A PLANE-SYMMETRIC UNIVERSE IN THE PRESENCE OF ZERO-MASS SCALAR FIELDS

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Abstract. A non-static plane-symmetric cosmological model in the presence of zero-mass scalar fields is obtained when the source of the gravitational field is a perfect fluid with pressure equal to energy density. Some properties of the model are discussed.

1. Introduction

The study of interacting fields, one of the fields being a zero-mass scalar field, is basically an attempt to look into the yet unsolved problem of the unification of the gravitational and quantum theories. Considerable interest has been focussed on a set of field equations representing zero-mass scalar fields coupled with the gravitational field for the past two decades. Bergmann and Leipnik (1957), Bramhachary (1960), Das (1962), Stephenson (1962), Gautreau (1969), Rao *et al.* (1972), Reddy and Rao (1983), Patel (1975), Chatterjee and Roy (1982), Singh (1980), and Reddy and Venkateswarlu (1987) are some of the authors who have studied various aspects of interacting fields in the framework of general relativity.

In this paper we present a non-static plane-symmetric universe in the presence of zero-mass scalar fields when the source of the gravitational field is a perfect fluid with pressure equal to energy density. Some physical properties of the cosmological model are also discussed. The model represents a plane-symmetric Zel'dovich universe in the presence of zero-mass scalar fields.

2. Field Equations and the Model

Einstein field equations corresponding to zero-mass scalar fields are given by

$$\begin{aligned} R_{ij} - \tfrac{1}{2}g_{ij}R &= -T_{ij} - (\phi_{,i}\phi_{,j} - \tfrac{1}{2}g_{ij}\phi_{,k}\phi^{,k}), \\ \phi_{;k}^{;k} &= 0; \end{aligned} \tag{1}$$

where ϕ is the zero-mass scalar field; T_{ij}, the stress energy tensor of the matter; and comma and semi-colon denote partial and covariant differentiation, respectively. We consider the Riemannian space-time described by the line element

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2\,d\theta^2 - S^2\,dz^2), \tag{2}$$

where r, θ, z are the usual cylindrical polar coordinates and h and S are functions of t only. It is well-known that this line element is plane-symmetric. The plane symmetry assumed implies that the scalar field ϕ shares the same symmetry. By a straightforward calculation we find that the non-vanishing components of the Einstein tensor, G^i_j for the metric (2), are

$$e^{2h}G^1_1 = G^2_2 e^{2h} = -\frac{S_{44}}{S} - M\,,$$

$$G^3_3 e^{2h} = \frac{2S_4 h_4}{S} - M\,, \tag{3}$$

$$G^4_4 e^{2h} = 2(h_{44} - h_4^2) - M\,;$$

where $M = 2h_{44} + h_4^2 + 2S_4 h_4/S$. A suffix 4 following an unknown function denotes hereafter an ordinary differentiation with respect to t.

The energy-momentum tensor T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)V_i V_j - pg_{ij}\,, \tag{4}$$

together with

$$g_{ij}V^i V^j = 1\,; \tag{5}$$

where V^i is the four-velocity vector of the fluid and p and ρ are the proper pressure and energy density, respectively. Using Equations (3), (4), and (5) we find that the field equations (1) for the line element (2) can be written as

$$e^{-2h}\left(\frac{S_{44}}{S} + 2h_{44} + h_4^2 + \frac{2S_4 h_4}{S} + \frac{\phi_4^2}{2}\right) + p = 0\,, \tag{6}$$

$$e^{-2h}\left(2h_{44} + h_4^2 + \frac{\phi_4^2}{2}\right) + p = 0\,, \tag{7}$$

$$e^{-2h}\left(3h_4^2 + \frac{2S_4 h_4}{S} - \frac{\phi_4^2}{2}\right) - \rho = 0\,, \tag{8}$$

and

$$\phi_{44} + \phi_4\left(\frac{S_4}{S} + 2h_4\right) = 0\,, \tag{9}$$

Equations (6)–(9) are four equations in five unknowns, h, S, p, ρ, and ϕ. For a complete determinacy of the system one more condition is needed. One way to obtain it is to use an equation of state. The other alternative is a mathematical assumption on the space-time and then to discuss the physical nature of the Universe. Here we shall confine ourselves to the equation of state

$$\rho = p\,, \tag{10}$$

representing a stiff matter or Zel'dovich fluid. The models with $\rho = p$ are important in relativistic cosmology for the description of very early stages of the Universe.

Subtracting Equation (7) from (6) we get

$$e^{-2h}\left(\frac{S_{44}}{S}+\frac{2S_4h_4}{S}\right)=0\,,$$

which on integration yields

$$e^{-2h}S_4 = C_1\,. \tag{11}$$

Adding Equations (6) and (8) and using (10) we obtain

$$e^{-2h}\left(\frac{S_{44}}{S}+2h_{44}+4h_4^2+\frac{4S_4h_4}{S}\right)=0\,,$$

which on integration gives

$$e^{-2h}S = C_2t + C_3\,. \tag{12}$$

Equation (9) can be readily integrated to give

$$e^{2h}S\phi_4 = C_4\,. \tag{13}$$

Equations (11)–(13) ultimately yield the exact solutions

$$\begin{aligned} &e^{2h} = (C_2t + C_3)^{1-C_1/C_2}\,, \qquad S = C_0(C_2 + C_3)^{C_1/C_2}\,,\\ &e^{\phi} = \phi_0(C_2t + C_3)^{C_4/C_2}\,; \end{aligned} \tag{14}$$

where C_i's and ϕ_0 are constants of integration. Without any loss of generality we can set $\phi_0 = 1$, $C_0 = 1$, $C_1 = a$, $C_2 = 1$, and $C_4 = b$, where a and b are constants. Thus we get the following model for a Zel'dovich universe in the presence of zero-mass scalar fields after a proper choice of coordinates:

$$\mathrm{d}s^2 = T^{1-a}\,(\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - T^{2a}\,\mathrm{d}z^2)\,. \tag{15}$$

3. Some Properties of the Model

The pressure p and the energy density ρ in the model are given by

$$p = \rho = T^{a-3}\left[\frac{(3+a)(1-a)}{4} - \frac{b^2}{2}\right], \tag{16}$$

and the zero-mass scalar field ϕ is given by

$$\exp(\phi) = T^b\,. \tag{17}$$

On physical grounds $\rho > 0$, $p > 0$ we are led to

$$(3+a)(1-a) > 2b^2 \quad \text{and} \quad T > 0\,.$$

When $b = 0$, the zero-mass scalar field ϕ vanishes and then the model (15) is identical to the stiff matter plane-symmetric universe of general relativity.

For $(3 + a)(1 - a) = 2b^2$, the model reduces to empty space-time in the presence of zero-mass scalar fields discussed by Patel (1975).

For $a = 1$ and $b = 0$, the model reduces to empty space-time discussed by Bera (1969).

Equation (15) is plane-symmetric as it obviously admits four-parameter group of motions. If $a = 0$, the model becomes conformal to flat space-time and reduces to a particular case of the Lemaître universe.

4. Conclusions

The plane-symmetric cosmological model obtained here represents a Zel'dovich universe in the presence of zero-mass scalar fields. Equation (15) serves as a generalisation of the vacuum models obtained by Bera (1969) and Patel (1975). The model is formally similar to the plane-symmetric cosmological model in Lyra's manifold discussed by Reddy and Innaiah (1986).

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