FREE CONVECTION EFFECTS ON THE FLOW PAST A MOVING VERTICAL INFINITE POROUS PLATE

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Abstract. An analysis of the effects of free convection currents on the flow field of an incompressible viscous fluid past an infinite porous plate, which is uniformly accelerated upwards in its own plane, is presented, when the fluid is subjected to a variable suction (or injection) velocity. It is assumed that this normal velocity at the porous plate varies as $t'^{-1/2}$, where t' denotes time. The equations governing the flow are solved numerically, using two-point boundary value shooting techniques.

1. Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate in its own plane was first studied by Stokes (1851). Because of its practical importance, it has been extended to bodies of different shapes by a number of researchers. Amongst them are Illingworth (1950), Stewartson (1951), Hall (1969), and Elliott (1969).

Soundalgekar (1977) has studied the free convection flow past an impulsively started vertical impermeable plate when it is cooled or heated by the free convection currents. Gupta *et al.* (1979) have investigated the unsteady flow and heat transfer in free convection flow past a vertical impermeable plate which is uniformly accelerated upwards in its own plane. Because of the importance of suction (or injection) for the boundary layer control in the field of fluid mechanics, we study here the effects of free convection currents on the flow field of an incompressible viscous fluid past an infinite porous plate, which is uniformly accelerated upwards in its own plane, when the fluid is subjected to a variable suction (or injection) velocity. It is assumed that the normal velocity at the porous plate varies as $t'^{-1/2}$, where t' is the time.

2. Analysis

In this work we study the two dimensional unsteady flow and heat transfer in free convection flow of an incompressible viscous fluid past a vertical porous plate, which is uniformly accelerated upwards in its own plane. We take the x'-axis along the plate in the upward direction and the y'-axis normal to the plate. The fluid and the plate are initially at rest and at some temperature T'_∞. But at $t' > 0$ the plate starts moving with velocity $u' = c't'$ (where c' is a positive constant) in its own plane and its temperature is instantaneously raised or lowered to $T'_w \gtrless T'_\infty$, which is thereafter maintained

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constant. Under these assumptions the problem is described by Equations (1)–(3) and the boundary conditions (4), as follows:

$$\frac{\partial v'}{\partial y'} = 0\,, \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\,\frac{\partial u'}{\partial y'} = g_{x'}\,\beta(T' - T'_\infty) + \nu\,\frac{\partial^2 u'}{\partial y'^2}\,, \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\,\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p}\,\frac{\partial^2 T'}{\partial y'^2}\,, \tag{3}$$

$$\begin{aligned} &t' \leqq 0: \quad u'(y',t') = 0, \qquad T'(y',t') = T'_\infty\,,\\ &t' > 0: \begin{cases} u'(0,t') = c't'\,, & T'(0,t') = T'_w\,,\\ u'(\infty,t') = 0\,, & T'(\infty,t') = T'_\infty\,, \end{cases} \end{aligned} \tag{4}$$

where u', v' are the velocity components in x' and y' direction, respectively, t' the time, $g_{x'}$ the acceleration due to gravity, β the coefficient of volumetric expansion, T' the fluid temperature inside the thermal boundary layer, ν the kinematic viscosity, ρ the density, κ the thermal conductivity, and c_p the specific heat of the fluid at constant pressure.

Equation (1) gives

$$v' = -\,v_0(t') = -\,\alpha\left(\frac{\nu}{t'}\right)^{1/2},$$

where α represents the velocity of suction or injection at the porous plate according as $\alpha > 0$ or $\alpha < 0$, respectively.

Using the non-dimensional transformations

$$\begin{aligned} &t = t'\left(\frac{c'^2}{\nu}\right)^{1/3}, \qquad y = y'\left(\frac{c'}{\nu^2}\right)^{1/3},\\ &u = \frac{u'}{(\nu c')^{1/3}}\,, \qquad T = \frac{T' - T'_\infty}{T'_w - T'_\infty}\,,\\ &\mathrm{Gr} = \frac{1}{c'}\,(T'_w - T'_\infty)g_{x'}\,\beta \quad \text{(Grashof number)},\\ &\mathrm{P} = \frac{\rho\nu c_p}{\kappa} \quad \text{(Prandtl number)}, \end{aligned} \tag{5}$$

equations (2) and (3) become

$$\frac{\partial u}{\partial t} - \alpha t^{-1/2}\,\frac{\partial u}{\partial y} = \mathrm{Gr}\,T + \frac{\partial^2 u}{\partial y^2}\,, \tag{6}$$

$$P \frac{\partial T}{\partial t} - \alpha P t^{-1/2} \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} . \tag{7}$$

The boundary conditions (4) reduce to

$$t > 0: \begin{cases} u(0, t) = t , & T(0, t) = 1 , \\ u(\infty, t) = 0 , & T(\infty, t) = 0 . \end{cases} \tag{8}$$

Solutions of Equations (6) and (7), for small t, can now be obtained if we put

$$u = tf(n), \qquad T = T(\eta) , \tag{9}$$

where

$$\eta = \frac{y}{2t^{1/2}} .$$

Introducing Equations (9) into Equations (6) and (7), we get

$$f'' + 2(\eta + \alpha)f' - 4f + 4\,\mathrm{Gr}\,T = 0 , \tag{10}$$

$$T'' + 2(\eta + \alpha)PT' = 0 . \tag{11}$$

The corresponding boundary conditions become

$$f(0) = 1 , \qquad T(0) = 1 , \qquad f(\infty) = 0 , \qquad T(\infty) = 0 . \tag{12}$$

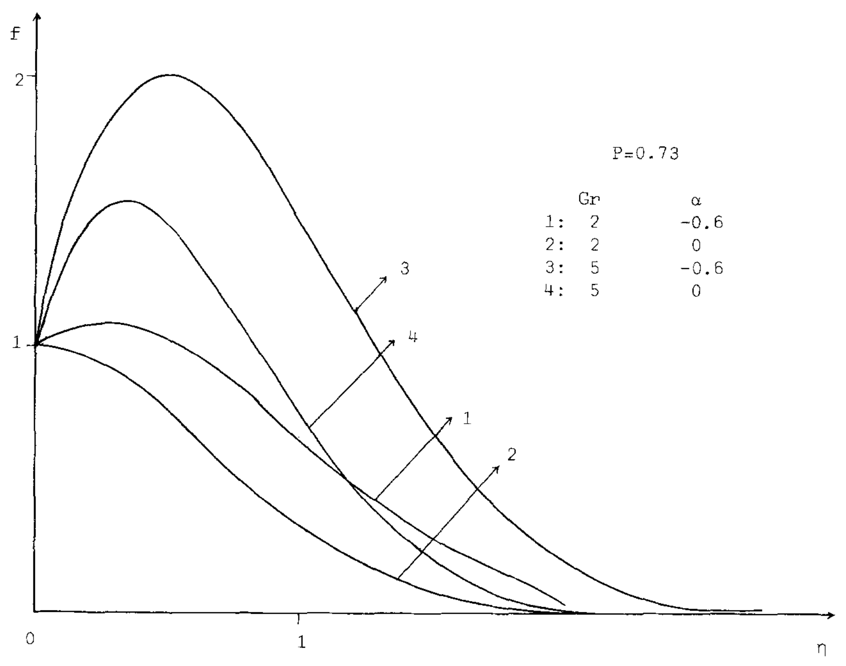

Fig. 1. Velocity profiles.

Equations (10) and (11) governed by the boundary conditions (12) are solved numerically, using two-point boundary-value shooting techniques.

From Figure 1, we observe that when the parameter α increases the velocity decreases. From Figure 2, we also observe that when the parameter α increases the temperature decreases.

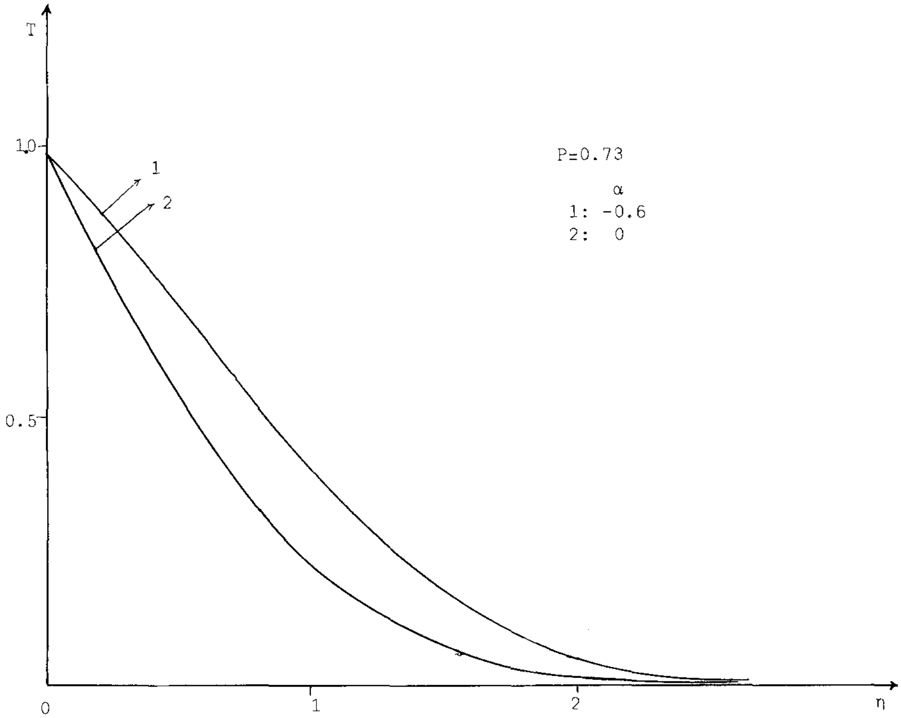

Fig. 2. Temperature profiles.

Once we know the velocity and temperature field, it is important to calculate the skin-friction and the rate of heat transfer which are given, in the dimensionless form, by the expressions (13) and (14), respectively

$$\tau = \tfrac{1}{2}t^{1/2}f'(0)\,, \tag{13}$$

$$\mathrm{Nu} = -\tfrac{1}{2}t^{-1/2}T'(0)\,, \tag{14}$$

where τ is the skin friction and Nu is the Nusselt number.

By considering Equation (13) and Table I, we find that when the parameter α increases the skin-friction decreases. By considering Equation (14) and Table II, we also observe that when the parameter α increases the heat transfer increases.

TABLE I

Values of the $f'(0)$

$P = 0.73$			
Gr = 2		Gr = 5	
$\alpha = -0.6$	$\alpha = 0$	$\alpha = -0.6$	$\alpha = 0$
0.754	0.175	4.40	3.82

TABLE II

Values of the rate of $T'(0)$

$P = 0.73$	
$\alpha = -0.6$	$\alpha = 0.0$
-0.484	-0.966

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