NEUTRON-PHONON INTERACTION IN NEUTRON STARS: PHONON SPECTRUM OF COULOMB LATTICE

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(Received 9 October, 1995; accepted 12 January, 1996)

Abstract. The phonon excitation spectrum of Coulomb lattice in the neutron star crusts is studied by solving Dyson's equation for phonons. It is shown that a strong renormalization of the phonon spectrum occurs at densities $\rho \geq \rho_s/4$ for the crustal matter compositions with spherical nuclei, which imply relatively small nuclear mass numbers and charges. It is shown that, the lattice becomes unstable against density fluctuations above a critical density of the order of $\sim \rho_s/3$, where $\rho_s \simeq 2.6 \times 10^{14}$ g cm^{-3} is the nuclear saturation density. The neutron quasiparticle spectrum and the virtual mass of a nucleus are briefly discussed.

1. Introduction

The ground state configuration and the equation of state of matter in the crusts of neutron stars for densities above the neutron drip density have been extensively studied in the late 60's and early 70's. The basic approaches and results were established in the pioneering works by Bethe, Börner and Sato (1970), Baym, Bethe and Pethick (1971), Buchler and Barkat (1971), Arponen (1972), Ravenhall, Bennett and Pethick (1972), Negele and Vautherin (1973), Sahakian (1974) and a comprehensive description of the subject now can be found in the book by Shapiro and Teukolsky (1983). According to the standard picture the matter in the density range $4 \times 10^{13} \leq \rho \leq 10^{14}$ g cm^{-3} is composed of neutron rich nuclei, relativistic electrons, and free neutron gas. The Fermi energies of fermions are of the order $1-20$ MeV, while the typical interior temperatures are $1-10$ keV, therefore the system is effectively at zero temperature. At the temperatures below the Lindemann melting temperature the nuclei are arranged in a bcc Coulomb lattice, while the low density neutron gas feels the intervening space between nuclear clusters. Because of weak screening the relativistic electrons are distributed almost uniformly. With increasing density neutron rich nuclei with larger mass numbers become successively stable in the β-equilibrated matter. The bound nucleons merge into the continuum state at densities close to the nuclear saturation density $\rho_s \sim 2.6 \times 10^{14}$ g cm^{-3}, when the adjacent nuclei come in contact. Though the equation of state of the matter was fairly good established in these studies, the ground state composition was subject to modification depending on the details of the form of the total energy functional adopted. Approaches based on the liquid-drop model of the nucleus (Bethe *et al.*,

Astrophysics and Space Science **236:** 267–276, 1996.

1970; Baym *et al.*, 1971; Buchler and Barkat, 1971; Arponen, 1971; Ravenhall *et al.*, 1972), used extrapolations from the semi-empirical mass formula of the Bethe-Weizsaecker type with corrections for the surface and Coulomb energies. The Hartree-Fock calculations of the finite nuclei by Negele and Vautherin (1973) allowed also for the nuclear shell effects. The total energy functional was commonly supplemented by a parametrization of the bulk neutron matter energy following from a certain many-body calculation.

It turned out that, the differences in the treatment of the nuclear surface energy within the differential Thomas-Fermi theory give rise to large differences in the parameters of the stable nuclei, like the mass number, charge, and the surface thickness. Since the Coulomb energy of a unit cell is coupled to the surface energy by a purely geometrical condition, these differences affected the predictions for Coulomb term of the total energy functional as well. For these reasons we shall further consider three different compositions, namely those of Baym *et al.* (1971, hereafter BBP), Arponen (1972, hereafter An), and Negele and Vautherin (1973, hereafter NV). The nuclei implied by the two latest compositions, in contrast to the first one, have considerably smaller mass numbers and charges, and resemble large laboratory nuclei. We shall see that, the coupling of the neutron liquid to the phonon modes of the nuclear lattice, which was commonly neglected in the previous studies, has different impact on these three compositions.

In the present paper it will be shown that the excitation spectrum of phonons can be strongly modified by the neutron-phonon coupling for models of matter composition in the neutron star's inner crusts with not very large nuclei. This modification can lead to a lattice instability under definite conditions.

It should be stressed that, the recent developments showed that in the high density region of the crust the nuclei might not be of the spherical shape, but rather they can assume peculiar and geometrically extended forms (Lorenz, Ravenhall and Pethick, 1993; Oyamatsu, 1993). These structures can support phonon modes which considerably differ from the phonon spectrum of a three-dimensional Coulomb crystal. Therefore, we will restrict here to the compositions with spherical nuclei and extend our results to nonspherical case in a subsequent study.

2. Formal Theory

The total Hamiltonian of the system in the second quantized form reads:

$$H = H_0 + H_{nph} + H_{nn}, \tag{1}$$

where

$$H_0 = \sum_{\mathbf{k}\sigma} \epsilon_0(\mathbf{k}) a^+_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{q<q_D} \omega_0(q) b^+_{\mathbf{q}s} b_{\mathbf{k}s}, \tag{2}$$

is the Hamiltonian of the noninteracting system,

$$H_{nph} = \sum_{\mathbf{k},\sigma,\mathbf{q},s} i\alpha(q) a^{+}_{\mathbf{k}+\mathbf{q}\sigma} a_{\mathbf{k}\sigma} (b_{\mathbf{q}s} + b^{+}_{-\mathbf{q}s}), \tag{3}$$

describes the neutron-phonon interaction, and

$$H_{nn} = \sum_{\mathbf{k}'\mathbf{k}\mathbf{p}\sigma} \langle \mathbf{k}' + \mathbf{p}, \mathbf{k} - \mathbf{p}\sigma | V | \mathbf{k}, \mathbf{p}\sigma \rangle a^{+}_{\mathbf{k}'+\mathbf{p}\sigma} a^{+}_{\mathbf{k}-\mathbf{p}\sigma} a_{\mathbf{k}\sigma} a_{\mathbf{k}'\sigma} \tag{4}$$

describes the neutron-neutron interaction due to the nuclear force. Here $a^{+}_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ are the creation and destruction operators of a neutron with wave vector $\mathbf{k}$ and spin σ, $\epsilon_0(\mathbf{k})$ is the free particle energy, $b^{+}_{\mathbf{q}s}$ and $b_{\mathbf{q}s}$ are the creation and destruction operators of a phonon with wave vector $\mathbf{q}$ and polarization s, $\omega_0(q)$ being the unperturbed phonon frequency, the sum over the wave numbers of phonons is restricted to the Debye wavelength q_D, V is the two-body neutron-neutron interaction due to the direct nuclear force. Neutron-phonon interaction, Equation (3), is treated in it's simplest form by utilizing the well-known Fröhlich Hamiltonian (Fröhlich, 1952). The coupling constant, $\alpha(q)$, in this model is given as

$$\alpha(q) = \left(\frac{q}{2M^* N_A c_s} \right)^{1/2} \Gamma_{nA}(q), \tag{5}$$

where M^* is the actual mass of nucleus including it's virtual mass, N_A is the number of nuclei in the given volume, and $\Gamma_{nA}(q)$ is the transition matrix for neutron-nucleus scattering as a function of momentum transfer q. We consider an isotropic crystal in which the phonon frequencies are separated into one longitudinal and two transverse modes and restrict to the normal processes only. In this case the neutrons interact only with longitudinally polarized phonons and the sum over phonon polarizations should be dropped. A linear acoustic phonon spectrum, $\omega_0(q) = c_s q$, where c_s is the nonrenormalized sound velocity is assumed. The unperturbed neutron quasiparticle spectrum is assumed to have the form $\epsilon_0(k) = k^2/2m_n$, where m_n is the bare mass of the neutron. The unperturbed longitudinal phonon frequency is assumed to be the plasma frequency modified by the screening of relativistic electrons:

$$\omega_0(q) = \left[\frac{4\pi Z^2 e^2 n_A}{M^*} \frac{q^2}{q^2 + k_{TF}^2} \right]^{1/2}, \tag{6}$$

where Z is the nuclear charge, k_{TF} is the relativistic Thomas-Fermi screening length, and n_A is the density of the nuclei.

In this work we shall apply the zero-temperature Green's functions formalism to the ground state of the system described by the Hamiltonian (1). Particularly, the neutron-phonon part of the interaction in the inner crust of a neutron star will be treated in a close analogy to the approach of Migdal (1958) to a strongly interacting electron-phonon system in metals at zero temperature, (see also Abrikosov, Gorkov, Dzyaloshinski, 1963). The Green's functions of neutrons and phonons can be introduced in terms of time-ordered products of creation and distraction operators in the standard way

$$G(\mathbf{k}\sigma, t) = -i\langle T a_{\mathbf{k}\sigma}(t) a^+_{\mathbf{k}\sigma}(0)\rangle, \quad D(\mathbf{q}, t) = -i\langle T b_{\mathbf{q}}(t) b^+_{\mathbf{q}}(0)\rangle. \tag{7}$$

Respective Dyson's equations in the momentum representation are

$$G(\mathbf{k}, \epsilon) = G^{(0)}(\mathbf{k}, \epsilon) + G^{(0)}(\mathbf{k}, \epsilon)[\Sigma_{nph}(\mathbf{k}, \epsilon) + \Sigma_{nn}(\mathbf{k}, \epsilon)]G(\mathbf{k}, \epsilon), \tag{8}$$

$$D(\mathbf{q}, \omega) = D^{(0)}(\mathbf{q}, \omega) + D^{(0)}(\mathbf{q}, \omega)\Pi(\mathbf{q}, \omega)D(\mathbf{q}, \omega), \tag{9}$$

where $G^{(0)}$ and $D^{(0)}$ are the Green's functions of noninteracting neutrons and phonons,

$$G^{(0)}(\mathbf{k}, \epsilon) = \frac{1}{\epsilon - \epsilon^0_{\mathbf{k}} + i\delta \mathrm{sign}(k - k_{Fn})}, \tag{10}$$

$$D^{(0)}(\mathbf{q}, \omega) = \frac{\omega_0^2(\mathbf{q})}{\omega^2 - \omega_0^2(\mathbf{q})}, \tag{11}$$

G and D are the full Green's functions of neutrons and phonons respectively. The self energy operators, Σ_{nph} due to the neutron-phonon interaction, Σ_{nn} due to the direct nuclear interaction, and the polarization operator Π are explicitly given by expressions:

$$\Sigma_{nph}(k) = i g_{nph} g_{nn} \int G(k - q)D(q)\Gamma_{nph}(k - q, k; q)\frac{d^4 q}{(2\pi)^4}, \tag{12}$$

$$\Sigma_{nn}(k) = -2i \int G(q)\Gamma_{nn}(k, q; k, q)\frac{d^4 q}{(2\pi)^4}, \tag{13}$$

$$\Pi(q) = -2i g_{nph} g_{nn} \int G(k)G(k - q)\Gamma_{nph}(k, k - q; q)\frac{dk^4}{(2\pi)^4}, \tag{14}$$

where four-momenta $k \equiv (\mathbf{k}, \epsilon)$ and $q \equiv (\mathbf{q}, \omega)$ are introduced. Here $\Gamma_{nph}(k, q)$ is the neutron-phonon interaction vertex function and the effective coupling constant is $g_{nph} = [\alpha^2(q)/\omega_0(q)]^{1/2}$. Vertex g_{nn} schematically takes into account neutron-neutron correlations, owing to the nuclear component of the interaction. At the densities of interest it is determined by the pion exchange mechanism. The self energy Σ_{nn} is given in the ladder approximation, where Γ_{nn} is the transition matrix in the particle-particle channel.

2.1. VERTEX FUNCTIONS AND NEUTRON QUASIPARTICLE SPECTRUM

The excitation spectrum of the neutron liquid is given by the poles of the retarded Green's function, which is the solution of the Dyson's Equation (8). If one neglects for a moment the neutron-phonon interaction the full retarded Green's function is given as

$$G(\varepsilon_k) = \frac{1}{\varepsilon(k) - \epsilon(k) - \Sigma_{nn}(\varepsilon(k)) + i\delta}, \tag{15}$$

and the quasiparticle energies are thus defined as

$$\varepsilon_k = \frac{k^2}{2m} + \Sigma_{nn}(\varepsilon(k)) \simeq v_F(p - p_F), \tag{16}$$

where $v_{Fn} \equiv k_F/m_n^*$ is the quasiparticle velocity on the Fermi surface and

$$\frac{m_n^*}{m_n} = \left(1 + \frac{m_n}{k}\frac{\partial\Sigma_{nn}(\varepsilon_k)}{\partial k}\bigg|_{k=k_F}\right)$$

is the neutron quasiparticle effective mass due to the nuclear interaction and k_F is the Fermi momentum. Further evaluation requires the solution of the nonrelativistic version of the Bethe-Salpeter equation for the scattering amplitude Γ_{nn}. We will not consider this problem here, (see e.g. Sedrakian *et al.*, 1994), but note that for the realistic neutron-neutron interactions the effective mass varies in a range $0.6 \leq m_n^*/m_n \leq 0.8$ for the densities of interest.

When the neutron-phonon interaction is switched on, neutron quasiparticle spectrum is further renormalized. In the low frequency limit, $\omega \ll \omega_D$, the real part of the neutron self-energy Σ_{nph} acquires the simple form $\Sigma_{nph} = -\lambda\omega$, where $\lambda \equiv \lambda_0/(1 - 2\lambda_0)$, (Abrikosov *et al.*, 1963). Therefore the neutron quasiparticle spectrum takes the form

$$\varepsilon(k) \simeq \tilde{v}_{Fn}(k - k_F), \quad \tilde{v}_{Fn} = \frac{v_{Fn}}{1 + \lambda}, \tag{17}$$

which shows that, the quasiparticle velocity on the Fermi surface is reduced due to the neutron-phonon interaction. This modification of quasiparticle spectrum can be accounted for by defining a neutron quasiparticle mass $m_n^{**} = (1 + \lambda)m_n^*$.

Let us turn to the vertex functions. It can be proved that Migdal's theorem is valid for the present system; (for the relevant parameters see the next section). According to this theorem the first order correction to the vertex function $\Gamma(k, q)$ is of the order of $g\lambda_0\omega_q^0/\epsilon_{Fn}$, where

$$\lambda_0 = \frac{\alpha^2(q)}{\omega_0(q)}\nu(\epsilon_{Fn}) \tag{18}$$

is the dimensionless neutron-phonon coupling constant, (the Fröhlich parameter), with $\nu(\epsilon_{Fn}) = m_n^* k_{Fn}/2\pi^2$ being the density of states of neutrons on the Fermi surface per direction of spin. Therefore, the neutron-phonon vertex function can be replaced by a contact interaction,

$$\Gamma_{nph} = g_{nph}\left[1 + O\left(\frac{\lambda_0\omega_0(q)}{\epsilon_{Fn}}\right)\right],$$

when the parameter $\lambda_0\omega_0(q)/\epsilon_F$ is small compared with unity. The remaining vertex g_{nn}, treated as a constant, schematically takes into account neutron-neutron correlations. This type of correlations were estimated on the basis of the Migdal's (1978) theory of finite Fermi systems, (see also Voskresensky, 1993). Because the phenomenological constants, (Landau-Migdal parameters), are not firmly known for pure neutron matter, this estimates led to values in the range $0.4 \leq g_{nn} \leq 1$. We shall further adopt the value $g_{nn} = 1$. Numerical results given in the next section can be appropriately scaled for other values of g_{nn}.

In closing of this section let us consider the hydrodynamically generated virtual mass of a nucleus immersed in the neutron liquid. We model the nucleus as an absolutely solid sphere of radius R_N, and assume that the neutron liquid is incompressible. A nucleus, subject to oscillations in the neutron fluid, will acquire a virtual mass due to the hydrodynamical interaction with the ambient fluid. In other words, in the reference frame moving with the nucleus, the neutron flow past the nucleus will induce a backflow of neutron fluid. The kinetic energy of the backflow will contribute to the energy of the system and can effectively be incorporated in the energy functional by introducing the virtual mass. The solution of the Laplas equation $\Delta\phi = 0$, with the boundary condition assuming liquid at rest at the infinity, (i.e. $r \gg R_N$), is $\phi = \mathbf{A} \cdot \boldsymbol{\nabla}(1/r)$. For the sphere of radius R_N $\mathbf{A} = (1/2)R_N^3\mathbf{u}$, where $\mathbf{u}$ is the macroscopic fluid velocity. Then the neutron fluid energy is $E = (\rho_n/2)[4\pi\mathbf{A}\cdot\mathbf{u} - (4\pi/3)R_N^3u^2] = (2\pi/3)\rho_n R_N^3 \cdot (u^2/2)$. The coefficient of $u^2/2$ can be identified as the virtual mass of the nucleus, which turns out to be equal to the half of the neutron fluid mass displaced by the nucleus. The hydrodynamical correction to the bare mass of the nucleus, M, can be expressed through the ratio $M^*/M = 1 + (\rho_n/2\rho_s)$, where we have assumed that the nucleon density in the nucleus is roughly equal to the saturation density of the nuclear matter.

2.2. Phonon Spectrum and Lattice Instability

To find the phonon dispersion relation we have to evaluate the polarization function, Equation (14). To this end, note that the neutron-phonon interaction affects only a narrow momentum and frequency ranges, $|\mathbf{k} - \mathbf{k}_F| \leq \omega_D/v_{Fn}$ and $\omega \leq \omega_D$, while the integration in Equation (14) involves much broader range of variables. Therefore $G(\mathbf{k},\omega)$ can be approximately replaced by $G^{(0)}(\mathbf{k},\omega)$, the corrections being small, of the order of $\lambda_0\omega_0(q)/\epsilon_{Fn}$. Then the evaluation of the polarization function is straightforward. In the limit of small energy transfer $\omega \ll qv_{Fn}$, one finds

$$\Pi(q,\omega) = -\lambda_0\left[f\left(\frac{q}{k_{Fn}}\right) + i\pi\frac{m_n^*|\omega|}{k_{Fn}q}\theta(2k_{Fn} - q)\right], \tag{19}$$

where

$$f(x) = 1 + \left[\frac{1-x^2}{2x}\right]\ln\left|\frac{1+x}{1-x}\right|$$

is the Lindhard function. From the Dyson Equation (9) it follows that

$$D(\mathbf{q},\omega)=\frac{\omega_0^2(q)}{\omega^2-\omega_0^2(q)[1+\Pi(q,\omega)]} \tag{20}$$

where the renormalized phonon spectrum is given by the real part of the pole of the $D(\mathbf{q},\omega)$ function

$$\omega^2(q)=\omega_0^2(q)[1+\ \mathrm{Re}\,\Pi(q,\omega)], \tag{21}$$

whereas it's imaginary part describes the damping of the phonon modes due to the neutron-phonon interaction. Using the fact that in the interval $0<x<1$ the $f(x)$ function can be approximated as $f(x)=2-x^2$, the relation (21) in the limit $x \ll 1$ reduces to

$$\omega(q)=\omega_0(q)\sqrt{1-2\lambda_0}. \tag{22}$$

It can be seen that the renormalization of the phonon spectrum is most important in the long wave length limit, $q \to 0$. Particularly, from Equation (22) it follows that the phonon frequencies are pure imaginary in this limit when $\lambda_0 \geq 0.5$, therefore any density fluctuation in the system will grow exponentially leading to a lattice instability. Introducing the energies of neutron-nucleus interaction, $\mathcal{E}_{nA}=|\Gamma_{nA}(q)|\sqrt{N_A}$, and Coulomb interaction between nuclei, $\mathcal{E}_{\mathrm{Coul.}}=4\pi Z^2e^2/(q^2+k_{TF}^2)$, the condition for the onset of lattice instability, $\lambda \geq 0.5$, can be written in a more transparent way

$$\frac{\mathcal{E}_{nA}^2}{\mathcal{E}_{\mathrm{Coul.}}\epsilon_{Fn}}\frac{n_n}{n_A} \geq \frac{4}{3}, \tag{23}$$

where n_n is the density of the neutron liquid. If the energy density scales are measured in the units of the Fermi energy density of the neutron liquid, $\epsilon_{Fn}n_n$, the condition (23) is the simple statement, that lattice becomes unstable when the ratio of the *energy density* of the Coulomb interaction between nuclei, $\mathcal{E}_{\mathrm{Coul.}}n_A/(\epsilon_{Fn},n_n)$, to that of neutron-nucleus interaction, $\mathcal{E}_{nA}n_n/(\epsilon_{Fn}n_n)$, is smaller than 3/4 of the energy density of the neutron-nucleus interaction itself. Thus, for a given composition, the stability of the lattice is essentially controlled by the ratio of the Coulomb energy of the lattice to the energy of the neutron-nucleus interaction.

3. Estimates

Next we shall estimate the neutron-nucleus interaction energy. The Born approximation can not be applied to neutron-nucleus scattering in the present case, because the typical optical potential depth is of the order of 40 MeV, while the neutron Fermi energies are less than 20 MeV. The exact amplitude Γ_{nA} can be calculated by solving the appropriate Bethe-Salpeter equation, or alternatively it can be determined from the differential neutron-nucleus scattering cross-section extracted from the experiment. Here the second alternative will be chosen. The relation of the transition matrix, $\Gamma_{nA}(q)$, to the differential cross-section is

$$|\Gamma_{nA}(q)|^2 = \left(\frac{m_n^*}{2\pi\hbar^2}\right)^2 \frac{d\sigma}{d\Omega}(\epsilon_{Fn}, \theta) \tag{24}$$

where θ is the scattering angle and is related to the momentum transfer as $q = 2k_{Fn}\sin(\theta/2)$. As shown above the instability of the lattice sets on first for the small transferred momenta, i.e. in the limit, $\theta \to 0^0$. Also, because the condition $q \leq q_D \ll k_{Fn}$ is fulfilled, it is sufficient to evaluate the scattering cross-section in the forward scattering limit ($\theta \to 0^0$). The parameter values in neutron star crust for compositions of BBP, NV, and An respectively are given in the table. For an order of magnitude estimate we first calculate the neutron-phonon coupling constant λ_0^C, using a typical constant value of the scattering cross section $d\sigma/d\Omega \sim 10$ barn/sr for $\theta = 0^0$. An improved differential neutron-nucleus cross-section at $\theta = 0^0$ as a function of nuclear mass number A was obtained by fitting to a sequence of experimental differential scattering cross-sections for neutron scattering from elements $^{91}Zr_{40}$, $^{118}Sn_{50}$, $^{180}Ta_{73}$, $^{207}Pb_{82}$ $^{208}Bi_{83}$ and $^{232}Th_{90}$ at the energy 7 MeV (Perey and Buck, 1962). The polynomial fit formula used reads

$$\frac{d\sigma}{d\Omega}(x) = (59.973 - 235.827x + 301.665x^2 - 115.805x^3)\ \text{barn}, \tag{25}$$

where $x = A/A_0$, A is the mass number of a nucleus and $A_0 = 232$. The fit completely covers the nuclei mass numbers present in the composition of NV. It should be stressed that, since the neutron-nucleus interaction is almost charge independent, the fact that in neutron stars one finds nuclei with considerable neutron excess compared with the laboratory ones, plays negligible role as far as the range of the mass numbers fitted are in the same range as those implied by the composition. In the energy range $4 \leq E \leq 20$ MeV the cross section increases with increasing energy of the incident neutron (Perey and Buck, 1962). In the essential energy range of interest the correction to the fit due to the energy dependence of the scattering cross-section is less than 10%, therefore we will use the fit at a fixed energy. To extend the results for nuclei with $A > 232$ an extrapolation is needed. Since for energies of interest the surface scattering is dominant, the hard sphere scattering argument was used in this case to obtain the cross-section by scaling the fit result for $^{207}Pb_{82}$ by a factor $(A/207)^{2/3}$; (here A is the mass number of a nucleus under consideration). The coupling constant λ_0 given in the table is calculated using the fit for the scattering cross-section. It should be noted that the virtual mass does not contribute to the neutron-phonon coupling constant, since $\lambda_0 \sim (M^* c_s^2)^{-1}$. (Typically the virtual mass of the nucleus is less than 20% of it's bare mass). As noted above, the neutron-neutron correlations can change the value of the constant by a factor of 2, however present knowledge of Landau parameters for the neutron matter does not allow to draw more definitive conclusions.

By inspecting the table it can be seen that, there are large differences in the numerical values of the neutron-phonon coupling constant for compositions considered. The origin of these differences is the very different predictions for the nuclear mass numbers and nuclear charges, especially between the composition of

Table I
Density dependence of parameter values and the neutron-phonon coupling in neutron star crust

n (cm^{-3})	A	k_{Fn} (fm^{-1})	q_D (fm^{-1})	ω_D (MeV)	ϵ_{Fn} (MeV)	c_s/c	λ_0^C	λ_0
3.81e+36	171	0.36	0.07	0.19	2.45	0.06	3.51e-04	5.36e-04
6.71e+36	191	0.44	0.08	0.21	3.41	0.06	5.36e-04	1.77e-03
9.15e+36	211	0.49	0.08	0.23	4.07	0.06	1.77e-03	5.10e-03
4.88e+37	546	0.86	0.12	0.38	10.40	0.06	5.10e-03	1.21e-02
7.93e+37	991	1.01	0.13	0.44	13.77	0.06	1.21e-02	2.09e-02
1.05e+38	1635	1.11	0.13	0.47	18.11	0.06	2.09e-02	2.90e-02
3.73e+36	159	0.45	0.08	0.23	2.60	0.015	0.005	0.002
5.77e+36	161	0.52	0.09	0.28	3.30	0.015	0.008	0.002
8.91e+36	164	0.61	0.09	0.28	4.20	0.015	0.010	0.003
2.04e+37	192	0.82	0.11	0.32	6.50	0.015	0.017	0.010
4.57e+37	183	1.09	0.11	0.28	10.9	0.013	0.038	0.018
7.89e+37	232	1.30	0.14	0.27	19.5	0.010	0.112	0.112
4.42e+36	145	0.48	0.13	0.40	3.08	0.015	0.03	0.01
8.60e+36	163	0.60	0.15	0.48	4.35	0.016	0.04	0.02
1.49e+37	185	0.73	0.17	0.54	5.60	0.016	0.06	0.05
2.37e+37	211	0.85	0.20	0.61	6.88	0.015	0.10	0.11
3.54e+37	247	0.97	0.23	0.66	8.23	0.015	0.17	0.19
5.04e+37	277	1.09	0.27	0.70	9.82	0.013	0.37	0.45
6.89e+37	349	1.21	0.34	0.68	11.8	0.010	1.03	1.46
7.99e+37	438	1.27	0.38	0.63	13.0	0.008	1.74	2.86
8.48e+37	482	1.29	0.41	0.60	13.6	0.007	2.44	4.27

The upper set of parameters corresponds to the composition of BBP, the intermediate one to NV, and the lower one to An.

BPP and that of NV and An. This can be traced back to the different treatments of the nuclear surface energy term. While for compositions of NV and An the nuclei resemble large laboratory nuclei ($A \sim 200$), with small nuclear charge ($Z \leq 50$) and rather small surface thickness, the nuclei in the composition of BPP have considerable bigger mass numbers and charges, as well as much larger surface thickness.

For the composition of NV neutron-phonon coupling plays a moderate role. It has a maximal effect at the last point where $\lambda_0 = 0.112$ and it does not change the sound velocity considerably. For Arponen's composition the phonon spectrum is essentially modified already for densities $\rho_s/4$ and the lattice becomes unstable against the density fluctuations at density $\rho_s/3$. This result indicates that the system can be unstable against phase transition to a new state with different parameters of the nuclear lattice structure, or it may be unstable against phase separation into a pure neutron and nuclear phases. For composition of BBP the neutron-phonon coupling is negligible. The different degrees of importance of the neutron-phonon coupling for compositions examined can be traced to the different density dependence of the Z/A ratio they imply.

4. Conclusions

To conclude, it was shown that the neutron-phonon interaction plays an important role for the Arponen's composition of neutron star crust, leading to a strong renormalization of phonon frequencies and to a lattice instability at densities of the order of third of nuclear saturation density. This effects plays a moderate role for the composition of Negele and Vautherin, which has similar characteristics of nuclei as Arponen's composition, but includes the nuclear shell effects. On the other hand the neutron-phonon interaction is found negligible for the composition of Baym *et al.* An extension of present results to the case of non-spherical nuclei and more precise treatment of the neutron-nucleus interaction will be given in a separate work.

Acknowledgements

The author is grateful to Professors C.J. Pethick and D.N. Voskresensky for discussions and comments, to Professor G. Röpke and Dr. D. Blaschke for their interest in this work, and to the referee for a number of useful suggestions. This work was in part supported by BMFT grant No RO140 and by a NASA grant NAGW-3591.

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