibrium and Adiabatic Exponents for Neutron Star Matter

In order to obtain properties of relativistic structures we choose some equation of state of general form

$$P = P(E) \quad \text{or} \quad P = P(\rho) \quad \text{or} \quad E = E(\rho)\,, \tag{1}$$

and solve it along with coupled differential equations for hydrostatic equilibrium

$$dP/dr = -(P + E)(4\pi P r^3 + m)/r(r - 2m)\,, \tag{2}$$

$$dm/dr = 4\pi E r^2\,, \tag{3}$$

$$dP/dr = -(\tfrac{1}{2})(P + E)\, d\nu/dr\,, \tag{4}$$

where E is the energy density; ρ, rest mass density; and ν appears in spherically-symmetric and static metric in Schwarzschild coordinates

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2\,, \tag{5}$$

where ν and λ are functions of r alone.

For neutron star matter the adiabatic exponents are defined as

$$\begin{aligned} &\Gamma_1 = (d\ln P/d\ln \rho)_s\,, \qquad \Gamma_2/(\Gamma_2 - 1) = (d\ln P/d\ln T)_s\,, \\ &\Gamma_3 - 1 = (d\ln T/d\ln\rho)_s\,, \quad \text{and} \quad \Gamma = (d\ln P/d\ln E)_s\,; \end{aligned} \tag{6}$$

where the subscript 's' stands for constancy of entropy.

The relationship among P, E, and ρ is given by the first law of thermodynamics as

$$(dE/d\rho)_s = (P + E)/\rho\,. \tag{7}$$

3. Equations of State in the Core and Envelope

From the equality $\Gamma = \Gamma_3$ we obtained

$$(dP/dE) = (P/E)(1 - (P/E))^{-1} \tag{8}$$

or

$$P = K \exp(-E/P) \tag{9}$$

(where K = constant) as the equation of state. We have solved this equation of state in the neutron star core for certain central conditions (Fuloria *et al.*, 1988).

On the other hand, if we consider Γ_3 = constant, we get an equation of state of he form

$$\mathrm{d}P/\mathrm{d}E = K'$$

or

$$P = K'(E - E_a)\,, \tag{10}$$

where K' = constant and E_a is the value of E when pressure vanishes, i.e., $E = E_a$ when $P_a = 0$, at the surface of the entire structure. Equation (10) has been used as equation of state in the envelope.

4. Solution for the Envelope

The values of two unknown constants K' and E_a in Equation (10) can be obtained from the continuity of (P/E) and $(\mathrm{d}P/\mathrm{d}E)$ at the core-envelope boundary.

If at this boundary we substitute

$$\begin{gathered} P = P_b\,, \qquad E = E_b\,, \qquad f(=P/E) = f_b(=P_b/E_b)\,, \\ \nu = \nu_b \quad \text{and} \quad r = r_b\,, \end{gathered} \tag{11}$$

we get,

$$K' = P_b/(E_b - P_b) \quad \text{and} \quad E_a = P_b\,. \tag{12}$$

On taking the initial values of different parameters as defined in Equation (11) and substituting the values of K' and E_a as obtained above, the coupled Equations (2)–(4) and equation of state given by Equation (10) in the envelope are computed till the pressure vanishes.

At the outer boundary of the envelope $r = a$ the solutions are continuous with Schwarzschild exterior solution: i.e.,

$$e^{\nu(a)} = e^{-\lambda(a)} = 1 - (2M/a) \quad \text{at} \quad r = a\,; \tag{13}$$

where M is the total mass of the structure and a the radius of the complete configuration.

The core-envelope models are constructed by taking core of different sizes for the following values of $f(=P/E)$ at the centre of the structure

$$f_0 = 1, 0.50, \tfrac{1}{3}, 0.20, 0.10\,. \tag{14}$$

At the extreme relativistic case (when the signal moves with the speed of light, i.e., $\mathrm{d}P/\mathrm{d}E = 1$) $P/E = 0.50$.

5. Static Parameters

The neutron star parameters have been calculated by taking the surface density $E_a = 2 \times 10^{14}\ \mathrm{g\ cm^{-3}}$ (Durgapal *et al.*, 1980a, b). This includes the calculations of surface and central red-shifts (z_a and z_0, respectively), mass ($M/M_\odot$), size (a km) and binding coefficient

$$b = \frac{M_r - M}{M_r}\,,$$

where the rest mass M_r is given by

$$M_r = \int_0^a 4\pi\rho r^2 e^{\lambda/2}\,\mathrm{d}r\,.$$

6. Pulsational Stability

The spherically-symmetric and static configuration can have a relativistic gravitational collapse before they attain a large vlaue of central red-shift. Hence, it is necessary to study the stability of these configurations towards radial perturbations. We have used the variational method (Chandrasekhar, 1964) to ascertain the pulsational stability and ω^2/E_0 has been computed.

7. Rotational Properties

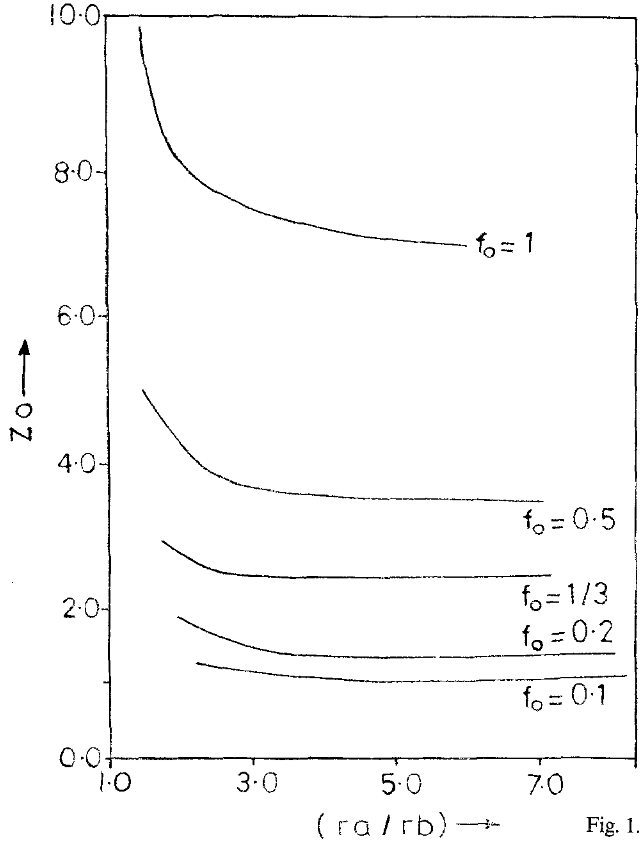
Using the theory of slow rotation (Chandrasekhar and Miller, 1974; Bonner, 1973; Irvine, 1978) we have calculated the drag of inertial frames associated with the configuration, and also their moment of inertia (I g cm^2) by assigning the surface density $E_a = 2 \times 10^{14}\ \mathrm{g\ cm^{-3}}$.

For different values of f_0, the variations of z_a, z_0, $M/M_\odot$, a, and b with the ratio (r_a/r_b) of the core size (r_b) and core-envelope size (r_a) have been given in Figures 1–5. Figure 6 shows variation of ω^2/E_0 and z_0. In Figures 7–9 we have shown variation of relative central drag (R.C.D.), relative surface drag (R.S.D.), and I with $\log E_0$. Variation of $\log E_0$ and (r_a/r_b) is given in Figure 10.

8. Discussion

The solution of equation of state corresponding to equality $\Gamma = \Gamma_3$ gives non-terminating solutions in the isothermal neutron star core. Hence, two density models for neutron star have been developed. In the envelope we have taken another equation of state corresponding to condition Γ_3 = constant.

The nature of various physical parameters is explained as follows while their variations are shown in Figures 1–10.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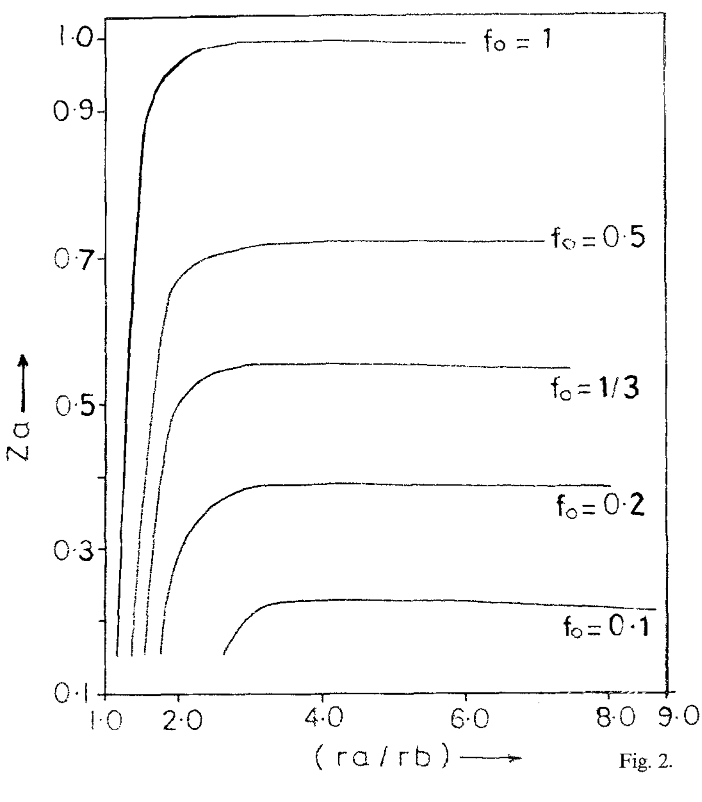
8·0
6·0
4·0
2·0
0·0
Zo
1·0
3·0
5·0
7·0
(ra / rb)
fo = 1
fo = 0·5
fo = 1/3
fo = 0·2
fo = 0·1

Fig. 1.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2·0
4·0
6·0
8·0
9·0
(ra / rb)
fo = 1
fo = 0·5
fo = 1/3
fo = 0·2
fo = 0·1

Fig. 2.

(i) The surface red-shift Z_a increases sharply but becomes constant for some value of r_a/r_b. For $r_a/r_b > 3$ the value of Z_a is almost constant (Figure 1).

(ii) The central red-shift Z_0 decreases sharply for higher value of f_0. For lower values of f_0 the variation is almost smooth and becomes almost constant for certain value of r_a/r_b (Figure 2).

(iii) The mass of neutron star model increases rapidly when $r_a/r_b < 2$; but then the increase in $M/M_\odot$ is smooth and more or less becomes constant for high r_a/r_b values. The maximum mass of the neutron star thus obtained is 6.3 $M/M_\odot$ (Figure 3).

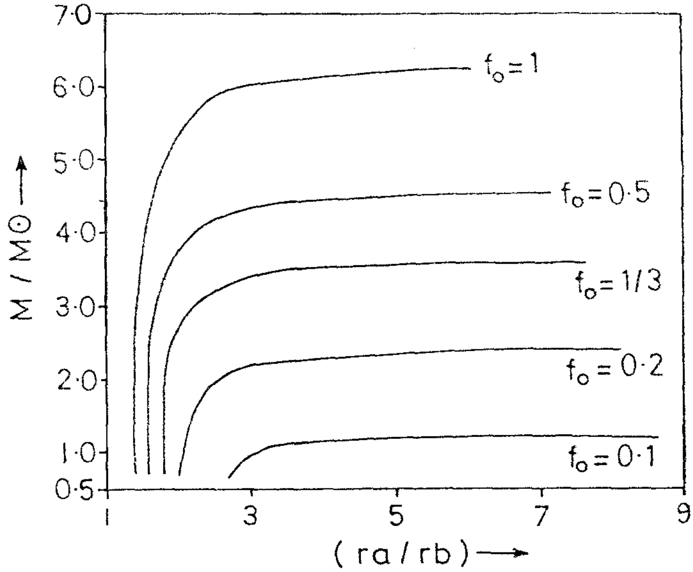


Fig. 3.

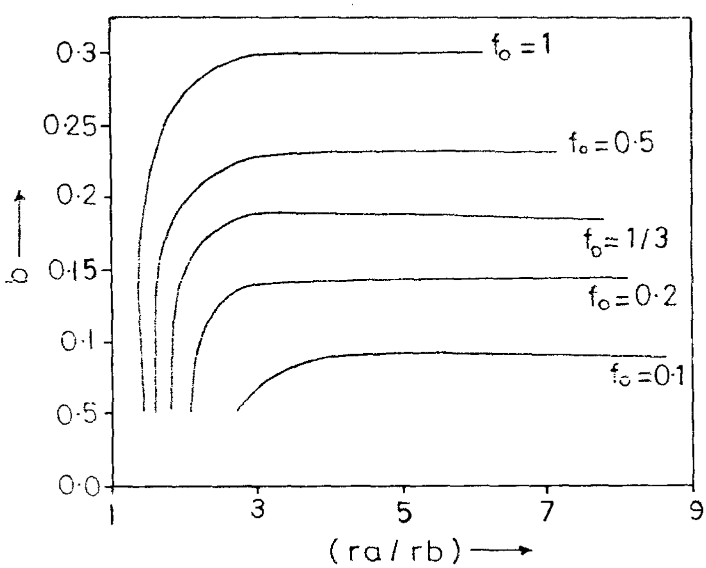


Fig. 4.

(iv) The binding coefficient b increases sharply but becomes constant for some value of r_a/r_b. For $r_a/r_b > 3$ the value of b is almost constant. For all the cases the neutron star structures are bound (Figure 4).

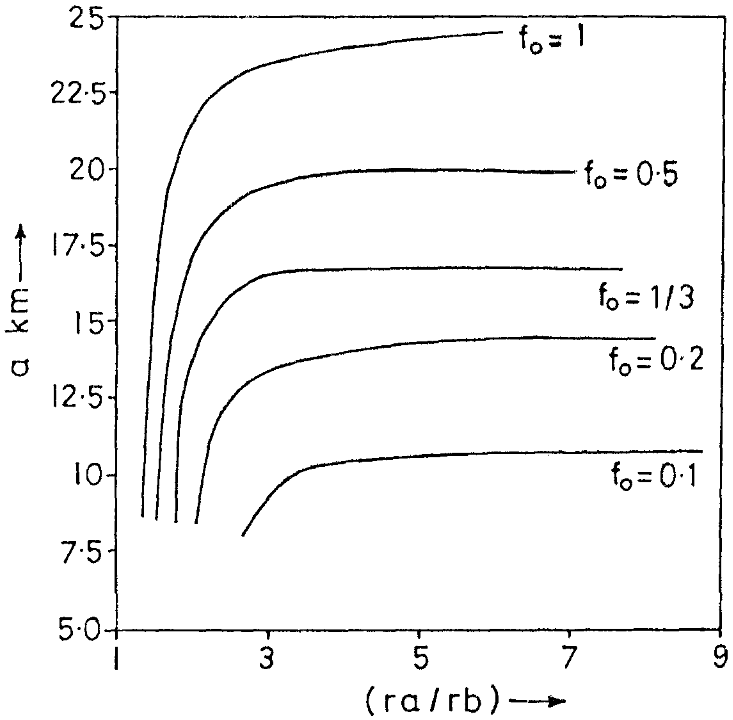


Fig. 5.

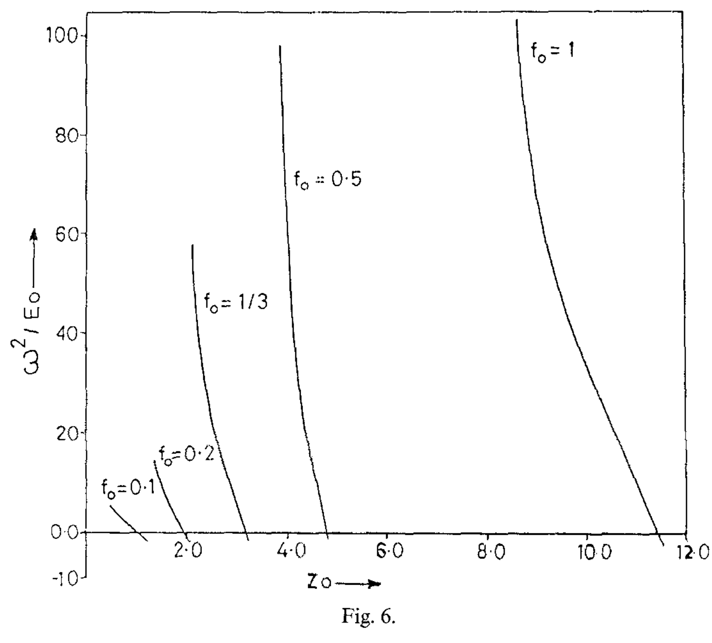


Fig. 6.

(v) The size of neutron stars (in km) increases very rapidly for higher values of f_0. For the low value of f_0 the variation in size is smooth. The maximum size of neutron star for assigned condition at the centre $f_0 = 1$ is 24.5 km (Figure 5).

(vi) With the increase of Z_0, ω^2/E_0 decreases and becomes negative at certain value of Z_0. At this stage the structure becomes pulsationally unstable (Figure 6).

(vii) The relative central drag increases with the increase of central density (Figure 7) but the relative surface drag decreases with increasing central density (Figure 8).

(viii) The moment of inertia decreases with increase in central density (Figure 9).

(ix) The central density decreases with the increase in (r_a/r_b) value (Figure 10).

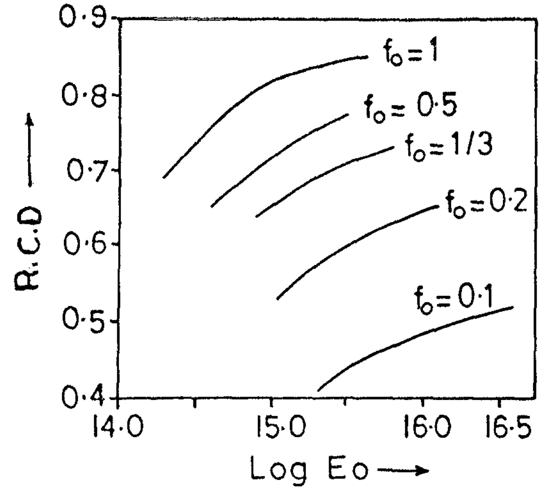


Fig. 7.

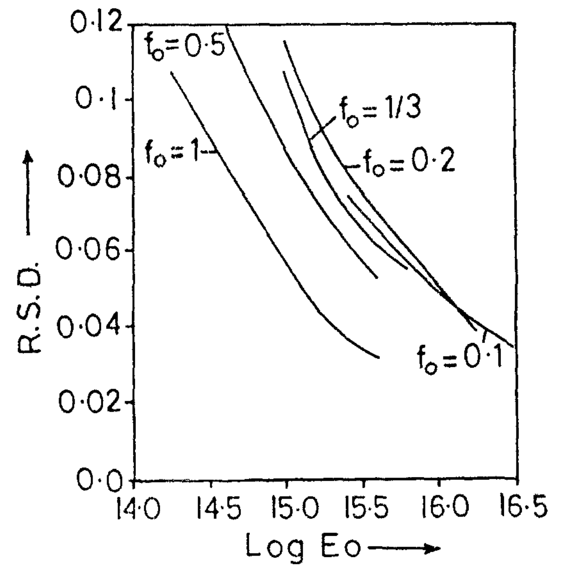
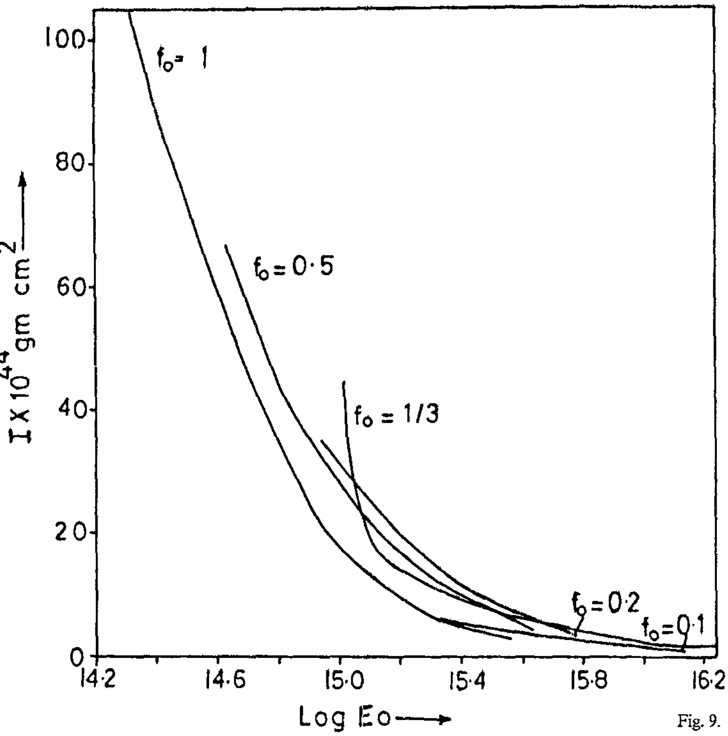


Fig. 8.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80
60
40
20
0
$I \times 10^{44}$ gm cm^2
$f_o = 1$
$f_o = 0{\cdot}5$
$f_o = 1/3$
$f_o = 0{\cdot}2$
$f_o = 0{\cdot}1$
14·2
14·6
15·0
15·4
15·8
16·2
Log Eo
Fig. 9.

Fig. 9.

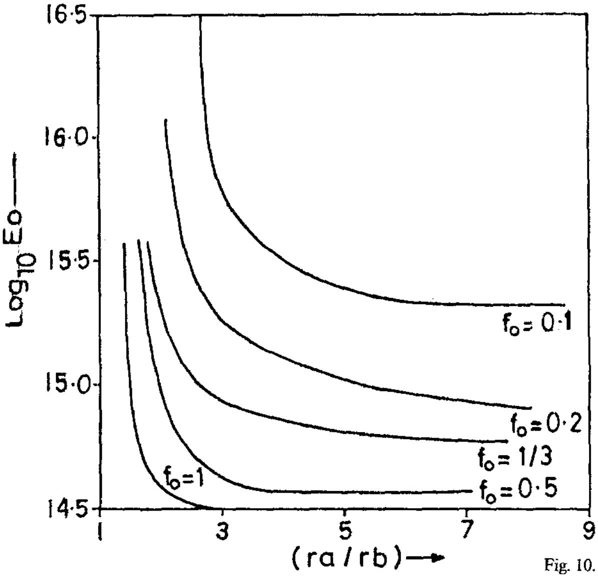
16·5
16·0
15·5
15·0
14·5
Log_{10} Eo
$f_o = 0{\cdot}1$
$f_o = 0{\cdot}2$
$f_o = 1/3$
$f_o = 0{\cdot}5$
$f_o = 1$
1
3
5
7
9
(ra / rb)
Fig. 10.

Fig. 10.

Acknowledgements

R. S. Fuloria and S. C. Pandey acknowledge their thanks to C.S.I.R. and I.S.R.O. for financial assistance.

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