THE OBLATE SPHEROIDS VERSION OF THE RESTRICTED PHOTOGRAVITATIONAL 2+2 BODY PROBLEM

T.J. KALVOURIDIS
National Technical University of Athens, Department of Engineering Science 5, Heroes of Polytechnion Ave., 157 73, Greece

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Abstract. The paper deals with the restricted photogravitational 2+2 body problem when the primaries are oblate spheroids. A study of the effect of the oblateness on the equilibrium positions and on the areas of the permissible motion of the minor bodies, is also made.

1. Introduction

In our previous article (Kalvouridis and Mavraganis, 1995), we have presented the photogravitational 2+2 body problem by considering spherical primaries P_i, $i = 1, 2$. The study of this model was based on the assumptions that one of the minor bodies, is so big that the radiation pressure acted upon it by the primaries be negligible compared with the gravitation and that the other minor body, say S_2, is so small to be acted by both the gravitational attractions of all other bodies and the radiation pressure of the primaries.

On the other hand it is known that the shape of the planets, mainly the more massive, differs from the spherical one and thus the oblateness in these cases couldn't be neglected. For example the polar and equatorial radii of Saturn are 60.400 and 54.600 km and those of Jupiter are 71.400 and 67.000 km respectively. Therefore it is important to investigate in what degree the oblateness affects the dynamical behaviour of the system.

We will assume hereafter that the equatorial planes of the primaries coincide with the plane of their motion. In the subsequent, we extract the equations of motion of the minor bodies S_i, $i = 1, 2$ and we study numerically the influence of the primaries' oblatenesses on the location of the equilibrium points and on the areas of the permissible motion of the small bodies. Some of the results obtained are exposed in tables and diagrams.

2. Equations and Integral of Motion

Among the existing formulas which describe the gravitational potential created by an oblate spheroid, that proposed by Mac Cuskey (1963, p. 164) approximates satisfactory and in a rather simple way, the behaviour of the natural bodies. If we denote R_{ie}, R_{ip}, $i = 1, 2$ the dimensionless equatorial and polar radii of the bodies P_i, $i = 1, 2$, and by,

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$$I_i = \frac{\mu_i}{5}(R_{ie}^2 - R_{ip}^2),\ i = 1,2$$

their oblatenesses, then the gravitational potentials of the primaries, according to Mac Cuskey's analysis, will be expressed with the general formula,

$$V_i = -\frac{M_i}{r_i} - \frac{I_i}{2r_i^3} + \frac{3I_i}{2r_i^5}z^2,\ i = 1,2$$

where M_i, $i = 1, 2$ are their reduced masses,

$$M_1 = 1 - \mu \text{ and } M_2 = \mu.$$

For the planar case ($z = 0$) the Lagrangian expressed in the synodic coordinate system $Oxyz$, takes the form,

$$L(x_i, y_i, \dot{x}_i, \dot{y}_i) = \sum_{i=1}^{2} \frac{\mu_i}{2}[(\dot{x}_i - \omega^* y_i)^2 + (\dot{y}_i + \omega^* x_i)^2]$$

$$+ \left\{ \mu_1 \sum_{i=1}^{2} q_i \left[\frac{M_i}{r_{i1}} + \frac{I_i}{2r_{i1}^3}\right] + \mu_2 \sum_{i=1}^{2} \left[\frac{M_i}{r_{i2}} + \frac{I_i}{2r_{i2}^3}\right] + \frac{\mu_1\mu_2}{\rho} \right\}. \tag{2.1}$$

In this expression, μ_i are the reduced masses of S_i, $i = 1, 2$, r_{ij}, $i, j = 1, 2$, are the distances between a primary and a minor body,

$$\left.\begin{aligned} r_{1i} &= [(x_i - \mu)^2 + y_i^2]^{1/2} \\ r_{2i} &= [(x_i + 1 - \mu)^2 + y_i^2]^{1/2} \end{aligned}\right\},\ i = 1,2$$

ρ is the distance between the bodies S_i,

$$\rho = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$$

and q_i are the radiation pressure parameters of both primaries on the smaller minor body S_2, with

$$q_i = 1 - \beta_i,$$

where β_i are the ratios of the radiation to gravitational forces. Here we assume that both ratios β_i are very small. The symbol ω^* denotes the mean motion of the oblate primaries,

$$\omega^* = \left[\omega_0^2 + \frac{3}{2}\sum_{i=1}^{2} \frac{I_i}{M_i}\right]^{1/2},$$

where $\omega_0 = 1$ is the mean motion of the spherical bodies.

The system is autonomous with four degrees of freedom and it is characterized by seven parameters, that is the mass parameters μ, μ_1, μ_2, the two radiation parameters q_i and the two oblatenesses I_i.

From (1.1) we easily come to the differential equations of motion,

Table Ia
Shift of the equilibrium locations of S_1 near L_1^P ($x_1 > x_1^P$, $I_1 = 0$)

I_2	μ 0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	0.12837 10^{0}	0.13810 10^{0}	0.10001 10^{0}	0.65216 10^{-1}	0.40515 10^{-1}	0.24478 10^{-1}
10^{-2}	0.57830 10^{-1}	0.12318 10^{0}	0.99165 10^{-1}	0.65179 10^{-1}	0.40513 10^{-1}	0.24477 10^{-1}
10^{-3}	0.10449 10^{-1}	0.71483 10^{-1}	0.92038 10^{-1}	0.64842 10^{-1}	0.40499 10^{-1}	0.24476 10^{-1}
10^{-4}	0.11609 10^{-2}	0.20470 10^{-1}	0.62796 10^{-1}	0.61891 10^{-1}	0.40374 10^{-1}	0.24470 10^{-1}
10^{-5}	0.11745 10^{-3}	0.29154 10^{-2}	0.25534 10^{-1}	0.47358 10^{-1}	0.39261 10^{-1}	0.24425 10^{-1}

Table Ib
Shift of the equilibrium locations of S_1 near L_1^P ($x_1 < x_1^P$, $I_1 = 0$)

I_2	μ 0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	0.12833 10^{0}	0.13803 10^{0}	0.99928 10^{-1}	0.65123 10^{-1}	0.40421 10^{-1}	0.24382 10^{-1}
10^{-2}	0.57816 10^{-1}	0.12314 10^{0}	0.99092 10^{-1}	0.65092 10^{-1}	0.40420 10^{-1}	0.24383 10^{-1}
10^{-3}	0.10446 10^{-1}	0.71465 10^{-1}	0.91991 10^{-1}	0.64767 10^{-1}	0.40411 10^{-1}	0.24383 10^{-1}
10^{-4}	0.11605 10^{-2}	0.20463 10^{-1}	0.62774 10^{-1}	0.61839 10^{-1}	0.40297 10^{-1}	0.24381 10^{-1}
10^{-5}	0.11742 10^{-3}	0.29141 10^{-2}	0.25522 10^{-1}	0.47332 10^{-1}	0.39204 10^{-1}	0.24345 10^{-1}

$$\left.\begin{aligned} \ddot{x}_i - 2\omega^* \dot{y}_i &= \frac{1}{\mu_i}\frac{\partial T^*}{\partial x_i} \\ \ddot{y}_i + 2\omega^* \dot{x}_i &= \frac{1}{\mu_i}\frac{\partial T^*}{\partial y_i} \end{aligned}\right\}, \quad i = 1, 2 \tag{2.2}$$

where,

$$T^* = \sum_{j=1}^{2} \mu_j \left\{ \frac{1}{2}\omega^{*2}(x_j^2 + y_j^2) + \frac{1}{2}\frac{\mu_{3-j}}{\rho} \right\} + \mu_1 \sum_{i=1}^{2} q_i \left(\frac{M_i}{r_{i1}} + \frac{I_i}{2r_{i1}^3} \right)$$

$$+\mu_2 \sum_{i=1}^{2} \left[\frac{M_i}{r_{i2}} + \frac{I_i}{2r_{i2}^3} \right]. \tag{2.3}$$

The function T^* does not depend explicitly on time, so the system (2.2) has a Jacobi integral,

$$\frac{1}{2}\sum_{i=1}^{2} \mu_i(\dot{x}_i^2 + \dot{y}_i^2) = T^* - C, \text{ where } C \text{ is a constant.} \tag{2.4}$$

3. Equilibrium Positions of the Minor Bodies S_i

The equilibrium positions of the minor bodies S_i, are the solutions of the algebraic system,

Table IIa

Shift of the equilibrium locations of S_1 near L_2^P ($x_1 > x_2^P$, $I_1 = 0$)

I_2 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.21437\ 10^{0}$	$0.47985\ 10^{0}$	$0.74496\ 10^{0}$	$0.88080\ 10^{0}$	$0.94455\ 10^{0}$	$0.97421\ 10^{0}$
10^{-2}	$0.73058\ 10^{-1}$	$0.22446\ 10^{0}$	$0.53759\ 10^{0}$	$0.78039\ 10^{0}$	$0.89776\ 10^{0}$	$0.95249\ 10^{0}$
10^{-3}	$0.13334\ 10^{-1}$	$0.83890\ 10^{-1}$	$0.22362\ 10^{0}$	$0.57152\ 10^{0}$	$0.79732\ 10^{0}$	$0.90570\ 10^{0}$
10^{-4}	$0.15158\ 10^{-2}$	$0.22149\ 10^{-1}$	$0.77449\ 10^{-1}$	$0.23544\ 10^{-1}$	$0.58829\ 10^{0}$	$0.80526\ 10^{0}$
10^{-5}	$0.15388\ 10^{-3}$	$0.32121\ 10^{-2}$	$0.26763\ 10^{-1}$	$0.64460\ 10^{-1}$	$0.24877\ 10^{0}$	$0.59621\ 10^{0}$

Table IIb

Shift of the equilibrium locations of S_1 near L_2^P ($x_1 < x_2^P$, $I_1 = 0$)

I_2 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.21437\ 10^{0}$	$0.47989\ 10^{0}$	$0.74502\ 10^{0}$	$0.88089\ 10^{0}$	$0.94463\ 10^{0}$	$0.97431\ 10^{0}$
10^{-2}	$0.73069\ 10^{-1}$	$0.22447\ 10^{0}$	$0.53763\ 10^{0}$	$0.78046\ 10^{0}$	$0.89784\ 10^{0}$	$0.95258\ 10^{0}$
10^{-3}	$0.13337\ 10^{-1}$	$0.83902\ 10^{-1}$	$0.22362\ 10^{0}$	$0.57156\ 10^{0}$	$0.79739\ 10^{0}$	$0.90579\ 10^{0}$
10^{-4}	$0.15162\ 10^{-2}$	$0.22156\ 10^{-1}$	$0.77460\ 10^{-1}$	$0.23543\ 10^{-1}$	$0.58833\ 10^{0}$	$0.80533\ 10^{0}$
10^{-5}	$0.15391\ 10^{-3}$	$0.32134\ 10^{-2}$	$0.26773\ 10^{-1}$	$0.64466\ 10^{-1}$	$0.24876\ 10^{0}$	$0.59626\ 10^{0}$

$$\left.\begin{array}{l}\dfrac{\partial T^*}{\partial x_i} = 0\\[2ex] \dfrac{\partial T^*}{\partial y_i} = 0\end{array}\right., \quad i = 1,2. \tag{3.1}$$

For the numerical investigation, we have followed the process which has been described analytically in our paper mentioned in the introduction. For the spherical case there are 14 equilibrium positions which are distributed near the five 'Lagrangian' points of the restricted 3-body photogravitational problem. Six of them lie on both sides of each collinear point and the rest of them are located close to the triangular Lagrangian points L_4^p and L_5^p, in equal pairs, on two approximately orthogonal directions.

Table IIIa

Shift of the equilibrium locations of S_1 near L_3^P ($x_1 > x_3^P$, $I_1 = 0$)

I_2 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.25548\ 10^{0}$	$0.60107\ 10^{0}$	$0.81191\ 10^{0}$	$0.91268\ 10^{0}$	$0.95951\ 10^{0}$	$0.98124\ 10^{0}$
10^{-2}	$0.44353\ 10^{-1}$	$0.26250\ 10^{0}$	$0.60298\ 10^{0}$	$0.81223\ 10^{0}$	$0.91272\ 10^{0}$	$0.95952\ 10^{0}$
10^{-3}	$0.48267\ 10^{-2}$	$0.45414\ 10^{-1}$	$0.26314\ 10^{0}$	$0.60317\ 10^{0}$	$0.81226\ 10^{0}$	$0.91272\ 10^{0}$
10^{-4}	$0.48703\ 10^{-3}$	$0.49394\ 10^{-2}$	$0.45511\ 10^{-1}$	$0.26320\ 10^{0}$	$0.60319\ 10^{0}$	$0.81226\ 10^{0}$
10^{-5}	$0.48747\ 10^{-4}$	$0.49838\ 10^{-3}$	$0.49498\ 10^{-2}$	$0.45521\ 10^{-1}$	$0.26321\ 10^{0}$	$0.60319\ 10^{0}$

Table IIIb

Shift of the equilibrium locations of S_1 near L_3^P ($x_1 < x_3^P$, $I_1 = 0$)

I_2 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.25544\ 10^{0}$	$0.60098\ 10^{0}$	$0.81180\ 10^{0}$	$0.91255\ 10^{0}$	$0.95938\ 10^{0}$	$0.98111\ 10^{0}$
10^{-2}	$0.44346\ 10^{-1}$	$0.26246\ 10^{0}$	$0.60290\ 10^{0}$	$0.81212\ 10^{0}$	$0.91259\ 10^{0}$	$0.95938\ 10^{0}$
10^{-3}	$0.48260\ 10^{-2}$	$0.45408\ 10^{-1}$	$0.26310\ 10^{0}$	$0.60309\ 10^{0}$	$0.81215\ 10^{0}$	$0.91260\ 10^{0}$
10^{-4}	$0.48696\ 10^{-3}$	$0.49387\ 10^{-2}$	$0.45505\ 10^{-1}$	$0.26317\ 10^{0}$	$0.60310\ 10^{0}$	$0.81215\ 10^{0}$
10^{-5}	$0.48740\ 10^{-4}$	$0.49831\ 10^{-3}$	$0.49491\ 10^{-2}$	$0.45514\ 10^{-1}$	$0.26317\ 10^{0}$	$0.60311\ 10^{0}$

Table IVa

Shift of the equilibrium locations of S_1 near L_1^P ($x_1 > x_1^P$, $I_2 = 0$)

I_1 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.24364\ 10^{-1}$	$0.10566\ 10^{-1}$	$0.49178\ 10^{-2}$	$0.22919\ 10^{-2}$	$0.10647\ 10^{-2}$	$0.49296\ 10^{-3}$
10^{-2}	$0.27243\ 10^{-2}$	$0.11872\ 10^{-2}$	$0.55704\ 10^{-3}$	$0.26086\ 10^{-3}$	$0.12148\ 10^{-3}$	$0.56312\ 10^{-4}$
10^{-3}	$0.27573\ 10^{-3}$	$0.12023\ 10^{-3}$	$0.56468\ 10^{-4}$	$0.26460\ 10^{-4}$	$0.12326\ 10^{-4}$	$0.57145\ 10^{-5}$
10^{-4}	$0.27607\ 10^{-4}$	$0.12038\ 10^{-4}$	$0.56546\ 10^{-5}$	$0.26498\ 10^{-5}$	$0.12344\ 10^{-5}$	$0.57230\ 10^{-6}$
10^{-5}	$0.27610\ 10^{-5}$	$0.12040\ 10^{-5}$	$0.56554\ 10^{-6}$	$0.26502\ 10^{-6}$	$0.12346\ 10^{-6}$	$0.57239\ 10^{-7}$

We have solved Equations (3.1) for various system configurations. The results show that the oblateness doesn't affect at all the number and the arrangement of the equilibrium locations, or their stability. It only resumes a slight shift toward the more massive and the more oblate primary. This shift can be considered as the sum of two small displacements. The first is accomplished together with the lagrangian points L^P, as if they constitute a rigid system. The second is a much smaller relative displacement, in which the equilibria of the minor bodies come closer to their neighbour lagrangian point L^P.

For all our applications we considered systems with constant $\mu_1 = 10^{-20}$, $\mu_2 = 10^{-15}$, $\beta_1 = 7.5\ 10^{-6}$, $\beta_2 = 10^{-4}$ and variable mass parameter μ and oblatenesses I_i, $i = 1, 2$. In the Tables I through VI we give the dimensionless shifts (in absolute values) of the collinear equilibrium positions of the body S_1

Table IVb

Shift of the equilibrium locations of S_1 near L_1^P ($x_1 < x_1^P$, $I_2 = 0$)

I_1 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.24371\ 10^{-1}$	$0.10573\ 10^{-1}$	$0.49246\ 10^{-2}$	$0.22988\ 10^{-2}$	$0.10716\ 10^{-2}$	$0.49984\ 10^{-3}$
10^{-2}	$0.27251\ 10^{-2}$	$0.11880\ 10^{-2}$	$0.55781\ 10^{-3}$	$0.26164\ 10^{-3}$	$0.12227\ 10^{-3}$	$0.57097\ 10^{-4}$
10^{-3}	$0.27581\ 10^{-3}$	$0.12031\ 10^{-3}$	$0.56546\ 10^{-4}$	$0.26539\ 10^{-4}$	$0.12406\ 10^{-4}$	$0.57942\ 10^{-5}$
10^{-4}	$0.27615\ 10^{-4}$	$0.12046\ 10^{-4}$	$0.56624\ 10^{-5}$	$0.26577\ 10^{-5}$	$0.12424\ 10^{-5}$	$0.58028\ 10^{-6}$
10^{-5}	$0.27618\ 10^{-5}$	$0.12048\ 10^{-5}$	$0.56632\ 10^{-6}$	$0.26581\ 10^{-6}$	$0.12426\ 10^{-6}$	$0.58037\ 10^{-7}$

Table Va
Shift of the equilibrium locations of S_1 near L_2^P ($x_1 > x_2^P$, $I_2 = 0$)

I_1	μ 0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	0.26752 10^{-1}	0.11075 10^{-1}	0.50337 10^{-2}	0.23225 10^{-2}	0.10768 10^{-2}	0.50107 10^{-3}
10^{-2}	0.33648 10^{-2}	0.13043 10^{-2}	0.58247 10^{-3}	0.26696 10^{-3}	0.12343 10^{-3}	0.57361 10^{-4}
10^{-3}	0.34616 10^{-3}	0.13289 10^{-3}	0.59206 10^{-4}	0.27113 10^{-4}	0.12531 10^{-4}	0.58226 10^{-5}
10^{-4}	0.34717 10^{-4}	0.13315 10^{-4}	0.59305 10^{-5}	0.27155 10^{-5}	0.12550 10^{-5}	0.58314 10^{-6}
10^{-5}	0.34727 10^{-5}	0.13317 10^{-5}	0.59315 10^{-6}	0.27159 10^{-6}	0.12552 10^{-6}	0.58323 10^{-7}

Table Vb
Shift of the equilibrium locations of S_1 near L_2^P ($x_1 < x_2^P$, $I_2 = 0$)

I_1	μ 0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	0.26744 10^{-1}	0.11067 10^{-1}	0.50267 10^{-2}	0.23156 10^{-2}	0.10699 10^{-2}	0.49417 10^{-3}
10^{-2}	0.33637 10^{-2}	0.13035 10^{-2}	0.58165 10^{-3}	0.26616 10^{-3}	0.12264 10^{-3}	0.56571 10^{-4}
10^{-3}	0.34605 10^{-3}	0.13281 10^{-3}	0.59124 10^{-4}	0.27031 10^{-4}	0.12450 10^{-4}	0.57424 10^{-5}
10^{-4}	0.34706 10^{-4}	0.13306 10^{-4}	0.59222 10^{-5}	0.27074 10^{-5}	0.12469 10^{-5}	0.57511 10^{-6}
10^{-5}	0.34716 10^{-5}	0.13309 10^{-5}	0.59232 10^{-6}	0.27078 10^{-6}	0.12471 10^{-6}	0.57520 10^{-7}

from the corresponding locations of the spherical case (the shifts of the body S_2 are quite similar).

4. Areas of the Permissible Motion

The usefulness of the Jacobi integral in clarifying certain general properties of the relative motion of a small body by the construction and investigation of zero-velocity curves in every problem of celestial dynamics was pointed out by many investigators in the past. Here we will confine our interest to the motions where both bodies $S_i, i = 1, 2$ start moving from the x-axis, i.e. with the initial conditions

$$x_{i0} \neq 0,\ y_{i0} = 0,\ \dot{x}_{i0} = 0,\ \dot{y}_{i0} \neq 0,\ i = 1, 2,$$

Table VIa
Shift of the equilibrium locations of S_1 near L_3^P ($x_1 > x_3^P$, $I_2 = 0$)

I_1	μ 0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	0.41549 10^{-2}	0.36644 10^{-3}	0.31942 10^{-4}	0.11021 10^{-5}	0.44025 10^{-5}	0.47309 10^{-5}
10^{-2}	0.52428 10^{-3}	0.44863 10^{-4}	0.39344 10^{-5}	0.96473 10^{-7}	0.49884 10^{-6}	0.53907 10^{-6}
10^{-3}	0.53844 10^{-4}	0.45892 10^{-5}	0.40272 10^{-6}	0.94253 10^{-8}	0.50604 10^{-7}	0.54717 10^{-7}
10^{-4}	0.53990 10^{-5}	0.45998 10^{-6}	0.40368 10^{-7}	0.94009 10^{-9}	0.50634 10^{-8}	0.54795 10^{-8}
10^{-5}	0.54005 10^{-6}	0.46008 10^{-7}	0.40376 10^{-8}	0.94229 10^{-10}	0.50659 10^{-9}	0.54797 10^{-9}

Table VIb
Shift of the equilibrium locations of S_1 near L_3^P ($x_1 < x_3^P$, $I_2 = 0$)

I_1 μ	0.1	0.01	0.001	0.0001	0.00001	0.000001
10^{-1}	$0.41660\ 10^{-2}$	$0.37648\ 10^{-3}$	$0.41887\ 10^{-4}$	$0.88327\ 10^{-5}$	$0.55313\ 10^{-5}$	$0.52029\ 10^{-5}$
10^{-2}	$0.52559\ 10^{-3}$	$0.46015\ 10^{-4}$	$0.50719\ 10^{-5}$	$0.10396\ 10^{-5}$	$0.63712\ 10^{-6}$	$0.59688\ 10^{-6}$
10^{-3}	$0.53978\ 10^{-4}$	$0.47062\ 10^{-5}$	$0.51817\ 10^{-6}$	$0.10588\ 10^{-6}$	$0.64689\ 10^{-7}$	$0.60574\ 10^{-7}$
10^{-4}	$0.54124\ 10^{-5}$	$0.47169\ 10^{-6}$	$0.51931\ 10^{-7}$	$0.10608\ 10^{-7}$	$0.64832\ 10^{-8}$	$0.60670\ 10^{-8}$
10^{-5}	$0.54139\ 10^{-6}$	$0.47179\ 10^{-7}$	$0.51940\ 10^{-8}$	$0.10607\ 10^{-8}$	$0.64824\ 10^{-9}$	$0.60685\ 10^{-9}$

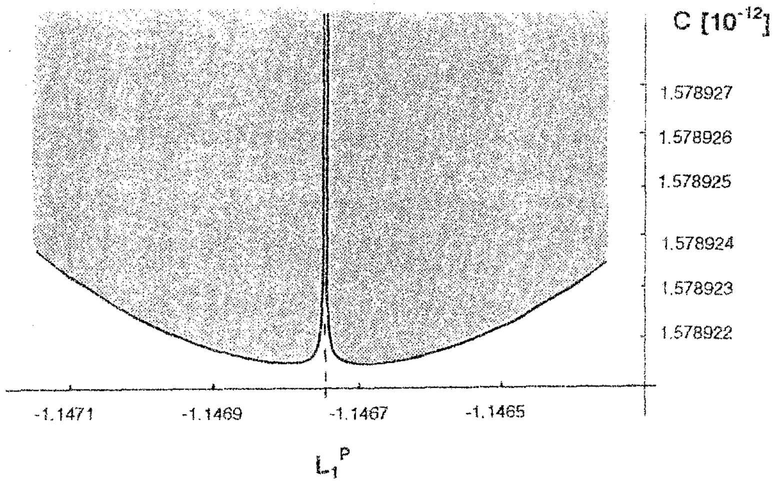

Figure 1. The permitted areas of motion of S_1 close to L_1^P. Mass parameter $\mu = 0.01$. Oblateness parameters: $I_1 = 10^{-4}$, $I_2 = 0$.

while their center of mass, rests on a Lagrangian collinear equilibrium (x_L^P, y_L^P). Since in all cases the coordinates of S_i, $i = 1, 2$ satisfy the relation of their mass center $\underline{r}_c = (x_c, y_c)$,

$$\sum_{i=1}^{2} \mu_i \underline{r}_i = \left(\sum_{i=1}^{2} \mu_i \right) \underline{r}_c$$

the integral of motion takes the form,

$$\frac{1}{2} \sum_{i=1}^{2} \mu_i (\dot{x}_{i0}^2 + \dot{y}_{i0}^2) = f(x_{10}; C) \geq 0,$$

which it is used to determine the region of the plane Oxy, within the body S_1 is permitted to move. Obviously there exist associate regions for the body S_2.

The Figures 1, 2 and 3 show the limit-curves (zero-velocity curves) which separate the areas of the permissible motion of S_1, from those where the motion

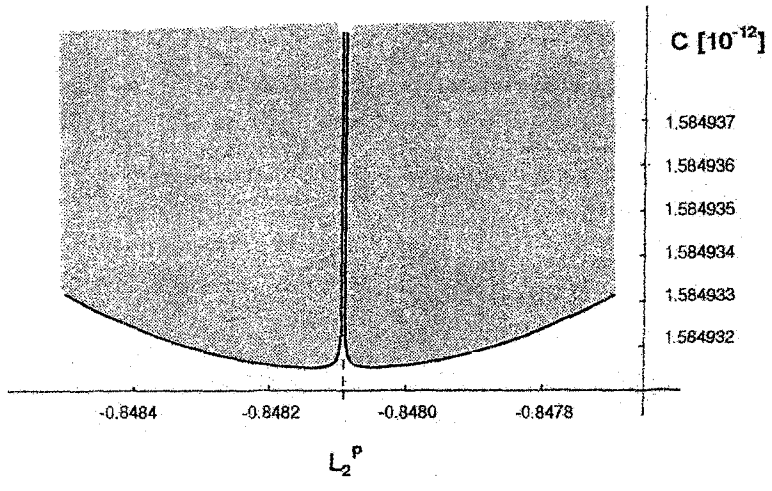

Figure 2. The permitted areas of motion of S_1 close to L_2^P. Mass parameter $\mu = 0.01$. Oblateness parameters: $I_1 = 10^{-4}, I_2 = 0$.

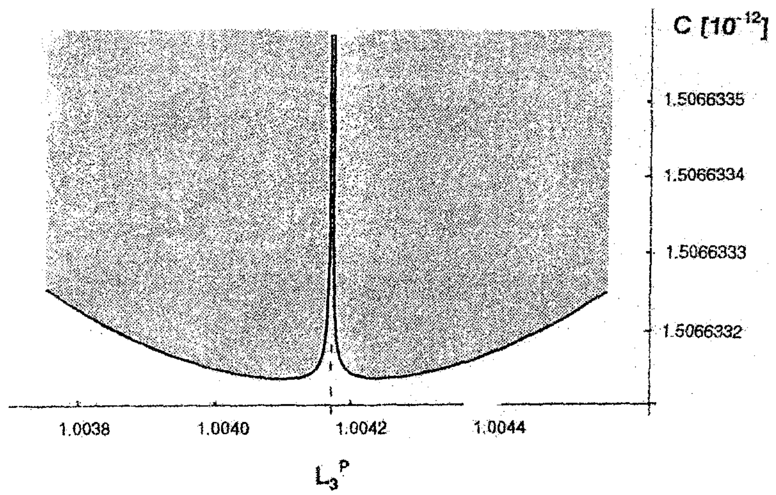

Figure 3. The permitted areas of motion of S_1 close to L_3^P. Mass parameter $\mu = 0.01$. Oblateness parameters: $I_1 = 10^{-4}, I_2 = 0$.

is not allowed (dark areas). The discontinuities which appear, correspond to the positions of the collinear equilibria of the restricted photogravitational three-body problem L_i^P, $i = 1, 2, 3$ and the extrema indicate the locations of the equilibrium points of the minor S_1. Here we note once again that the two minor bodies are assumed to be very close together and so according to Whipple (1984), only a very small area surrounding each 'Lagrangian' equilibrium must be considered.

5. Conclusions

From a careful inspection of the material exposed in the Tables I through VI we can conclude that for those systems where the oblate body is the less massive primary P_2 (that is $I_1 = 0$), the absolute displacements of the equilibrium locations are meaningful even for $\mu = 10^{-5}$ and $I_2 = 10^{-5}$. But for $I_2 < 10^{-10}$ they are almost zero. For those systems where the oblate body is the more massive primary P_1 (that is $I_2 = 0$) the for $\mu < 10^{-2}$ and $I_1 < 10^{-4}$ the displacements become ignorable.

Obviously the zero-velocity curves are getting shifted in a similar way towards the more massive and the more oblate body.

When speaking of our planetary system, we note that if we consider as the primaries the Sun (P_1) and a planet (P_2) then we will account that $I_1 = 0$ and $I_2 \neq 0$, but if we consider as the primaries a planet and its natural satellite, then will have $I_2 = 0$ and $I_1 \neq 0$. For the systems of the first group $\mu < 0.001$ and $I_2 < 2\ 10^{-13}$, so it is out of the question to speak of a noticeable influence of the oblateness. For the systems of the second group $\mu < 0.01$ and $I_1 < 0.004$ and there are some cases where the oblateness effect although very small must be taken into account.

References

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