PRODUCTION AND PROPAGATION OF COSMIC RAY H^2 AND He^3 NUCLEI

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Abstract. The results of detailed calculations on the production of H^2 and He^3 nuclei by cosmic ray protons and helium nuclei in interstellar medium are presented. The flux and energy spectra of these nuclei as well as those of cosmic ray H^1 and He^4 nuclei in the vicinity of the Earth are calculated. For this purpose the source spectra are assumed to be in the form of a power law in total energy per nucleon with an additional velocity dependent term. This spectrum denoted as Fermi Spectrum, is about mid-way between the power law spectrum in rigidity and in total energy per nucleon. The fluxes are calculated taking into account: (1) energy dependent cross-sections of thirteen nuclear reactions of cosmic ray protons and helium nuclei with interstellar H^1 and He^4 leading to the production of H^2 and He^3 nuclei, (2) angular distributions and kinematics of these reactions, (3) ionization loss of the primary and secondary nuclei in interstellar medium, (4) elastic collisions of cosmic ray protons and helium nuclei, (5) distributions of cosmic ray path-lengths in interstellar space as in gaussian and exponential forms, and (6) interplanetary modulation of cosmic rays from the numerical solution of the complete Fokker-Planck equation describing the diffusion, convection and adiabatic deceleration of cosmic ray nuclei in the solar system. On comparing the calculated values of H^2/He^4 and $He^3/(He^3+He^4)$ as a function of energy with the observed data of several investigators, it is found that agreement between the calculated values and most of the observed data is obtained on the basis of: (a) source spectrum in the form of 'Fermi' Spectrum, (b) distribution of path-lengths as in the gaussian form with a mean value of 4 g cm^{-2} of hydrogen or as in exponential form with leakage path length of 4 g cm^{-2}.

1. Introduction

It is now well recognized that the energy spectra of low energy cosmic ray nuclei and the relative abundances of some of the elements and isotopes measured near the Earth are markedly different from those believed to be present in the source region. For example the abundances of nuclei of H^2, He^3, Li, Be, B, F, in cosmic rays are much higher than their universal abundances (Cameron, 1968) and these nuclei are believed to be produced in the inelastic collisions of the primary cosmic ray nuclei with the matter present in their path between the source and the Earth. Further the passage of cosmic ray nuclei through interplanetary space reduces their intensities and energies due to interactions with solar wind and consequent diffusion, convection, and adiabatic deceleration. Therefore in order to obtain an understanding of the relative abundances of these nuclei at low energies it is necessary to take into account the effects of ionization loss and spallation process in the interstellar medium as well as the effects of energy loss and the intensity reduction in the interplanetary space. It is shown in this paper that detailed calculations of the intensities and energy spectrum of H^2 and He^3 nuclei as well as those of H^1 and He^4 nuclei and the comparison with observational data yield information on the characteristics of the source spectrum of cosmic rays at low energies and the propagation in the interstellar medium.

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Earlier calculations on the production of H^2 and He^3 nuclei were made by several authors (Biswas *et al.*, 1967a; Meyer *et al.*, 1968; Biswas *et al.*, 1970; Ramaty and Lingenfelter, 1969; Biswas and Ramadurai, 1971). However, since the calculations were published the experimental data on the abundances of H^2 and He^3 nuclei changed rather significantly, necessitating a re-examination of the whole problem. For example, it is concluded by Hsieh and Simpson (1969) that the results of Ramaty and Lingenfelter (1969) did not agree with their new experimental observations. Comstock *et al.* (1972) and Meyer (1972) calculated the H^2/He^4 and $He^3/(He^3$ and $He^4)$ ratios including, in an approximate way, the effects of adiabatic deceleration in interplanetary space. They concluded that the determination of H^2/He^4 ratios near the Earth, imposes severe restrictions on the spectra of cosmic ray nuclei at the sources and that the source spectrum is best described by a spectrum which is midway between a power spectrum in rigidity and in total energy per nucleon. Such a spectral form was obtained on theoretical grounds by Ramadurai and Biswas (1972) and is denoted as 'Fermi Spectrum'. Hence we have used such a form of source spectrum in the present calculations. In the earlier works of Meyer *et al.* (1968) and Biswas *et al.* (1967a) the important effects of elastic collisions of cosmic ray protons and helium nuclei with interstellar H^1 and He^4 nuclei at low energies were neglected. Hence it is necessary to make a detailed investigation incorporating all these modifications including the effects of adiabatic deceleration in interplanetary space in a rigorous manner. This is attempted in the present work.

In the present calculations the source spectrum is assumed to be 'Fermi' type which is equivalent to a power law in total energy per nucleon with an additional velocity term. Calculations are made taking into account (a) elastic collisions of cosmic ray protons and helium nuclei with interstellar matter (Ramadurai, 1970), (b) inelastic collisions of protons and helium nuclei leading to the production of H^2 and He^3 nuclei, including the effects of energy dependent cross sections, kinematics and angular distributions of the secondaries in the nuclear reactions, (c) ionization loss of the primary and the secondary particles, (d) the distribution of path-lengths of cosmic ray particles in the form of: (1) a gaussian with a standard deviation, σ, as 0.633 $\bar{x}$ where $\bar{x}$ is the mean quantity of matter traversed in propagation, and (2) an exponential type with a leakage mean free path of 4 g cm^{-2}; and (e) interplanetary modulation of cosmic ray intensities from numerical solution of the complete Fokker-Planck equation describing the diffusion, convection and adiabatic deceleration. The intensities and energy spectra of H^1, H^2, He^3 and He^4 nuclei in the vicinity of the Earth are calculated and H^2/He^4 and He^3/He^4 ratios are obtained. These results are compared with experimental observations and the source and propagation characteristics are deduced.

2. Procedures of Calculations

2.1. Source Spectrum

In this work source spectrum of protons and He-nuclei is assumed to be of the form

$$N(W)\,\mathrm{d}W = K(Mc^2)^{1/a\beta T}\frac{1}{a\beta T}\frac{\mathrm{d}W}{W^{1+1/a\beta T}}, \tag{1}$$

where W = total energy per nucleon, βc = velocity of the particle and a and T are acceleration parameter and mean accelerating time respectively, and $1/aT$ is chosen as 1.65 corresponding to the observed energy spectrum of relativistic cosmic ray nuclei. This is denoted as Fermi spectrum, as deduced by Ramadurai and Biswas (1971; 1972). This spectral form is shown in Figure 1 and is intermediate between the power law in rigidity and in total energy per nucleon. Calculations were made earlier by us with $W^{-2.6}$ spectrum and we find the present form of source spectrum as more appropriate than the other two forms.

2.2. Elastic collisions

Even though the elastic collisions do not change the internal structure of the nuclei they lead to a redistribution of the energy of the colliding and the collided particles. The energy change ΔE suffered by the colliding particle can easily be shown (e.g.

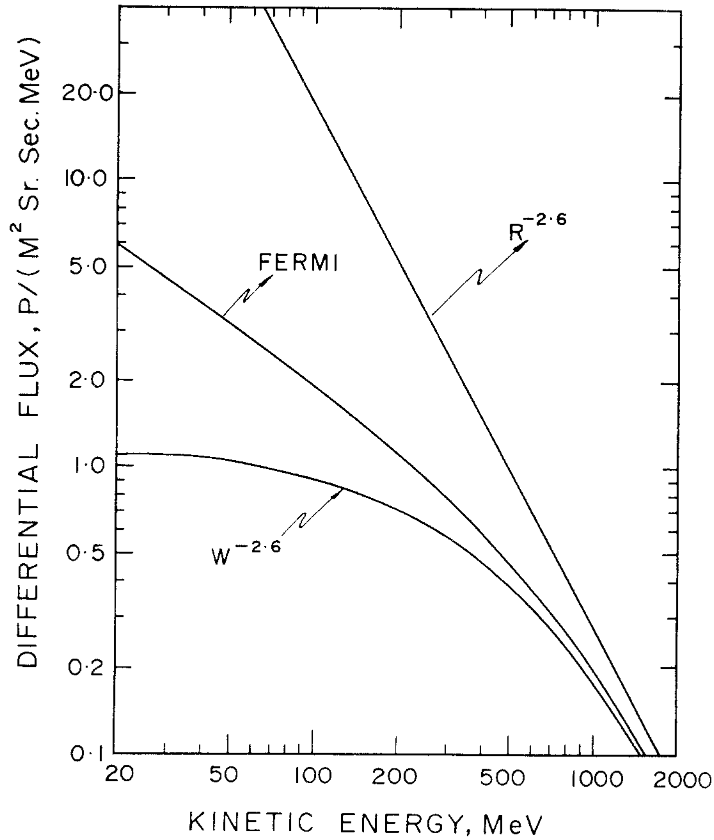

Fig. 1. The source spectra as 'Fermi type' spectrum with $1/a\beta T = 1.65$ for $\beta \approx 1$, and as power law in total energy per nucleon $W^{-2.6}$, and in rigidity $R^{-2.6}$.

Kompaneyets, 1961) to be

$$\Delta E = E_p - E_1 = \frac{m_t (E_p^2 - m_p^2 c^4)}{m_p^2 c^2 + m_t^2 c^2 + 2 m_t E_p} (1 - \cos\theta^*), \tag{2}$$

where E_1 = energy of the particle after collision, E_p = energy of the particle before collision, m_t = mass of the target nuclei which is assumed to be at rest, θ^* = C.M. angle into which the particle is scattered.

From the above relation it follows that the energy change will be maximum in collisions of particles with nearly equal mass. The interstellar matter consists mostly of hydrogen with a small proportion of helium ($\sim 12\%$) (Greenberg, 1968). Hence the only elastic collisions of importance will be the collisions of primary cosmic ray protons and helium nuclei with interstellar hydrogen. Since the helium abundance is small the effects of collisions with interstellar helium is not very significant and hence this has been neglected in this calculation. In case of protons an interesting aspect is the transfer of energy to the interstellar protons which are nearly at rest. The gain of energy by the target proton converts it into a low energy cosmic ray particle. Thus in case of protons the elastic collisions play an important part and is considered in some details.

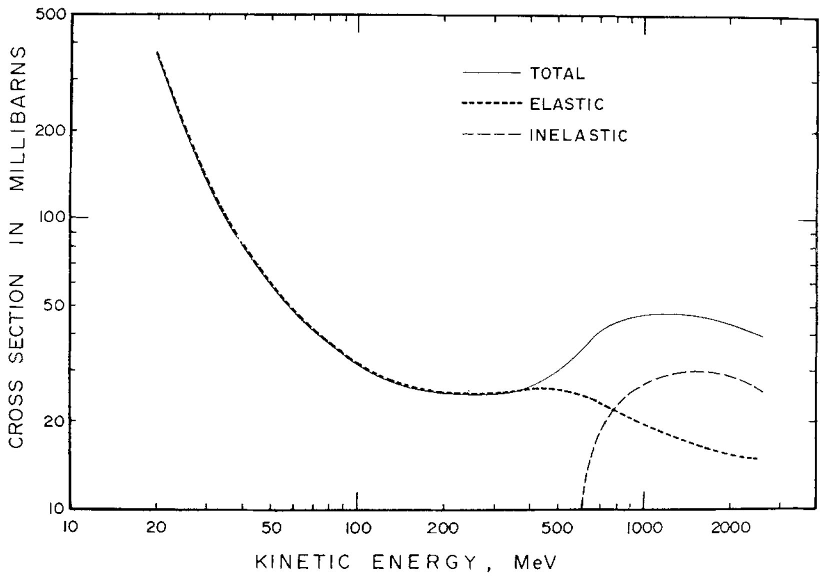

Fig. 2. The total elastic and inelastic cross-sections for the $p-p$ collisions. The data are taken from Preston (1962) and Lindenbaum (1957).

The cross-sections for the elastic $p-p$ collisions change very rapidly with energy (Lindenbaum, 1957) as seen in Figure 2. Below 500 MeV n^{-1} the only collisions taking place in the case of $p-p$ collisions are elastic collisions. At higher energies inelastic processes tend to dominate. Further, the angular distribution of the scattered particle also vary greatly at different energies. This can be seen from Figure 3 (a and b), where

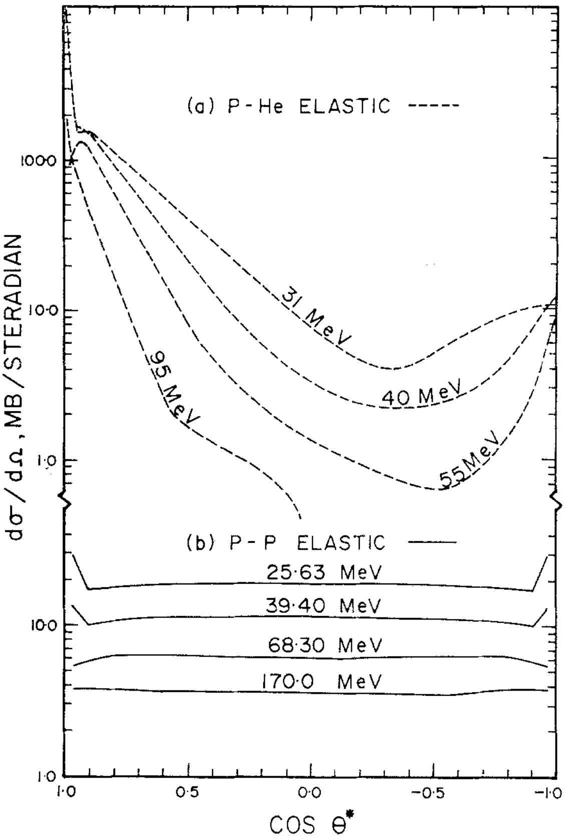

Fig. 3. The angular distribution of protons in $p-He^4$ and $p-p$ elastic collisions. The experimental data for $p-He^4$ collisions are taken from the works of Bunch *et al.* (1964), Brussel and Williams (1957), Hayakawa *et al.* (1964), and Selove and Teem (1958). The experimental data for $p-p$ collisions are taken from Preston (1962).

the measured differential cross-sections are plotted at various C.M. angles. The energy change will be minimum if all the particles are scattered in the forward direction as seen from Equation (2). In the case of $p-p$ elastic collisions the differential cross-section is almost isotropic, which indicates large changes in energy of the scattered particle.

The shape of the angular distribution of the $p-p$ scattering curve arises from the combination of the coulomb and nuclear scattering. At very small angles coulomb scattering dominates. Since both the scattering and scattered particles are of the same mass the angular distribution is symmetric around 90° in the centre of mass system (Figure 3b). The differential cross-section is given by

$$\frac{d\sigma}{d\Omega} = A(E)\,\mathrm{cosec}^4\,\frac{\theta^*}{4} + K(E), \tag{3}$$

where the first term on the right-hand side represents the coulomb scattering and the second the nuclear scattering. The energy dependence of A as well as K can be estimated from theory as well as experiment (Preston, 1962). We have fitted the experimental observations by an expression of the form

$$\frac{d\sigma}{d\Omega} = 0.164E^{-2} \operatorname{cosec}^4 \frac{\theta^*}{4} + \begin{cases} 437.5E^{-1} & \text{for} \quad E \leqslant 110\,\text{MeV}, \\ 3.7 & \text{for} \quad E > 110\,\text{MeV}. \end{cases} \quad (4)$$

$$Mb/Sr$$

In case of the angular distribution of He^4-p elastic scattering we assume that for the same energy per nucleon this is same as that of $p-$He elastic scattering as shown in Figure 3a. In this case we can see clearly the change in angular distribution with energy. Only at low energies the angular distribution is markedly different from the forward peak. Hence the energy changes are less than those in the case of $p-p$ collisions. In this case no simple expression is available to fit the experimentally observed angular distributions. Therefore for every energy region mentioned in the Figure 3a we have fitted different power laws for the angular distribution over different ranges of $\cos\theta^*$, thus spanning the entire range of $\cos\theta^*$, from $+1.0$ to -1.0. In order to determine $d\sigma/d\Omega$ for any energy region and angle, interpolation was done using the appropriate curves. The appropriate values of $d\sigma/d\Omega$ for $p-p$ and He$-p$ elastic scattering were used in the diffusion equations as discussed later.

2.3. Inelastic collisions

The inelastic collisions of primary cosmic ray protons and alpha particles with interstellar hydrogen and helium produce H^2 and He^3 nuclei in a number of reactions as

TABLE I

Inelastic collisions of cosmic ray protons and α-particle with interstellar hydrogen and helium nuclei

(a) Reactions Producing Deuterons
(1) $p+p \rightarrow H^2+\pi^+$
(2) $p+He^4 \rightarrow H^2+He^3+(\pi)$
(3) $p+He^4 \rightarrow H^2+2p+n+(\pi)$
(4) $p+He^4 \rightarrow H^2+H^2+p+(\pi)$
(5) $\alpha+H^1 \rightarrow H^2+He^3+(\pi)$
(6) $\alpha+H^1 \rightarrow H^2+2p+n+(\pi)$
(7) $\alpha+H^1 \rightarrow H^2+H^2+p+(\pi)$

(b) Reactions Producing He^3 and H^3
(1) $p+He^4 \rightarrow He^3+H^2+(\pi)$
(2) $p+He^4 \rightarrow He^3+p+n+(\pi)$
(3) $p+He^4 \rightarrow H^3+2p+(\pi)$
(4) $\alpha+H^1 \rightarrow He^3+H^2+(\pi)$
(5) $\alpha+H^1 \rightarrow He^3+p+n+(\pi)$
(6) $\alpha+H^1 \rightarrow H^3+2p+(\pi)$

given in Table I. H^3 isotopes produced in some of the reactions are included in He^3 production because all H^3 isotopes having a half-life of ~ 12 yr decay to He^3 in interstellar space.

The cross-section for the production of deuterons from $p-p$ collisions have been well studied from threshold to very high energies (see for a summary, Ramaty and Lingenfelter, 1969, hereafter also called R-L, 1969). These are shown in Figure 4 along

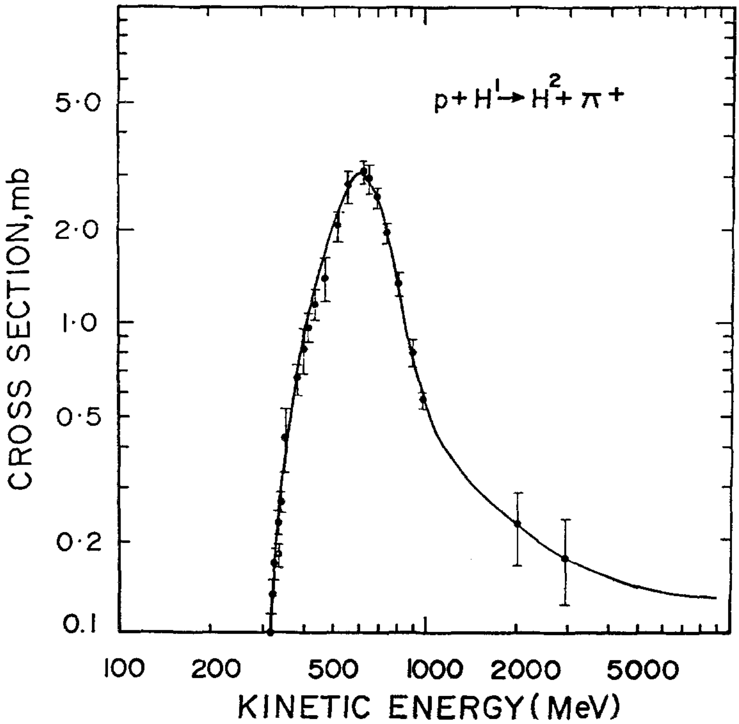

Fig. 4. The cross-sections for the production of deuterons by the pick-up reaction $p + H^1 \rightarrow H^2 + \pi^+$. For references see summary by R-L (1969).

with a smooth curve used in our present calculation. The cross-section for production of deuterons and He^3 (and H^3) from $p-He^4$ interactions, which are the same as He^4-p interaction at the same energy per nucleon, were measured by various groups. These are shown in Figures 5 and 6. The cross-sections are fairly well-known at lower energies although at high energies these have fairly large uncertainties. However, the trend of the variation of cross-sections can be approximately obtained from the data. At higher energies we have made use of the neutron induced reaction cross-sections. The use of these neutron cross-sections can be justified by the fact that at ~ 90 MeV the cross-section for the reaction $p + He^4 \rightarrow He^3 + d$ is 15 ± 3 *mb* (Selove and Teem, 1958) which is very close to the value of 12 ± 2.5 *mb* for the neutron induced reaction $n + H^4 \rightarrow d + H^3$ (Tannenwald, 1953).

At higher energies where pion production becomes important it is necessary to

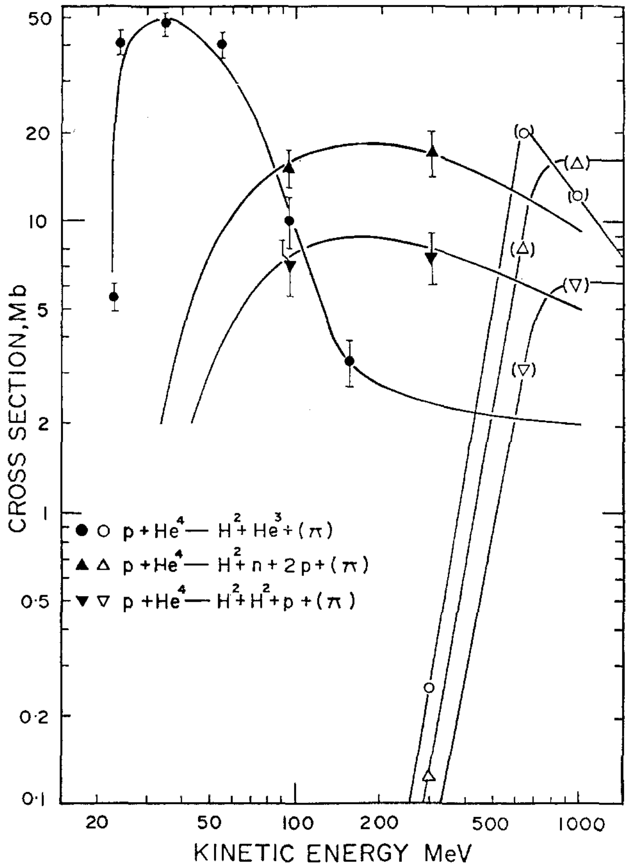

Fig. 5. The cross-sections for the production of deuterons from various reactions. For references see summary by R-L (1969).

separate out the contribution due to any particular mode. Moreover the experiments of Kozadaev *et al.* (1960) as well as that of Riddiford and Williams (1960) did not distinguish between reactions having two, three or more charged particles in the final state. In the absence of any other data in the high energy region we have to analyse these data to obtain a rough estimate of the individual cross-section. Such an analysis was done by Ramaty and Lingenfelter (1969) and we have used some of their estimates in the Figures 5 and 6.

In the present calculations we have neglected the contributions from the interactions of cosmic ray helium nuclei with the interstellar helium nuclei because no experimental $\alpha-\alpha$ cross-sections are available at present. As the amount of helium nuclei in interstellar space is less than 12%, the uncertainties in the calculated production rates of H^2 and He^3 nuclei are expected to be rather small.

Since C, N, O nuclei are much less abundant in interstellar space their contribution to the production of deuterons and helium 3 is very small at low energy (<1 GeV N^{-1}) as shown by (R-L, 1969), hence we have ignored these interactions in our calculations.

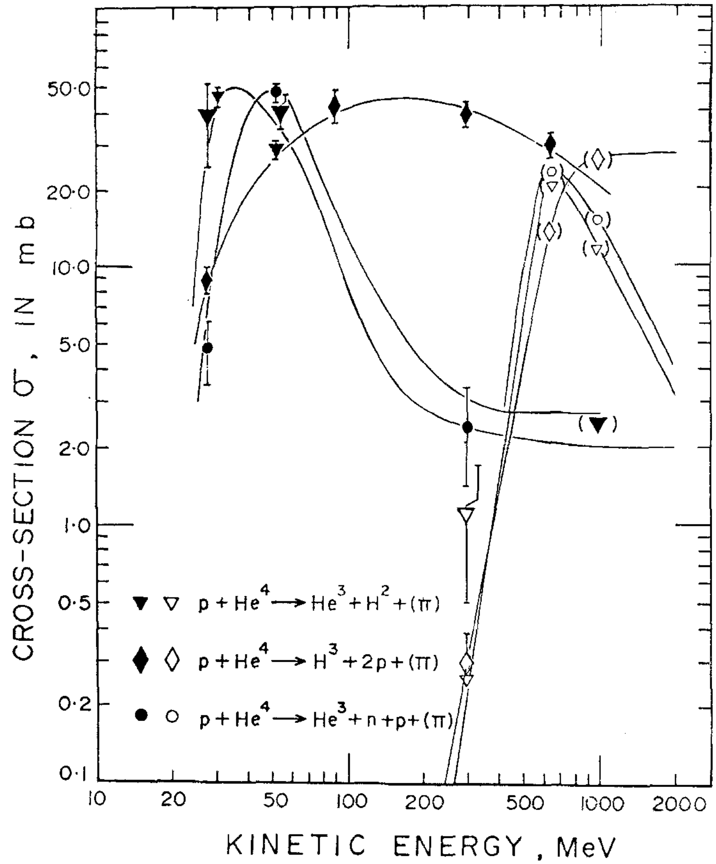

Fig. 6. The cross-sections for the production of He^3 nuclei from various reactions. For references see Biswas *et al.* (1967a) and R-L (1969).

2.3.1. *Kinematics and Angular Distribution of Secondary Particles*

In order to calculate the flux of secondary H^2 and He^3 nuclei in a given energy interval produced by cosmic ray protons and helium nuclei in the reactions given in Table I, it is necessary to derive, firstly, the information on the kinematics of the nuclear reactions, because there exists a range of energy of the primary particle which can produce the secondary particle in a given energy interval at a particular angle of emission. These are calculated using relativistic kinematics and are given in Appendix A. Since the angle of emission of a secondary particle is dependent on the angular distribution of the secondaries in a given reaction, it is necessary to calculate the probability distribution function, $F(\gamma, \gamma_s)$, from the measured angular and energy distributions of the secondaries. The function $F(\gamma, \gamma_s)$ denotes the probability that a particle of Lorentz factor γ produces a secondary particle of Lorentz factor γ_s in a given reaction. The details of the procedure and calculations are given in Appendix B. In these calculations we have followed the procedure given by Ramaty and Lingenfelter (1969).

However, a few modifications of the angular distribution functions given by R-L were found necessary in order to match observational data and these have been incorporated in our calculations as additional refinements, as given in Appendix B. On comparing the total production rates of H^2 and He^3 nuclei from our calculations with those of R-L calculations using the same input spectrum, we find satisfactory agreement between the two sets of calculations (see Appendix B). The probability distribution functions $F(\gamma, \gamma_s)$ derived in this section are used in the diffusion equations as discussed below.

2.4. Diffusive propagation in interstellar medium

The connections between the intensities of various nuclear species in a given energy interval in nearby interstellar space and the intensities of these at the point of origin are described on the basis of diffusive propagation in interstellar medium (and in the source region) incorporating source composition, source spectrum, composition of interstellar medium, elastic collisions, inelastic collisions, kinematics and angular distributions, ionization loss of the primaries and secondaries and the models of distributions of path-lengths as in the form of (a) slab, (b) gaussian, and (c) exponential types. The methods of calculations and the results on the fluxes of nuclei are briefly described in this section.

The abundances of nuclei of the source region of cosmic rays are assumed to be $H^1:H^2:He^3:He^4 = 100:0:0:15.7$ at the same total energy per nucleon. This value is so chosen as to match the calculated p/He ratio with the measured value at very high energies where interplanetary propagation effects are negligible.

The source spectrum of H^1 and He^4 nuclei are assumed to be modified total energy per nucleon spectrum as given by Equation (1).

Composition of interstellar medium is known mainly in the vicinity of some hot stars (Unsöld, 1960). From several theoretical and phenomenological arguments Greenberg (1968) estimated the abundance ratio of $H^1:He^4$ as 100:12 by number in interstellar medium and we have used this ratio in our calculations. The abundances of nuclei of higher charges are small and are neglected in the present work.

The parameters corresponding to the elastic and inelastic collisions and probability distribution functions of nuclear reactions, derived earlier are used in propagation calculations as follows.

2.4.1. *Slab and Gaussian Distribution of Path-Lengths*

In these models the diffusion equations are derived by modifying the earlier expressions given by Biswas *et al.* (1966) as follows:

$$N_p(E_2, x + \Delta x)\,\mathrm{d}E_2 = 2\pi \frac{\mathrm{d}E_3}{\mathrm{d}E_2}\,\mathrm{d}E_2 \int_{-1}^{+1} N_p(E_3, x)\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,pp(E_3) \times$$

$$\times\, n_{\mathrm{H}}\mathrm{d}(\cos\theta^*) + N_p(E_1, x)\left\{1 - \frac{\Delta x}{\Lambda_p(E_1)}\right\}\frac{\mathrm{d}E_1}{\mathrm{d}E_2}\,\mathrm{d}E_2,$$

$$N_{He^4}(E_3, x+\Delta x)\, dE_2 = 2\pi \frac{dE_3}{dE_2} dE_2 \int_{-1}^{+1} N_{He^4}(E_3, x) \frac{d\sigma}{d\Omega} \alpha_p(E_3) \times$$

$$\times\, n_H \, d(\cos\theta^*) + N_{He^4}(E_1, x) \left\{ 1 - \frac{\Delta x}{\Lambda_{He^4}(E_1)} \right\} \frac{dE_1}{dE_2} dE_2, \tag{6}$$

$$N_{H^2}(E_2, x+\Delta x)\, dE_2 = N_{H^2}(E_1, x) \left\{ 1 - \frac{\Delta x}{\Lambda_{He^2}(E_1)} \right\} \frac{dE_1}{dE_2} dE_2 +$$

$$+ \sum_j dE_2 \int N_j(E, x)\, dE n_k \sigma_j(E)\, F_j(E, E_2) \tag{7}$$

$$N_{He^3}(E_2, x+\Delta x)\, dE_2 = N_{He^3}(E_1, x) \left\{ 1 - \frac{\Delta x}{\Lambda_{He^3}(E_1)} \right\} \frac{dE_1}{dE_2} dE_2 +$$

$$+ \sum_j dE_2 \int N_j(E, x)\, dE n_k \sigma_j(E)\, F_j(E, E_2) \tag{8}$$

In these expressions, $N_i(E_1, x)$ represents the flux of nuclei of type i (H^1, He^4, H^2 and He^3) of energy between E_1 and $E_1 + dE_1$ at a 'distance' of x g cm^{-2} of interstellar matter from the source and $N_i(E_2, x+\Delta x)$ the flux of the nuclei of energy between E_2 and $E_2 + dE_2$ at a 'distance' of $x+\Delta x$ g cm^{-2}. The energy E_2 is related to E_1 by the ionization loss suffered by the ith type of nuclei in traversing the thickness of Δx g cm^{-2} of interstellar matter and is calculated by using the range energy relation in the form $E = C\,[E, Z, A].\, x^{n(E)}$, where E is the energy per nucleon in MeV n^{-1} of the particle of charge Z, and mass number A, C is a constant depending on the energy and $Z^2 A^{-1}$ of the particle and n is the exponent which is a function of energy E, x is the amount of matter, mainly hydrogen, in g cm^{-2} traversed by the particle. The constants C and n are evaluated by fitting the range energy curves given by Barkas (1962) for various nuclei in neutral hydrogen. $N_i(E_3, x)$ is the flux of particles at x g cm^{-2} from the source in the energy interval E_3 and $E_3 + dE_3$, where E_3 is related to the energy E_1 of the particle of $x+\Delta x$ g cm^{-2} by the relation $E_p = E_3 -$ (the energy loss due to ionization in traversing Δx g cm^{-2}) and E_p is related to E_1 by expression (2).

In Equations (5) and (6), $d\sigma_{jp}/d\Omega(E)$ is the differential elastic scattering cross-section of jth type of nuclei (p or α) at energy E, n_H, the number of hydrogen nuclei in Δx g cm^{-2} of interstellar matter, Λ_i, the total attenuation mean free path of the ith type of nuclei in g cm^{-2} which is related to the total attenuation cross-section σ_i by $\Lambda_i = (N\sigma_i)^{-1}$ where N is the Avogadro number.

The first term on the right-hand side in Equations (5) and (6) represents the contribution to p and He flux from p and He nuclei at higher energies scattered elastically by interstellar hydrogen. In case of protons contribution of interstellar hydrogen acquiring the requisite energy by elastic collision process of the primary cosmic ray protons is also included by using appropriate quantities. In Equation (7) and (8) the second term on R.H.S. represent the production of H^2 and He^3 nuclei and the spread in the velocity of the secondary is taken into account by the probability distribution function $F(E, E_2)$ as derived in Appendix B. The limits of integrations are described in Appendix A. The summation is done over all the reactions specified in Table I; σ_j represent the cross-section for the reaction j and n_k, the number of target nuclei per Δx g cm^{-2}. The first term on r.h.s. denotes the loss of nuclei due to collision processes and is determined by the corresponding attenuation mean free path, Λ_i.

The fluxes of H^1, He^4, H^2, and He^3 nuclei are calculated at various depths from 0 to 10 g cm^{-2} in steps of 0.1 g cm^{-2} by successive iterations. The results thus obtained from the slab model are used in the calculations on the basis of gaussian distribution of path-lengths.

In the model of gaussian distribution of path-lengths (Balsubrahmanyan *et al.*, 1964) the mean deviation of path lengths, σ is given by $\sigma = 0.633\ x$ where $\bar{x}$ is the mean amount of matter traversed. The probability of a particle traversing the amount of matter between x and $x + \mathrm{d}x$ is given by the normalised gaussian distribution

$$P(x)\,\mathrm{d}x = \left\{\left(\frac{5}{\pi}\right)^{1/2} / 1.886\bar{x}\right\} \exp\left\{-\frac{5}{4}\frac{(x - x)^2}{\bar{x}^2}\right\} \mathrm{d}x\,. \tag{9}$$

By use of this distribution function the flux of any nuclei of type j, after a traversal of a mean distance of $\bar{x}$ g cm^{-2} is evaluated by using the expression

$$N_j(E, \bar{x})\,\mathrm{d}E = \mathrm{d}E \int_0^\infty N_j'(E', x)\,P(x)\,\mathrm{d}x\,, \tag{10}$$

where $N_j'(E', x)$ is the flux of j type of nuclei at a distance of x g cm^{-2} from the source calculated from the slab model, and E' is the energy at x g cm^{-2} from the source.

2.4.2. *Exponential Distribution of Path-Lengths*

In this model it is assumed that the distribution of the amount of matter traversed by a particle is of exponential type and the energy spectrum of nuclei at any point in space is given in the steady state by the following expression, as deduced by several authors (see, e.g., Cowsik *et al.*, 1967; Gloeckler and Jokipii, 1970)

$$N_j(E)\,\mathrm{d}E = \mathrm{d}E \int P_j\left[E_j'(E, x)\right] \frac{b_j(E_j')}{b_j(E)} \exp\left[-\int_0^x \mathrm{d}y \times \times \left\{\frac{1}{\Lambda_j\{E_j'(E, y)\}} + \frac{1}{\lambda_1\{E_j'(E, y)\}}\right\}\right] \mathrm{d}x\,, \tag{11}$$

where $P_j[E_j'(E, x)]$ is the number of nuclei of type j of energy between E_j' and $E_j' + \mathrm{d}E_j'$, E_j' being the energy of the particle at x g cm^{-2} from the point of observation where the particle has an energy E; $b_j(E_j')/b_j(E)$ is the Jacobian to take care of the spread of energy interval due to traversal of x g cm^{-2}; $b_j(E)$ is the rate of ionization loss per g cm^{-2}, of the particle of type j at energy E. The distribution in the amount of matter traversed is represented by the exponential term which includes two terms,

$\Lambda_j[E'(E, y)]$ the attenuation mean free path and $\lambda_1[E'(E, y)]$ the leakage mean free path representing the loss of particles from the galactic boundary. The production term $P_j(E')$ consist of two terms

$$P_j[E'(E, x)]\,\mathrm{d}E' = Q_j(E')\,\mathrm{d}E' + \\ + \mathrm{d}E' \sum_i \int Q_i(E')\, n\, \sigma_i(E)\, F_i(E, E')\,\mathrm{d}E\,, \qquad (12)$$

where the first term represents the contribution from the cosmic ray sources, and the second term due to the production from collisions of primary cosmic ray nuclei in interstellar medium. In this model the cosmic ray sources are assumed to be distributed uniformly in the galaxy. In case of protons and helium nuclei the production term is modified to take into account the contribution from the elastic collisions of nuclei of higher energy as given by

$$P_j[E'(E, x)]\,\mathrm{d}E' = Q_j(E')\,\mathrm{d}E' + \mathrm{d}E' \times \\ \times \int_{-1}^{+1} Q_j(E')\, n_{\mathrm{H}} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E')\,\mathrm{d}(\cos\theta^*)\,. \qquad (13)$$

In case of H^2 and He^3 nuclei, the summation of the second term on the right-hand side of Equation (12) is taken over all reactions given in Table I.

The galactic equilibrium spectra of H^1, He^4, H^2, and He^3 nuclei are thus calculated on the basis of this model.

2.5. Interplanetary diffusion, convection and adiabatic energy loss

It is now well known that the intensity and energy spectra of cosmic ray nuclei in the nearby interstellar space are significantly modified in the inner solar system by the influence of solar wind and this can be described in terms of diffusion, convection, and adiabatic deceleration of cosmic ray particles (see, e.g., Parker, 1969; Jokipii, 1971). It has been shown by several authors (see, e.g., Gleeson and Urch, 1971) that in a steady state condition and for a spherically symmetric configuration the equation describing the passage of cosmic ray particles through the inner solar system can be given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 UV - r^2 k\frac{\partial U}{\partial r}\right) = \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha T U), \qquad (14)$$

where $U(r, T)$ is the differential number density of cosmic ray nuclei of kinetic energy T at a distance r from the Sun, $V(r)$ the solar wind velocity at r, $k(V, T)$ the diffusion coefficient, and $\alpha=(T+2E_0)/(T+E_0)$, where E_0 is the rest energy of the particle.

Although several approximate analytical solutions of Equation (14) were obtained by several authors, none of these can be applied in the entire energy interval of interest in galactic cosmic rays, and hence, numerical solutions have been derived (Goldstein *et al.*, 1970; Gleeson and Urch, 1971). In this work we use the scheme of Gleeson and Urch (1971) with the following assumptions: (1) The velocity of solar wind $V(r)=400$

$[1-\exp\{(1-r)/15\}]$ km s^{-1}, where r is the distance from the Sun in units of solar radius; (2) the diffusion coefficient $k=6.0\times10^{21}$ $(1/r^2)\,p\beta$ cm^2 s^{-1}, where r is in unit of UA and p the particle rigidity in GV and βc the particle velocity, (3) the outer boundary of the modulating region is taken as 10 UA and it is assumed that there is no modulation of particles of energy 1000 GeV and higher. We derived the numerical solution of Equation (14) using the programme of Urch and Gleeson (1972). The spectra of various nuclei at nearby interstellar space calculated earlier were used as input spectra and the energy spectra of H^1, He^4, H^2 and He^3 nuclei at 1 UA were derived.

3. Results and Discussions

The calculated energy spectra of protons and He^4 nuclei at nearby interstellar space and at 1 UA during a period near solar minimum (1965) using Equations (5) and (6)

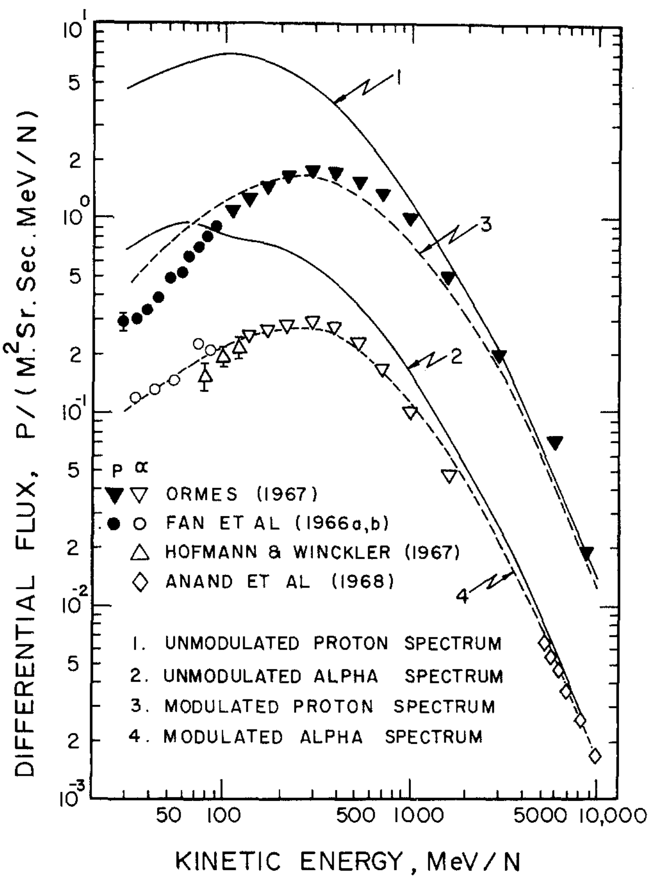

Fig. 7. The experimentally measured energy spectra of P- and He^4-nuclei at a period near solar minimum ($\sim$ 1965) is shown along with the calculated spectra obtained from a gaussian distribution of path-lengths with $\bar{x}=4.0$ g cm^{-2}, taking into account the solar modulation. The spectra at local interstellar space are also shown.

and a gaussian distribution of path-lengths with mean path-length $\bar{x} = 4.0$ g cm^{-2} are shown in Figure 7, along with experimentally measured spectra. The calculated and measured spectra are normalised at 50 GeV n^{-1} where modulation is negligible. It is seen in the figure that there is good agreement between the calculated and measured spectra of both protons and He-nuclei. In case of proton spectrum better agreement at energies less than 100 MeV n^{-1} may be obtained by using a different exponent of the rigidity dependence of the diffusion coefficient as mentioned by several authors (e.g. Ramadurai and Biswas, 1971; Burger and Tanaka, 1970).

The energy spectra of protons and He nuclei were also calculated using an exponential distribution of path-lengths with a leakage mean free path of 4 g cm^{-2} and these are also in reasonable agreement with the measured spectra – similar to those given by several earlier investigators (Meyer, 1970; R-L, 1969, and Comstock *et al.*, 1972). Hence these are not shown here. We wish to mention that because of the large modulation effect at low energies it is not possible from the observations of only

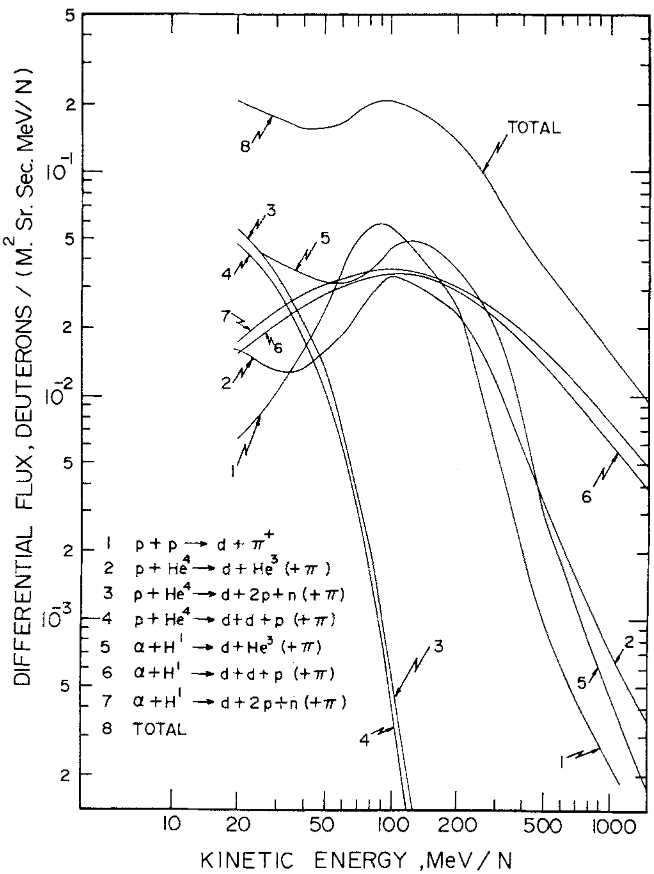

Fig. 8. The calculated fluxes of deuterons produced from various reactions and the differential energy spectrum at local interstellar space. The distribution of path-lengths is assumed to be a gaussian with a mean value of 4 g cm^{-2} and source spectrum as in the form of 'Fermi' type.

protons and helium nuclei to distinguish between the source spectrum in the form of total energy per nucleon and the modified total energy per nucleon spectra ('Fermi' spectrum) used here. The results on H^2 and He^3 nuclei are needed for this purpose as discussed below.

The calculated differential fluxes and energy spectra of H^2 and He^3 nuclei in nearby interstellar space are shown in Figures 8 and 9, using a gaussian distribution of path-lengths with a mean value $\bar{x}=4$ g cm^{-2}. The contributions of various reactions to the total differential flux are also shown in Figures. It is seen in Figure 8 that in 20–50 MeV n^{-1} energy range contributions of the six reactions, 2 to 7, are comparable and that of (1) is small whereas in 50–200 MeV n^{-1} interval the two cosmic ray proton induced reactions, (1) and (2) and three He^4 induced reactions, 5 to 7, make dominant contribution. At high energies >500 MeV n^{-1}, deuteron flux arise essentially from the two reactions 6 and 7 involving multibody breakup of cosmic ray He^4 nuclei. In case of He^3, at all energies the three reactions 4 to 6 by cosmic ray He^4 always dominate

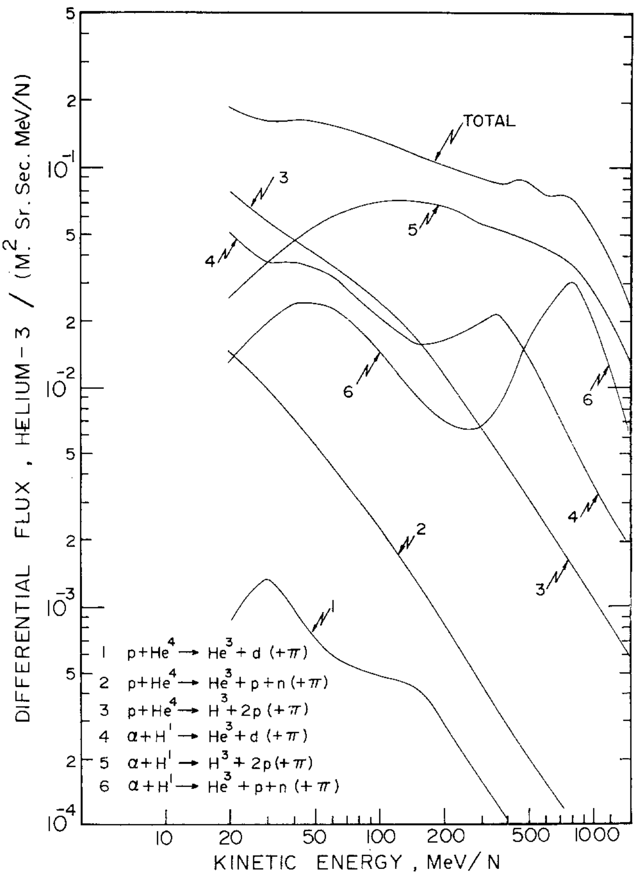

Fig. 9. The calculated fluxes of He^3 (and tritons) produced from various reactions and the differential energy spectrum at local interstellar space. The distribution of path-lengths is assumed to be a gaussian with a mean value of 4 g cm^{-2} and the source spectrum as 'Fermi' type.

(Figure 9), only the reaction (3) involving multibody breakup of interstellar helium by primary protons make significant contribution at energies less than 300 MeV n^{-1}.

The calculated ratios of H^2/He^4 as a function of energy in local interstellar space and at 1 UA are shown in Figure 10 using gaussian distribution of path-lengths with mean values of $\bar{x}=4$ and 5.5 g cm^{-2}. The value of $\bar{x}$ is chosen as 4 g cm^{-2} so as to obtain the best fit to the observed data of *both* H^2/He^4 and $He^3/(He^3+He^4)$ ratios. Solar modulation correction is made corresponding to the period close to minimum solar activity. The calculated curve for H^2/He^4 ratio at 1 UA using exponential distribution of path-lengths with a leakage mean free path of 4 g cm^{-2} is shown in the figure. The experimentally measured ratios in the vicinity of the Earth by different investigators are shown in the figure. We may mention that although experimental ratios were measured at different times, most of the measurements refer to the period of low solar activity similar to that used for the calculated curve. On comparing the

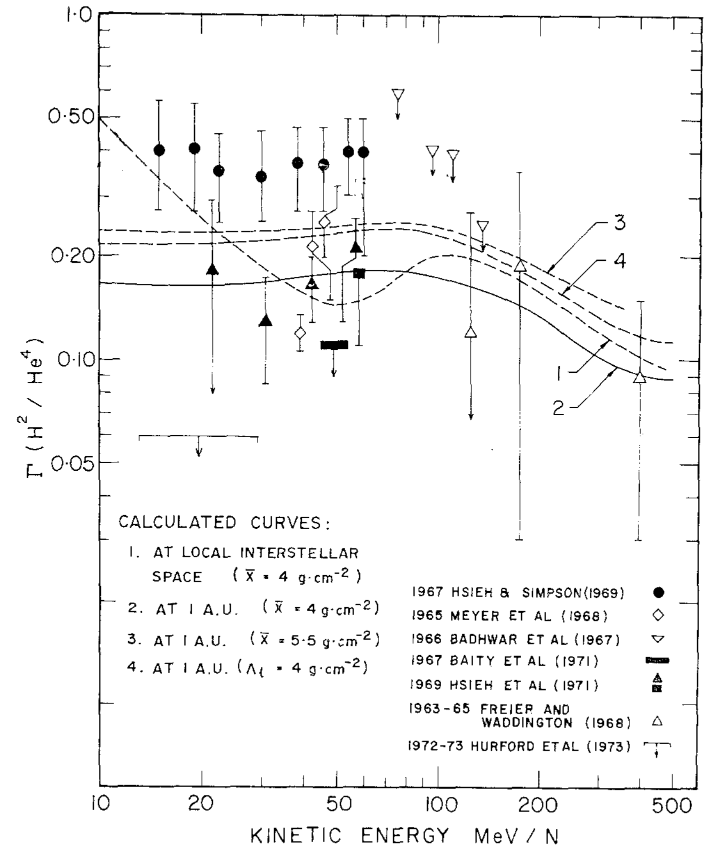

Fig. 10. The calculated ratios of H^2/He^4 nuclei as a function of energy at local interstellar space (curve 1) and at 1 UA are shown using a gaussian distribution of path-lengths with a mean value of $\bar{x} = 4$ g cm^{-2} (curve 2) and $\bar{x} = 5.5$ g cm^{-2} (curve 3). The calculated ratios at 1 UA using an exponental distribution of path-lengths with a leakage mean free path of 4 g cm^{-2} are shown by curve 4. The experimentally measured ratios near the Earth by various authors are shown. (The years of measurements are indicated before the names of the authors.)

calculated and measured data in Figure 10, we note that: (i) there exists a large scatter of the observed data points obtained in different experiments. It is not known at present to what extent the differences are due to experimental effects or due to solar modulation effects; (ii) both propagation models predict very similar values of the ratio of H^2/He^4 at 1 UA and their energy dependence as shown by curves (2) and (4) in Figure 10; (iii) The calculated ratios and their energy dependence as given by the curves (2) and (4) agree well with the experimental data of Meyer *et al.* (1968) and Hsieh *et al.* (1971); (iv) The recent ratios of Teegarden *et al.* (1973) are significantly smaller than the calculated values; possible interpretations are discussed later.

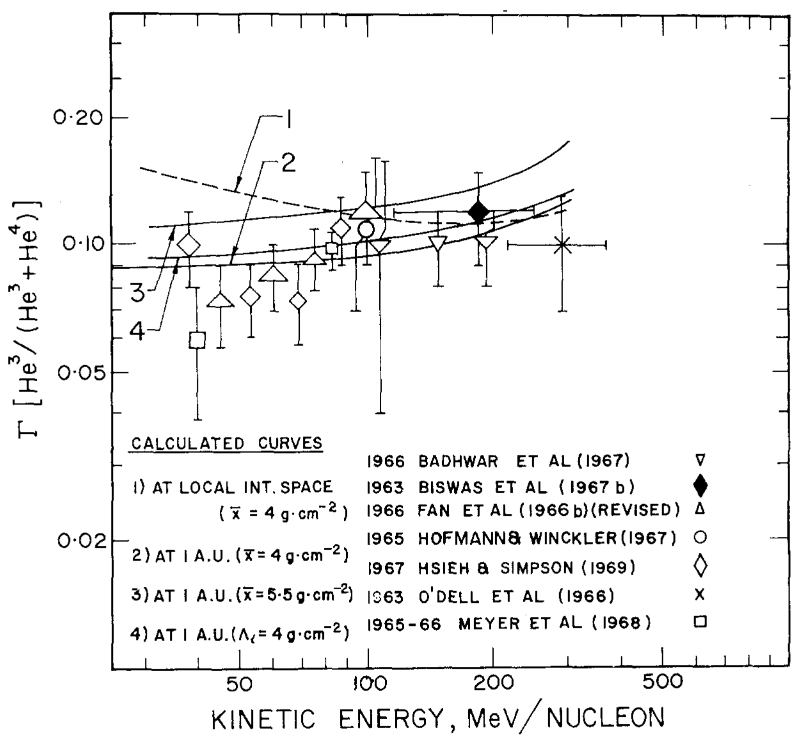

Fig. 11. The calculated ratios of $He^3/(He^3+He^4)$ nuclei as a function of energy at local interstellar space (curve 1) and at 1 UA are shown using a gaussian distribution of path-lengths with a mean value of $\bar{x}=4$ g cm^{-2} (curve 2) and $\bar{x}=5.5$ g cm^{-2} (curve 3). The calculated ratios at 1 UA using an exponential distribution of path-lengths with a leakage mean free path of 4 g cm^{-2} are shown by curve (4). The experimental ratios near the Earth by various authors are shown.

The calculated ratios of $He^3/(He^3+He^4)$ as a function of energy and the measured values by different investigators are shown in Figure 11. In this figure it is seen that: (i) There exists good agreement between the experimental data of different investigators which were measured during the period of low solar activity in 1963–67. These indicate that the variations in the ratios due to differences in the solar modulation during this period are probably equal to or smaller than the statistical uncertainties of the values; (ii) The calculated curve (2) for the mean path-length $\bar{x}=4$ g cm^{-2} with gaussian distribution of path-lengths is in excellent agreement with the measured data and their energy dependence in the energy interval 40–400 MeV nucleon^{-1}. It is also seen that a value of $\bar{x} = 4$ g cm^{-2} gives a better fit to the data than that of $\bar{x} = 5.5$ g cm^{-2}; (iii) The calculated curve for the ratio at 1 UA is a little lower than that at nearby interstellar space indicating that solar modulation effect near solar minimum on the ratio is not large in this energy range and is reasonably interpreted in the present ideas of solar modulation in this energy range.

We now discuss briefly some of the significant aspects of the calculations and the comparison with experimental data. In these calculations the adjustable parameters are (a) the source spectrum of cosmic ray H^1 and He^4 nuclei; (b) propagation model and the mean quantity of material traversed in interstellar medium (and in source region) and (c) the solar modulation parameters. The experimentally measured parameters are the fluxes and energy spectra of H^1, H^2, He^3, and He^4 nuclei. Therefore by fitting the calculated results to the observed parameters of all the four components simultaneously, the adjustable parameters can be evaluated and a self-consistent model can be obtained as discussed below. Such a procedure has been discussed earlier by Simpson (1972).

As regards the source spectrum, simultaneous observations of fluxes of H^2 and He^3 isotopes together with those of H^1 and He^4 nuclei impose severe restrictions on the form of source spectrum as was noted earlier. This arises from the fact that at energies less than ~ 50 MeV n^{-1} deuterons in the local interstellar space are produced by higher energy protons and helium nuclei. (This is because of the kinematics of the nuclear reactions.) Hence, at local interstellar space a change in the source spectra of protons and helium nuclei produces far less changes in the fluxes of deuterons compared to those in protons and He^4 nuclei. Consequently H^2/He^4 ratios at local interstellar space will critically depend on the source spectra of He^4 nuclei. It is found that source spectrum assumed in the form of power law in rigidity or in total energy per nucleon can not explain both H^2/He^4 and He^3/He^4 data and it is necessary to use a source spectrum which is intermediate between the above two forms, as the one used in this work (see, e.g., Ramadurai and Biswas, 1971, 1972). Similar ideas were suggested by Comstock *et al.* (1972) and Meyer (1972).

Regarding the problem of solar modulation it is well known that the spectral forms of proton and helium nuclei at the Earth, particularly at low energies are very sensitive to the solar modulation. In this work solar modulation of the local interstellar spectra are derived on the basis of numerical solution of the Fokker-Planck equation using the formulation of Goldstein *et al.* (1970) and Gleeson and Urch (1971), and these give reasonably good agreement with the spectral form of *both* proton and helium nuclei at solar minimum. Therefore we feel that modulation effect is taken into account adequately for this period for H^1, H^2, He^3 and He^4 nuclei. The good agreement between the calculated and measured energy dependence of $He^3/(He^3 + He^4)$ confirm the above conclusion. In case of H^2/He^4 ratios calculated values are in fair agreement with many of the data points. The values of H^2/He^4 ratios measured in 1973 by Teegarden *et al.* (1973) in 20–40 MeV n^{-1} interval are significantly lower. Possible explanation seems to be due to either a change in the character of modulation during this period or an additional quiet time solar He^4 flux at low energies. The latter possibility is suggested because H^2 flux was normal and He^4 was enhanced. Similar suggestion was made by Hurford *et al.* (1973) to interpret their upper limit value. Further observations are necessary for clarification.

In considering the interstellar propagation it is shown that observed data on H^2/He^4 and $He^3/(He^3+He^4)$ can be explained by a gaussian distribution of path-lengths with a mean path-length of 4 g cm^{-2} of hydrogen or by an exponential distribution of path-lengths with a leakage path-length of 4 g cm^{-2}. The path-lengths derived in this work are in good agreement with those estimated from other secondary components of cosmic rays (see, e.g., Shapiro and Silberberg, 1970). The observations of H^2 and He^3 nuclei do not distinguish between the two models of path-length distributions, because, the yields of H^2 and He^3 arise primarily from long path-lengths and hence are insensitive to the distribution of short path-lengths. The study of the spallation products of Fe nuclei can yield information on short path-lengths, and it was found from these studies (Shapiro and Silberberg, 1971; Chohan *et al.*, 1973) that a distribution having a dearth of short path-lengths can explain main features. Therefore we have used a gaussian distribution of path-lengths and it is found such a model is consistent with H^2 and He^3 observations. A path-length distribution similar to that of a gaussian may arise if cosmic ray particles traverse significant amount of material near cosmic ray sources. Several such models are under consideration at present (e.g. Meneguzzi, 1973; Cowsik and Wilson, 1973) and the constraints derived from the H^2 and He^3 nuclei may be incorporated in cosmic ray models.

Acknowledgements

The authors are thankful to Drs R. Ramaty, L. Fisk, J. P. Meyer, L. J. Gleeson, and I. Urch for valuable discussions. One of the authors, S. Biswas, acknowledges the NASA-NAS Senior Research Associateship at Goddard Space Flight Center, Greenbelt, Maryland, U.S.A., during the tenure of which a part of the work was done.

Appendix A

Kinematics of Nuclear Reactions

As the energy of the secondary particle is dependent on the angle of emission, there exists a range of energy for the primary particle, which is calculated using relativistic kinematics.

Let γ_s and γ_s^* be the Lorentz factor of the secondary particle of mass m_s in the Lab. and C.M. systems and let this particle be emitted in the direction $\cos\theta^*$ in the C.M. System. Then γ_s and γ_s^* are related through the Lorentz factor of the centre of Mass γ_c by

$$\gamma_s = \gamma_c \gamma_s^* + \{(\gamma_c^2 - 1)(\gamma_s^{*2} - 1)\}^{1/2} \cos\theta^* . \tag{A.1}$$

The Lorentz factor of the centre of mass γ_c is uniquely related to the energy E of the primary particle of mass m_i and the mass of the target, m_t, assumed to be at rest by

$$\gamma_c = \frac{\gamma m_i + m_t}{E'} \quad \text{where} \quad \gamma = \frac{E + m_i}{m_i}, \tag{A.2}$$

$$E' = (m_i^2 + m_t^2 + 2\gamma m_i m_t)^{1/2}. \tag{A.3}$$

In the two-body reactions γ_s^* is uniquely given by law of conservation of mass and energy

$$\gamma_s^* = \frac{E'^2 + m_s^2 - m_r^2}{2m_s E'}, \tag{A.4}$$

where m_r is the mass of the particles other than the secondary. This is the maximum possible energy of the secondary particle in the C.M. System and hence let us call this γ_m^*

The range of primary energies clearly depends on γ_s^* and $\cos\theta^*$ and these ranges can be obtained by the two criteria $|\cos\theta^*| \leqslant 1$ and $\gamma_s^* \leqslant \gamma_m^*$. It follows from (A 1) that the lower limit is given by $\cos\theta^* = 1$ and $\gamma_s^* = \gamma_m^*$. In the case of upper limits a few cases have to be distinguished.

If γ_s is a particular value $\gamma_s(\infty)$ given by $\frac{1}{2}(m_s/m_t + m_t/m_s)$ then the upper limit becomes infinite. If $\gamma_s < \gamma_s(\infty)$, in the case of two-body reactions the upper limit is found by imposing the condition $\gamma_s^* = \gamma_m^*$ and $\cos\theta^* = -1$. For multi-body reactions in the case $m_t > m_s$ the upper limit becomes infinite. However, if $m_t < m_s$ the upper limit is given by the condition $\gamma_s^* = \gamma_m^*$ and $\cos\theta^* = -1$ regardless of the number of particles in the final state. For further details see Ramaty and Lingenfelter (1969).

Appendix B

Angular Distribution of the Secondary Particles and their Probability Distribution $F(\gamma, \gamma_s)$

In Appendix A the range of energy of the primary particle for producing a secondary particle of a given energy depending on the angle of emission was estimated. The angle of emission itself is dependent on the angular distribution $P(\gamma, \gamma_s^*, \cos\theta^*)$ of the secondaries in any reaction at any given energy. The probability that a particle of Lorentz factor γ produces a secondary particle with Lorentz factor γ_s is found to be

$$F(\gamma, \gamma_s) = (\gamma_c^2 - 1)^{-1/2} \int P(\gamma, \gamma_s^*, \cos\theta^*)\,(\gamma_s^{*2} - 1)^{-1/2}\,\mathrm{d}\gamma_s^*. \tag{B.1}$$

The necessary expressions of $P(\gamma, \gamma^*, \cos\theta^*)$ which are obtained form phenomenological considerations of the various experimental observations are given in Table B.I.

In the case of two-body reactions (1–5 in Table B.I.) the relation between $\cos\theta^*$ and γ_s^* is unique and hence Equation (B.I) becomes

$$F(\gamma, \gamma_s) = \{(\gamma_c^2 - 1)\,(\gamma_m^{*2} - 1)\}^{-1/2}\,P(\gamma, \cos\theta^*). \tag{B.2}$$

However, when pions are produced as in the case of reactions (2 and 4) the deuteron angular distribution is taken the same as in reaction (1) with $m_i = m_t = m_p$ and $m_r = m_\pi$. This is done so because the results from the experiment of Riddiford and Williams (1960) show that the pions are produced as if from the collisions of free nucleons. It follows that He^3 is produced at rest in the C.M. system in these reactions and $\gamma_s^* = \gamma_c$.

In the $p\alpha$ reactions (6–9) it is inferred from the works of Innes (1957) that deuterons as well as He^3 are produced predominantly with low energies. This can happen if the secondaries are produced in the C.M. system in backward directions with γ_s^* close to γ_c. This is incorporated in the angular distributions given in Table B.I. The same distribution holds good even when pions are produced.

TABLE B.I

Assumed angular distribution functions $P(\gamma, \gamma^*{}_s, \cos\theta^*)$

Reaction	$P(\gamma, \gamma_s^*, \cos\theta^*)$	Ref. to Experimental Measurement
1. $H^1(P, H^2)\,\pi^+$	$(\frac{1}{3} + \cos^2\theta^*)$	Several authors, see summary by Rosenfeld (1954)
2. $He^4(P, H^2)\,He^3$	$\dfrac{\exp\{-(1-\varepsilon\cos\theta^*)/\cos\theta_0\}}{\{1-\exp(-2/\cos\theta_0)\}\cos\theta_0}$	Bunch *et al.* (1964)
3. $He^4(P, He^3)\,H^2$	where	
4. $H^1(\alpha, H^2)\,He^3$	$\cos\theta_0 = \begin{cases}\exp\{-9.38(\gamma-1)\} \text{ for } \gamma \leqslant \gamma_{200} \\ \exp\{-9.38(\gamma_{200}-1)\} \text{ for } \gamma > {}_{200}\end{cases}$	Selove and Teem (1958) Hayakawa *et al.* (1964)
5. $H^1(\alpha, He^3)\,H^2$	$\varepsilon = \begin{cases}+1 & \text{for } (2)\,\&\,(5) \\ -1 & \text{for } (3)\,\&\,(4)\end{cases}$	(1)
6. $He^4(P, H^2)\,2pn$	$\dfrac{\exp\{-(1+\cos\theta^*)/\cos\theta_0\}}{\cos\theta_0\{1-\exp(-2/\cos\theta_0)\}}\,\delta(\gamma_s^* - \langle\gamma_s^*\rangle)$	Innes (1957)
where		
7. $He^4(P, 2H^2)\,p$	$\langle\gamma_s^*\rangle = \gamma_m^*$ for $\gamma_c < \gamma_{cz}$	
	$\langle\gamma_s^*\rangle = \gamma_c$ for $\gamma_c \geqslant \gamma_{cz}$	(2)
	$\cos\theta_0 = \dfrac{E_{0d}}{m_p\{(\gamma_c^2-1)(\langle\gamma_s^*\rangle^2-1)\}^{1/2}}$	
	$E_{0d} = (1/0.063)$ MeV n^{-1}.	

Table B.I. (Continued)

8. $He^4(P, H^3)\,2p$ 9. $He^4(P, He^3)\,pn$	$\frac{\exp\{-(1+\cos\theta^*)/\cos\theta_0\}}{\cos\theta_0\{1-\exp(-2/\cos\theta_0)\}}\,\delta(\gamma_s^*-\langle\gamma_s^*\rangle)$ $\langle\gamma_s^*\rangle=\gamma_m^*$ for $\gamma_c\leqslant\gamma_{cz}$ $\langle\gamma_s^*\rangle=\gamma_c$ for $\gamma_c>\gamma_{cz}$ $\cos\theta_0=\begin{cases}\frac{E_{0t}}{m_p\{(\gamma_c^2-1)(\langle\gamma_s^*\rangle^2-1)\}^{1/2}} & \text{for } \gamma_c<\gamma_{cz}\\ \frac{E_{0t}}{m_p\{(\gamma_c^2-1)(\langle\gamma_s^*\rangle^2-1)\}^{1/2}} & \text{for } \gamma_{cz}<\gamma_c<\gamma_{150}\\ \frac{E_{0t}}{m_p\{(\gamma_{150}^2-1)(\gamma_z^{*2}-1)\}^{1/2}} & \text{for } \gamma_c>\gamma_{150}\end{cases}$ $E_{0t}=1/0.27\,(\text{MeV } n^{-1})$	Tannenwald (1953) Innes (1957) (3)
10. $H^1(\alpha, H^2)\,2pn$ 11. $H^1(\alpha, 2H^2)\,p$	$\frac{\exp\{-(1-\cos\theta^*)/\cos\theta_0\}}{\cos\theta_0\{1-\exp(-2/\cos\theta_0)\}}\,\delta(\gamma_s^*-\langle\gamma_s^*\rangle)$ $\langle\gamma_s^*\rangle=\gamma_m^*$ for all energies $\cos\theta_0=\begin{cases}\frac{E_{0d}}{m_p\{(\gamma_c^2-1)(\langle\gamma_s^*\rangle^2-1)\}^{1/2}} & \text{for } E<40\text{ MeV } n^{-1}\\ \frac{E_{0d}}{m_p\{(\gamma_{cz}'^2-1)(\gamma_z^{*2}-1)\}^{1/2}} & \text{for } E\geqslant 40\text{ MeV n}^{-1}\end{cases}$	(4)
12. $H^1(\alpha, H^3)\,2p$ 13. $H^1(\alpha, He^3)\,pn$	$\frac{\exp\{-(1-\cos\theta^*)/\cos\theta_0\}}{\cos\theta_0\{1-\exp(-2/\cos\theta_0)\}}\,\delta(\gamma_s^*-\langle\gamma_s^*\rangle)$ $\langle\gamma_s^*\rangle=\gamma_m^*$ for all energies $\cos\theta_0=\begin{cases}\frac{E_{0t}}{m_p\{(\gamma_c^2-1)(\langle\gamma_s^*\rangle^2-1)\}^{1/2}} & \text{for } \gamma_{cp}\geqslant\gamma_{c2p}\\ \frac{E_{0t}}{m_p\{(\gamma_c^2-1)(\gamma_z^{*2}-1)\}^{1/2}} & \text{for } \gamma_{cp}>\gamma_{c2p}\end{cases}$	(5)

(1) γ_{200} represents the Lorentz factor of the primary particle with energy 200 MeV n^{-1}.
(2) γ_{cz} is the Lorentz factor of the center of mass for which $\gamma_c=\gamma^*_m$.
(3) γ_{150} is the Lorentz factor of the center of mass in the case of αp interaction at an energy/nucleon corresponding to the $p\alpha$ interactions for $\gamma_c=\gamma_{cz}$, and, γ^*_z is the Lorentz factor of the secondary He^3 when $\gamma_c=\gamma_{cz}$.

(4) γ'_{cz} is the Lorentz factor of the center of mass in the αp interactions for the same energy/nucleon at which $\gamma_c = \gamma_{cz}$ in the $p\alpha$ interactions.
(5) γ_{cp} is the Lorentz factor of the C.M. in the case of $p\alpha$ interactions for the energy of protons corresponding to the same energy per nucleon of the α-particles and γ_{c2p} is the Lorentz factor of the centre of mass in the case of $p\alpha$ interactions for which $\gamma_c = \gamma_{cz}$. γ'_c is the Lorentz factor of the centre of mass in the case of αp interactions corresponding to γ_{c2p}. γ^*_z is the Lorentz factor of the secondary He^3 or protons in the centre of mass for which $\gamma_c = \gamma'_c$.

In the case of αp interactions (10–13) the criteria adopted was that the angular distributions will be similar to the ones for $p\alpha$ interactions except for the fact that here γ_s^* can never equal γ_c and, hence, γ_s^* is always taken as γ_m^*. The additional check here has been that the secondary production rate at high energies should be equal to the ones calculated without, taking into account the angular distribution. This is so because at high energies the secondaries are produced with almost the same energy per nucleon as the primary particle.

The angular distributions given in Table B.I. correspond closely to those given by Ramaty and Lingenfelter (1969), except for the modifications in the expressions for the normalising factor $\cos\theta_0$ in the expressions in Table B.I. These modifications are necessary on two grounds. (a) In the case of $He^4(p, H^2)\, He^3$ the total cross-section

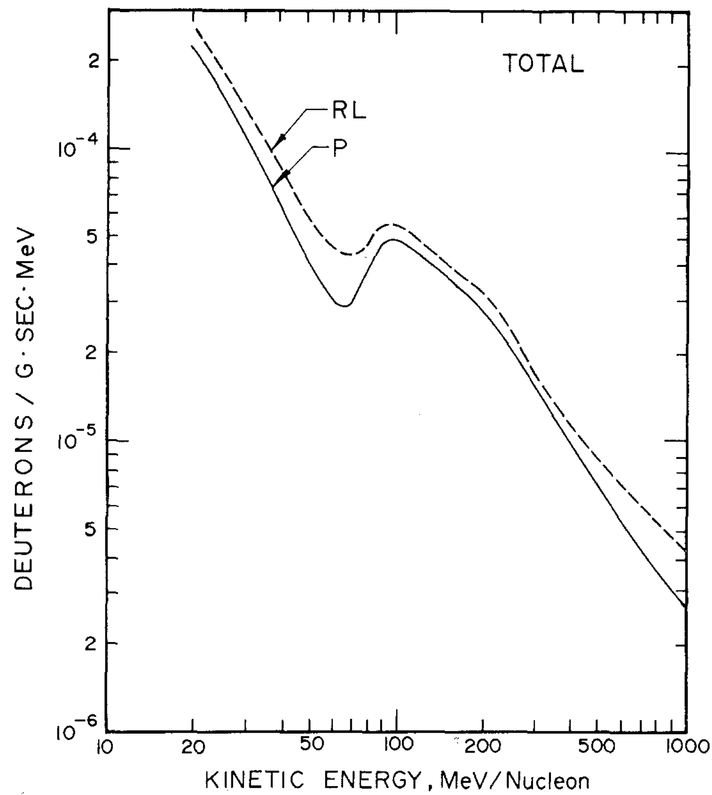

Fig. B.1. The total rate of production of deuterons per gm of interstellar material, calculated according to the distribution function used in this work and by Ramaty and Lingenfelter (1969) (R-L).

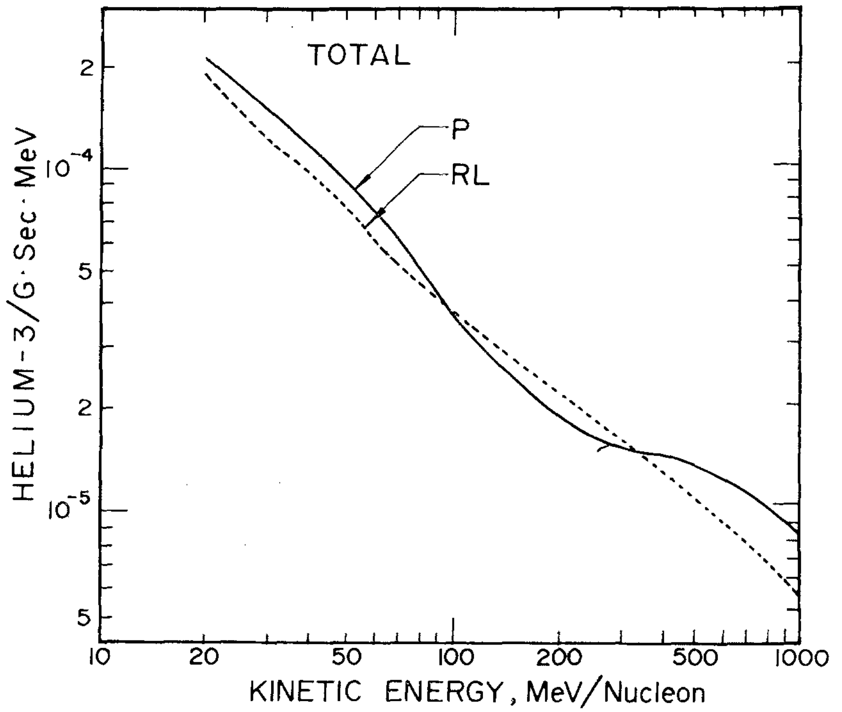

Fig. B.2. The total rate of production of $He^3(+H^3)$ per gm of interstellar material as calculated according to parameters used in this work and by R-L (1969).

becomes a constant after 200 MeV. If this normalisation is not brought about the total cross-section will go on decreasing with energy, (b) in the case of $He^4(p, H^3)\,2p$ and $He^4(p, He^3)\,pn$-reactions the distribution functions used in the present calculations are in accord with the experimentally measured energy spectra of H^3 nuclei in the case of the $He^4(n, n)\,pH^3$-reaction which is equivalent to the $He^4(p, p)\,pH^3$-reaction. This is not so with the original expressions of R-L (1969). Moreover, in case of αp-interactions, the criteria adopted here together with detailed kinematical calculations yield the flux values of H^2 nuclei at high energies, in agreement with those obtained with direct calculations neglecting angular distribution affects, since at high energies the angular distribution effects are extremely small.

We have compared, the total rate of production of deuterons and helium-3 nuclei using the distribution functions given here with those given by R-L (1969), using the same equilibrium spectra of P and He-nuclei. These are shown in Figures B.1. and B.2. It is seen that the over-all agreement is quite good. The differences in the details of the production rates arise primarily on account of modifications introduced in this work which are found necessary.

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