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Sound Propagation, Density, and Viscosity in Liquid ^{3}He*

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We have measured the temperature dependence of the velocity and the attenuation of sound in the range 60–700 mK. The maximum in the sound velocity observed by Abraham and Osborne is confirmed. The viscosity deduced from the attenuation measurements is in good agreement with that of Abel, Anderson, and Wheatley but differs from that of Betts, Keen, and Wilks. The sound velocity as a function of pressure at 150 mK was also measured and from this the low-temperature pressure dependence of the density was determined.

1. INTRODUCTION

In this paper we present direct measurements of the velocity and attenuation of sound in liquid ^{3}He at the vapor pressure in the temperature range 60–700 mK; measurements were also made of the sound velocity as a function of pressure at 150 mK.

Recently, the density of ^{3}He as a function of pressure and temperature was measured by Abraham and Osborne.[1] They noticed that at the vapor pressure, the velocity of sound as a function of temperature exhibits a maximum at about 515 mK. Even though the precision of measurement was about 0.1 %, the accuracy of the velocity of sound is much lower because it was obtained by taking derivatives of the experimental data. The present measurements confirm the existence of a maximum in the velocity of sound, although at a lower temperature and of smaller magnitude.

An unexplained difference exists between the viscosity calculated from the ultrasonic measurements of Abel, Anderson, and Wheatley[2] and the

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viscosity measurement of Betts, Keen, and Wilks[3] (BKW). The temperature dependence of the attenuation of sound presented here is in agreement with that shown by the measurements of Abel *et al.*[2] The normalization procedure used to obtain the absolute attenuation, in order to calculate viscosity, will be discussed later.

2. EXPERIMENT AND DISCUSSION

The technique used in this experiment is completely described in previous communications.[4–6] A lower frequency transducer than that used in previous work was employed (5.485 MHz) in order to have a measurable signal through a 1.01-cm path at 60 mK. However, the lower frequencies used required a longer delay line.* The signal-to-noise ratio was poorer than in previous experiments due partially to the poorer impedance match to the lower frequency transducers. In addition, a small signal leaked through the quartz spacer; this was troublesome at the high attenuations. Measurements were made at two frequencies, 5.485 and 16.524 MHz, to verify the results. In principle, the acoustic interferometer employed here is capable of a precision of 2×10^7. In this experiment that resolution was not achieved because of the aforementioned interference and the poorer signal-to-noise ratio. At the vapor pressure the resolution was 2×10^5, or a velocity change of ± 0.4 cm/sec. For the measurements as a function of pressure, the precision was limited by the Heise gauge used, which could be read only to ± 0.2 psi, which resulted in an uncertainty of ± 10 cm/sec.

*Ad-Yu Electronics, Inc., Passaic, New Jersey, model 20A2CX.

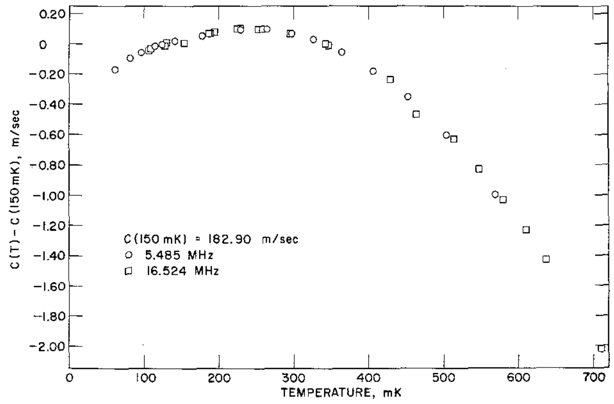

Fig. 1. The velocity of sound in liquid ^{3}He at the vapor pressure as a function of temperature. The data were normalized at 182.36 m/sec at 485 mK.

TABLE I
Experimental Values of Sound Velocity in ^{3}He Under Pressure

P, atm	c, m · sec^{-1}
$f = 5.485$ MHz	
0.51	192.1
0.74	196.2
1.39	206.5
2.72	224.9
4.83	249.2
8.02	279.3
13.69	321.5
13.74	321.5
18.78	352.2
21.91	368.6
26.16	389.3
$f = 5.485$ MHz	
0.57	193.7
1.29	205.2
2.10	216.4
3.19	230.8
4.62	247.3
6.19	262.6
8.57	284.1
11.46	306.1
15.99	335.8
19.73	357.5
24.12	379.5
26.33	390.3
28.23	398.7
$f = 16.524$ MHz	
0.51	192.0
0.68	194.5
1.36	205.2
2.04	215.3
2.72	224.9
3.74	237.0
5.58	256.7
6.80	268.2
8.45	282.8
10.34	298.0
12.66	314.8
15.58	333.6
18.58	351.2
21.43	366.6
23.27	375.7
25.10	384.3
27.22	394.3
28.31	399.0
28.72	401.0

This experiment measures only the change in transit time of the acoustic signal from which a velocity change can be calculated. In order to obtain absolute velocities, we have normalized our data to the measurements of Laquer, Sydoriak, and Roberts.[7] The entry in their Table III at 485 mK, $c = 182.36$ m/sec, was taken as the normalizing value. In Fig. 1 we show the velocity of sound in liquid ^{3}He as a function of temperature. The maximum is clearly evident; however, it occurs at 250 mK and the velocity drops by 0.2% at 60 mK. Furthermore, the maximum does not coincide with the density maximum. These results are in qualitative agreement with those of Abraham and Osborne.[1]

Measurements of the velocity as a function of pressure were made at 150 mK. This temperature was selected because there was very little temperature dependence above 3 atm, as shown by the work of Abraham and Osborne, and a stronger signal was obtained at 16.524 MHz. The experimental data as a function of pressure at 150 mK are presented in Table I. One can readily derive the density as a function of pressure from the velocity measurements through the relation

$$(\partial P/\partial \rho)_S = c^2 \tag{1}$$

At 150 mK, $C_P/C_V \cong 1$, so that the isothermal and isentropic sound velocities are almost identical. The analysis used here to obtain $\rho(P) - \rho_0$, the density at pressure P minus the density at the vapor pressure, differs slightly from that reported in Ref. 6. As seen in Fig. 2 a plot of c vs. $\rho - \rho_0$ for ^{4}He

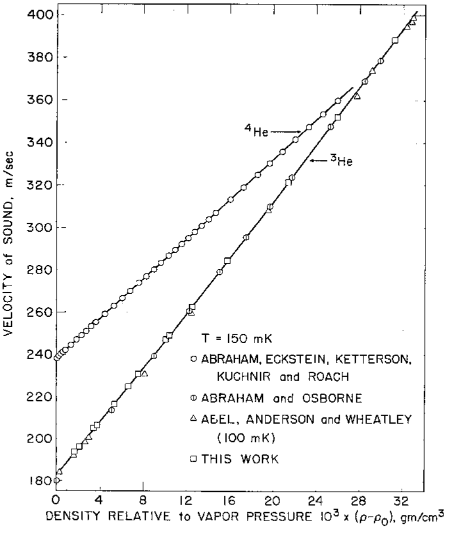

Fig. 2. Velocity of sound in liquid ^{3}He and liquid ^{4}He as a function of density relative to the density at the vapor pressure. The value of $c_3(\rho_0)$ is 182.9 m/sec.

yields a straight line to a high order of approximation, and a similar plot for ^{3}He shows a very slight curvature. Therefore, to fit the ^{3}He data, the velocity of sound was fitted to a second-degree polynomial:

$$c = c_0 + A_1(\rho - \rho_0) + A_2(\rho - \rho_0)^2 \tag{2}$$

From Eqs. (1) and (2) the expression for P is

$$\begin{aligned} P = {} & c_0^2(\rho - \rho_0) + c_0A_1(\rho - \rho_0)^2 + \tfrac{1}{3}[A_1^2 + 2c_0A_2](\rho - \rho_0)^3 \\ & + \tfrac{1}{2}A_1A_2(\rho - \rho_0)^4 + \tfrac{1}{5}A_2^2(\rho - \rho_0)^5 \end{aligned} \tag{3}$$

Equations (2) and (3) together constitute a relation between c and P through the parameter $\rho - \rho_0$. A least-squares fit, as described in Ref. 6, yielded for A_1 and A_2 the values 6.2441×10^5 cm^4/g·sec and 1.06777×10^6 cm^7/g^2·sec, respectively. For c_0 we used 182.9 m/sec, obtained by adding the measured delay between 485 and 150 mK to the Laquer, Sydoriak, and Roberts[7] value at 485 mK, 182.36 m/sec, and for ρ_0 the value 0.08191 g/cm^3 (Ref. 1).

Figure 2 shows the velocity of sound in both ^{3}He and ^{4}He as a function of $\rho - \rho_0$. We emphasize again that a nearly linear relationship exists in both cases. Also plotted are the experimental points of Abel, Anderson, and Wheatley[2] (AAW) and Abraham and Osborne[1] (AO). Though the measurements of AAW were made at 100 mK the negligible temperature dependence of the velocity permits us to make the comparison at 150 mK. It can be seen that the measurements of AAW are in fair agreement with these results and that the derived values of AO are in excellent agreement except at the vapor pressure, where it is only fair. The smooth values of P, c, and $\rho - \rho_0$ are given in Table II.

The attenuation measurements were made from 60 mK upward at 5.485 MHz and from 100 mK upward at 16.524 MHz. Because of the low frequency we could not use our continuously variable (wave guide beyond cutoff) attenuator but had to employ a step attenuator instead.* This resulted in a loss of precision so that the attenuation measurements could be made only to ±0.2 dB. As was pointed out in Ref. 4, it is unreliable to measure the height of successive echoes to arrive at a value for the absolute attenuation. Since only changes in the attenuation as a function of temperature were determined, a normalization procedure was required.

The attenuation measurements were normalized with the well-known classical hydrodynamic formula:

$$\alpha = 2\omega^2\eta/3\rho c^3$$

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*Weinschel Engineering, Gaithersburg, Md., model 64. A 10dB fixed-pad attenuator was placed before and after the step attenuator to insure a proper impedance match and thus a correct reading.

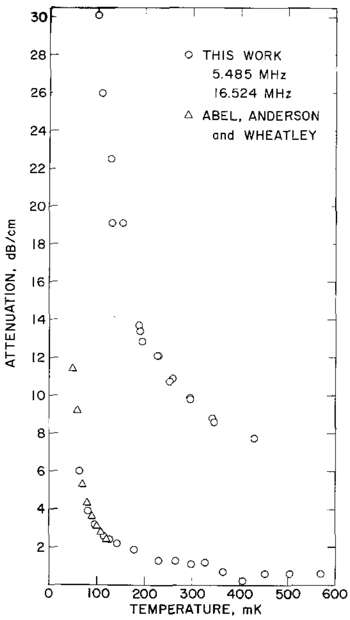

Fig. 3. The absolute attenuation of sound in liquid ^{3}He as a function of temperature at the vapor pressure and at two frequencies.

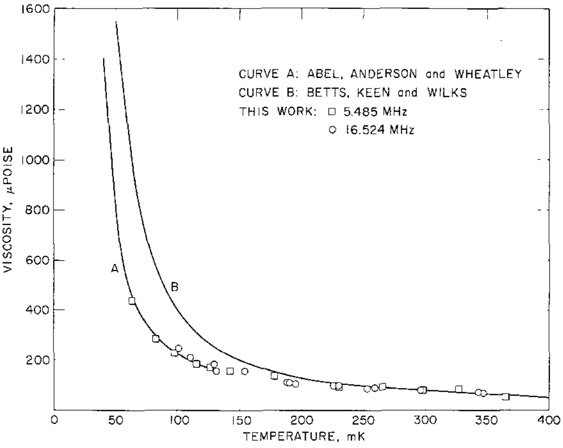

Fig. 4. The viscosity of liquid ^{3}He as a function of temperature. The data points, this work; curve A, Abel, Anderson, and Wheatley; curve B, Betts, Keen, and Wilks.

TABLE II
Pressure, Sound Velocity and Density Change, Calculated from Eqs. (2) and (3)

P, atm	ρ, g·cm^{-3}	$\Delta\rho$, g·cm^{-3}	c, m·sec^{-1}
0.0	0.08191	0	182.90
1.00	0.08467	0.00276	200.21
2.00	0.08702	0.00511	215.08
3.00	0.08908	0.00717	228.23
4.00	0.09093	0.00902	240.08
5.00	0.09261	0.01070	250.93
6.00	0.09416	0.01225	260.97
7.00	0.09559	0.01368	270.33
8.00	0.09693	0.01502	279.12
9.00	0.09820	0.01629	287.43
10.00	0.09939	0.01748	295.31
11.00	0.10052	0.01861	302.82
12.00	0.10160	0.01969	310.00
13.00	0.10263	0.02072	316.89
14.00	0.10362	0.02171	323.50
15.00	0.10457	0.02266	329.88
16.00	0.10549	0.02358	336.04
17.00	0.10637	0.02446	342.00
18.00	0.10722	0.02531	347.77
19.00	0.10804	0.02613	353.37
20.00	0.10884	0.02693	358.81
21.00	0.10962	0.02771	364.11
22.00	0.11037	0.02846	369.26
23.00	0.11110	0.02919	374.29
24.00	0.11182	0.02991	379.20
25.00	0.11251	0.03060	384.00
26.00	0.11319	0.03128	388.68
27.00	0.11386	0.03195	393.27
28.00	0.11450	0.03259	397.76
29.00	0.11514	0.03322	402.16
30.00	0.11576	0.03385	406.47

where α, ω, η, ρ, and c are, respectively, the attenuation in cm^{-1}, the circular frequency, the viscosity in poise, the density in g/cm^3, and the sound velocity in cm/sec. The normalizing value for η was 44 μP at 550 mK taken from Betts, Osborne, Welber and Wilks,[8] which is 0.61 dB/cm at 5.485 MHz and 5.5 dB/cm at 16.524 MHz. Since our precision is ± 0.2 dB (5.485 MHz) it makes little difference whether we normalized at 600 or 1000 mK or whose experimental values are taken. The normalization constant is only additive and will not change the shape of the curve $\alpha(T)$. The attenuation data for both frequencies are plotted in Fig. 3. Also plotted are the data of AAW normalized to agree with our data at 90 mK. AAW measure 5.2 dB/cm at 90 mK and this normalization corresponds to subtracting 2.0 dB/cm from each of their values.

More informative than the attenuation plot is the plot of viscosity vs. temperature illustrated in Fig. 4. The AAW renormalized data are plotted as curve A and BKW data as curve B in order to simplify comparison. The points shown were calculated from our attenuation measurements at 5.485 and 16.524 MHz. The agreement between the two frequencies is very good as is the agreement with AAW. Both sets of acoustic measurements disagree, however, with BKW. Our normalization of the AAW data results in $\eta(T) \propto T^{-2.4}$ at low temperatures. This indicates that the asymptotic T^{-2} behavior of the Landau theory must occur at temperatures below ~45 mK.

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