BIANCHI TYPE-VI_0 MODELS IN SELF-CREATION COSMOLOGY

(Letter to the Editor)

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Abstract. Spatially-homogeneous and anisotropic Bianchi type-VI_0 cosmological models are obtained, in Barber's second self-creation theory of gravitation, both in vacuum and in the presence of perfect fluid with pressure equal to energy density. Some properties of the model are discussed.

1. Introduction

Barber (1982) proposed two self-creation cosmologies by modifying the Brans and Dicke (1961) theory and general relativity. These modified theories create the Universe out of self-contained gravitational and matter fields. Recently, Brans (1987) has pointed out that Barber's first theory is not only in disagreement with experiment but is actually inconsistent, in general. Barber's second theory is a modification of general relativity to a variable G-theory. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples to the trace of the energy-momentum tensor. Hence, the field equations in Barber's second theory are

$$R_{ij} - \tfrac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} \tag{1}$$

and

$$\Box\phi = \frac{8\pi}{3}\lambda T, \tag{2}$$

where λ is a coupling constant to be determined from experiments. The measurements of the deflection of light restricts the value of the coupling to $|\lambda| \lesssim 10^{-1}$. In the limit $\lambda \to 0$ this theory approaches the standard general relativity theory in every respect. Barber (1982) and Soleng (1987) have discussed the Friedmann–Robertson–Walker solutions in Barber's second theory of gravitation.

In this paper, we present Bianchi type-VI_0 cosmological solutions in Barber's second theory of gravitation both in vacuum and in the presence of perfect fluid with pressure equal to energy density. The models represent Bianchi type-VI_0 vacuum and anisotropic Zel'dovich universes in the self-creation cosmology.

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2. Field Equations and the Models

The line element for the spatially-homogeneous Bianchi type-VI_0 universe can be written as

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + B^2(t)\,e^{-2qx}\,dy^2 + C^2(t)\,e^{2qx}\,dz^2\,, \tag{3}$$

where A, B, and C are cosmic-scale functions, and q is a non-zero constant.

The energy-momentum tensor T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)V_iV_j + pg_{ij}\,; \tag{4}$$

together with

$$g_{ij}V^iV^j = -1\,, \tag{5}$$

where V^i is the four-velocity vector of the fluid and p and ρ are the proper pressure and energy density, respectively. By use of the co-moving coordinates the field equations (1) and (2) for the metric (3) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{q^2}{A^2} = -8\pi\phi^{-1}p\,, \tag{6}$$

$$\frac{C_{44}}{C} + \frac{A_{44}}{A} + \frac{C_4A_4}{CA} - \frac{q^2}{A^2} = -8\pi\phi^{-1}p\,, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{q^2}{A^2} = -8\pi\phi^{-1}p\,, \tag{8}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA} - \frac{q^2}{A^2} = 8\pi\phi^{-1}\rho\,, \tag{9}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0\,, \tag{10}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi\lambda}{3}\,(\rho - 3p)\,. \tag{11}$$

Again using correspondence to general relativity and defining equivalent densities and pressures as

$$\rho_{\text{eq}} = \rho/\phi\,, \tag{12}$$

$$p_{\text{eq}} = p/\phi\,, \tag{13}$$

we can write the energy-conservation equation of general relativity (cf. Soleng, 1987) in the form

$$\left(\frac{\rho}{\phi}\right)_4 + \left(\frac{\rho}{\phi} + \frac{p}{\phi}\right)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0\,, \tag{14}$$

where the subscript 4 denotes ordinary differentiation with respect to t.

Equation (9) readily gives

$$B = \gamma C;$$

γ being an integration constant. Without loss of generality we take $\gamma = 1$. Hence, we have

$$B = C. \tag{15}$$

If we introduce the transformation

$$A = e^{\alpha}, \qquad B = e^{\beta}, \qquad dt = AB^2\, dT, \tag{16}$$

the field equations (6)–(10) and Equation (13) reduce to

$$2\beta'' - 2\alpha'\beta' - \beta'^2 + q^2 e^{4\beta} = -8\pi\phi^{-1} p\, e^{2\alpha+4\beta}, \tag{17}$$

$$\alpha'' + \beta'' - 2\alpha'\beta' - \beta'^2 - q^2 e^{4\beta} = -8\pi\phi^{-1} p\, e^{2\alpha+4\beta}, \tag{18}$$

$$2\alpha'\beta' + \beta'^2 - q^2 e^{4\beta} = 8\pi\phi^{-1}\rho\, e^{2\alpha+4\beta}, \tag{19}$$

$$\phi'' = \frac{8\pi\lambda}{3}(\rho - 3p)\, e^{2\alpha+4\beta} \tag{20}$$

and

$$\left(\frac{\rho}{\phi}\right)' + \left(\frac{\rho}{\phi} + \frac{p}{\phi}\right)(\alpha' + 2\beta') = 0; \tag{21}$$

where a dash denotes differentiation with respect to T.

Since the field equations (17)–(21) are highly nonlinear we consider the following physically important cases and obtain the corresponding models of the Universe.

2.1. Vacuum model

When $\rho = p = 0$, the field equations (17)–(21) reduce to vacuum case which give the solution

$$\begin{aligned} A &= \exp(\alpha) = \exp\left[\frac{q^2}{8a_2^2}\exp(4a_2 T) + a_3 T\right], \\ B &= C = \exp(\beta) = \exp[a_2 T], \end{aligned} \tag{22}$$

where the constants a_2 and a_3 are related by

$$2a_2 a_3 + a_2^2 = 0, \qquad a_2 \neq 0. \tag{23}$$

The corresponding vacuum model can now be written in the form

$$ds^2 = -\exp\left[\frac{q^2}{4a_2^2}\exp(4a_2 T) + (4a_2 + 2a_3)T\right] dT^2 +$$

$$+ \exp\left[\frac{q^2}{4a_2^2}\exp(4a_2T) + 2a_3T\right]dx^2 +$$

$$+ \exp[2(a_2T - qx)]\,dy^2 + exp[2(a_2T + qx)]\,dz^2\,; \tag{24}$$

with the scalar field given by

$$\phi = a_1T + b_1\,, \tag{25}$$

in which a_1 and b_1 are constants of integration.

Equation (24) represents the Bianchi type-VI_0 spatially-homogeneous and anisotropic vacuum universe in Barber's second theory. When $q = 0$, Equation (24) gives us the Bianchi type–I vacuum model in this theory. Also it is clear from (25) that when $a_1 = 0$, ϕ = constant and, hence, the model (24) reduces to general relativistic case. It is obvious from Equation (24) that the model has no singularity at $T = 0$. The anisotropic expansion of the Universe with time is evident from Equation (24). Also, the model (24) is formally similar to the Bianchi type-VI_0 vacuum model obtained by Reddy and Venkateswarlu (1989) in Dunn's (1974) scalar-tensor theory of gravitation.

2.2 Zel'dovich universe

When $\rho = p$, the field equations (17)–(21) reduce to the stiff fluid case and yield an exact solution given by

$$A = \exp(\alpha) = \exp[\tfrac{1}{2}(q/2b)^2\exp(4bT) + cT]\,,\quad b \neq 0\,,$$
$$B = C = \exp(\beta) = \exp[bT]\,, \tag{26}$$

$$\phi = \phi_0\exp[(\tfrac{16}{3}\pi\lambda)^{1/2}T]\,, \tag{27}$$

where b, c, and ϕ_0 are constants of integration.

Thus we get the following model for Zel'dovich universe in Barber's second self-creation theory:

$$ds^2 = -\exp[(q/2b)^2\exp(4bT) + (4b + 2c)T]\,dT^2 +$$
$$+ \exp(q/2b)^2\exp(4bT) + 2cT]\,dx^2 +$$
$$+ \exp[2(bT - qx)\,dy^2 + \exp[2(bT + qx)]\,dz^2\,, \tag{28}$$

with the scalar field given by Equation (27).

The pressure p and the energy density ρ in the model (28) are given by

$$p = \rho = -\phi_0\exp[(\tfrac{16}{3}\pi\lambda)^{1/2}T]\exp[-(q/2b)^2\exp(4bT) - (4b + 2c)T]\,. \tag{29}$$

For the reality of p and ρ, the condition $p > 0$, $\rho > 0$ to hold, it is necessary that $\phi_0 < 0$. Here the pressure, density, and the scalar field are not singular at $T = 0$. The volume element in the model (28) is

$$(-g)^{1/2} = \exp[(q/4b)^2\exp(4bT) + (b + \tfrac{1}{2}c)T]\,, \tag{30}$$

which shows the expansion of the Universe with time. When $\lambda \to 0$, the model (28) represents Bianchi type-VI_0 universe for stiff fluid with ϕ = constant. When $q = 0$, Equation (28) reduces to Bianchi type-I Zel'dovich universe in Barber's second theory.

3. Concluding Remarks

Cosmological models both in vacuum and in the presence of stiff fluid have been obtained in Barber's second self-creation theory with the aid of a spatially-homogeneous and anisotropic Bianchi type-VI_0 metric. The corresponding Bianchi type'I models in this theory have been found as special cases. It is observed that the models presented have no singularities at $T = 0$.

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