VISCOUS FLUID UNIVERSE FILLED WITH STIFF FLUID IN GENERAL RELATIVITY

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Abstract. We have investigated two stiff-fluid models in which the material distribution is that of viscous fluid. In the first model, the coefficient of shear viscosity is assumed to be constant while in the second model the coefficient of shear viscosity is proportional to the rate of expansion in the model. The paper also discusses some physical and geometrical aspects of the model. The behaviour of the model in absence of viscosity is also discussed.

1. Introduction

Stiff-fluid models are interesting in the sense that for such models speed of sound is equal to the velocity of light and its governing equations have the same characteristics as those of gravitational field. Johari *et al.* (1981) have investigated spatially-homogeneous and anisotropic models containing a barotropic fluid. Mohanty *et al.* (1982) have obtained a cylindrically-symmetric model for Zel'dovich fluid distributions and with $\Lambda = 0$ in general relativity. A class of shear-free perfect fluid models with equation of state in general relativity is investigated by Collins and White (1984). An exact solution for non-static perfect fluid spheres with shear and equation of state is obtained by Vanden and Wils (1985). Lorentz (1982) has obtained an exact Bianchi type-I cosmological model for dust distribution with electromagnetic field where $\Lambda \neq 0$. Prakash and Roy (1979) have investigated an anisotropic incoherent fluid cosmological model in general relativity. General solutions for spatially-homogeneous cosmological models representing incoherent matter have been obtained by Heckmann and Schucking (1962). Roy and Singh (1977) have obtained a non-static plane-symmetric space-time filled with disordered radiation. A non-static cylindrically-symmetric space-time with two degrees of freedom representing a distribution of disordered radiation, has been obtained by Bali (1985).

In this paper we have obtained two stiff-viscous fluid models in general relativity. In both models the coefficient of bulk viscosity is assumed to be zero and $\Lambda = 0$. In the first model the coefficient of shear velocity is assumed to be constant and in the second model the coefficient of shear viscosity is assumed to be proportional to scalar of expansion θ. Both models represent expanding shearing non-rotating and non-degenerate Petrov type-I in general. The flow vector is geodetic. Some other physical and geometrical aspects of the models are also discussed.

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We consider the space-time in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2, \tag{1.1}$$

where A, B, C are functions of t alone.

We assume coefficient of bulk viscosity to be zero. Thus energy-momentum tensor for viscous distribution is given by

$$T_i^j = (\varepsilon + p)v_i v^j + p g_i^j + M_i^j,$$

where M_i^j is the energy-momentum tensor given by Landau and Lifshitz (1963) as

$$M_i^j = -(v_{i;}^j + v_{;i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j) + \tfrac{2}{3}\eta v_{;l}^l (g_i^j + v_i v^j), \tag{1.2}$$

together with

$$g_{ij} v^i v^j = -1, \tag{1.3}$$

ε is the density; p, the pressure; η, the coefficient of shear viscosity; and v^i the flow vector satisfying Equation (1.3). We assume the coordinates to be co-moving, so that

$$v^1 = 0 = v^2 = v^3 \quad \text{and} \quad v^4 = A^{-1}.$$

The field equations

$$R_i^j - \tfrac{1}{2} R g_i^j = -8\pi T_i^j \tag{1.4}$$

for the line-element (1.1) lead to

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC}\right] = 8\pi\left[p - 2\eta\frac{A_4}{A^2} + \frac{2}{3}\eta v_{;l}^l\right], \tag{1.5}$$

$$\frac{1}{A^2}\left[-\frac{C_{44}}{C} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2}\right] = 8\pi\left[p - 2\eta\frac{B_4}{AB} + \frac{2}{3}\eta v_{;l}^l\right], \tag{1.6}$$

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2}\right] = 8\pi\left[p - 2\eta\frac{C_4}{AC} + \frac{2}{3}\eta v_{;l}^l\right], \tag{1.7}$$

$$\frac{1}{A^2}\left[\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC}\right] = 8\pi\varepsilon. \tag{1.8}$$

The suffix 4 after A, B, and C denotes ordinary differentiation with respect to t.

2. Solution of Field Equations

For a complete determination of the set (1.5)–(1.8) we need an extra condition. We assume that the model is filled with stiff fluid of viscous fluid distribution so that we have $\varepsilon = p$. From Equations (1.5) and (1.8), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} = \frac{16\pi\eta A}{3}\left[2\frac{A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right]. \tag{2.1}$$

From Equations (1.5) and (1.7), we have

$$\left(\frac{A_4}{A}\right)_4 + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_{44}}{B} - \frac{B_4C_4}{BC} = 16\pi\eta A\left(\frac{B_4}{B} - \frac{A_4}{A}\right) \tag{2.2}$$

and

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 16\pi\eta A\left(\frac{C_4}{C} - \frac{B_4}{B}\right). \tag{2.3}$$

Equations (2.2), (2.3), and (2.4) lead to

$$A = LB\left(\frac{B}{C}\right)^K \tag{2.4}$$

and

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 16\pi\eta LB\left(\frac{B}{C}\right)^K\left(\frac{C_4}{C} - \frac{B_4}{B}\right), \tag{2.5}$$

where L and K are constants of integration.

Equations (2.1) and (2.4) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} = \frac{16\pi\eta L(2K+1)}{3}B\left(\frac{B}{C}\right)^K\left[\frac{B_4}{B} - \frac{C_4}{C}\right], \tag{2.6}$$

where two cases arise:

Case (*i*) η = constant.

Case (*ii*) η/θ = constant = l (say).

We consider both the cases one by one.

Case (*i*). When η = constant

From Equations (2.5) and (2.6), we have

$$(BC)_{44} = -\frac{2K+1}{3}(CB_4 - BC_4)_4\,, \tag{2.7}$$

which leads to

$$(BC)_4 = -\frac{2K+1}{3}(CB_4 - BC_4) + M\,, \tag{2.8}$$

M being constant of integration.

Equation (2.8) is not integrable in general. Here we assume $M = 0$ or $K = -\frac{1}{2}$, when we take $M = 0$, we find that the scalar of expansion vanishes and reality conditions $\varepsilon + p > 0$ and $\varepsilon + 3p > 0$ are also not satisfied. The metric in this case reduces to the form

$$\mathrm{d}S^2 = \frac{L^2R^3}{T^2}\left[\mathrm{d}X^2 - \frac{\mathrm{d}T^2}{\left(\log \frac{P}{T^\gamma}\right)^2}\right] + T^{2(K-1)/(2K+1)}\,\mathrm{d}y^2 + T^{2(K+2)/(2K+1)}\,\mathrm{d}z^2\,, \tag{2.9}$$

where R and P being constant of integration and

$$\gamma = 16\pi\eta L R^{3/2}\,.$$

Now we consider the second possibility: i.e.,

$$K = -\tfrac{1}{2}\,. \tag{2.10}$$

Hence, Equation (2.8) leads to

$$\mu = Mt + N\,, \tag{2.11}$$

where N is constant of integration and $BC = \mu$.

If we put $BC = \mu$, $B/C = \nu$, and $K = -\frac{1}{2}$ in Equation (2.5) and by use of Equation (2.11), we have

$$\frac{\left(\frac{\mu\nu_4}{\nu}\right)_4}{\frac{\mu\nu_4}{\nu}} = -\,16\pi\eta L(Mt + N)^{1/2}\,, \tag{2.12}$$

which leads to

$$\frac{\nu_4}{\nu} = \frac{P}{\mu}\,e^{-32/3\,\pi\eta L(\mu)^{3/2}}\,. \tag{2.13}$$

From Equations (2.11) and (2.13), we have

$$\nu = \exp\int \frac{P}{Mt + N}\,e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}\,\mathrm{d}t\,. \tag{2.14}$$

Hence,

$$A^2 = L^2(Mt + N)\,, \tag{2.15}$$

$$B^2 = (Mt + N)\,\nu\,, \tag{2.16}$$

$$C^2 = \frac{Mt + N}{\nu}\,. \tag{2.17}$$

Thus the metric reduces to the form

$$dS^2 = L^2(Mt + N)(dx^2 - dt^2) + (Mt + N)\nu\, dy^2 + \frac{(Mt + N)}{\nu} dz^2, \quad (2.18)$$

where ν is determined from Equation (2.14).

In the absence of viscosity, i.e., when $\eta \to 0$, we have from Equation (2.14)

$$\nu = R(Mt + N)^{P/M}, \quad (2.19)$$

where R is constant of integration.

Hence, in the absence of viscosity the metric reduces to the form

$$dS^2 = L^2(Mt + N)(dx^2 - dt^2) + (Mt + N)^{(P/M)+1} dy^2 + \\ + (Mt + N)^{1-(P/M)} dz^2. \quad (2.20)$$

3. Some Physical and Geometrical Features

The pressure and density for model (2.18) are given by

$$8\pi p = 8\pi\varepsilon = \frac{1}{4L^2(Mt + N)^3}[3M^2 - P^2 e^{-64/3\,\pi\eta L(Mt+N)^{3/2}}]. \quad (3.1)$$

The model has to satisfy the reality condition given by Ellis (1971) as

(i) $\varepsilon + p > 0$.

(ii) $\varepsilon + 3p > 0$.

Both the conditions leads to

$$e^{(64/3)\pi\eta L(Mt+N)^{3/2}} > \frac{P^2}{3M^2}. \quad (3.2)$$

The scalar of expansion θ and the non-vanishing components of shear tensor σ_{ij} are given by

$$\theta = \frac{3M}{2L}(Mt + N)^{-3/2} \quad (3.3)$$

and

$$\sigma_{11} = \frac{(1 - l)M}{2(Mt + N)^{1/2}}, \quad (3.4)$$

$$\sigma_{22} = \frac{\nu}{2L(Mt + N)^{1/2}}[M(1 - l) + P e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}], \quad (3.5)$$

$$\sigma_{33} = \frac{1}{2L(Mt + N)^{1/2}\nu}[M(1 - l) - P e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}] \quad (3.6)$$

Hence, the coefficient of shear (σ) for model (2.18) is given by

$$\sigma^2 = \frac{1}{4L^2(Mt+N)^3}\left[3M^2(1-l)^2 + 2P^2\, e^{-64/3\,\pi\eta L(Mt+N)^{3/2}}\right]. \tag{3.7}$$

The non-vanishing components of conformal curvature tensor are given by

$$C^{12}_{12} = C^{34}_{34} = \frac{1}{24L^2(Mt+N)^3}\left[-6MP\{1+16\pi\eta L(Mt+N)^{3/2}\}\times \times\, e^{-32/3\,\pi\eta L(Mt+N)^{3/2}} - 2P^2\, e^{-64/3\,\pi\eta L(Mt+N)^{3/2}}\right], \tag{3.8}$$

$$C^{13}_{13} = C^{24}_{24} = \frac{1}{24L^2(Mt+N)^3}\left[6MP\{1+16\pi\eta L(Mt+N)^{3/2}\}\times \times\, e^{-32/3\,\pi\eta L(Mt+N)^{3/2}} - 2P^2\, e^{-64/3\,\pi\eta L(Mt+N)^{3/2}}\right], \tag{3.9}$$

$$C^{14}_{14} = C^{23}_{23} = \frac{1}{24L^2(Mt+N)^3}\left[4P^2\, e^{-64/3\,\pi\eta L(Mt+N)^{3/2}}\right]. \tag{3.10}$$

The model in general represents expanding, shearing, and non-rotating universe in general. The coefficient of viscosity does not affect expansion of the model. The expansion in the model stops for large values of T. The model also represents Petrov type-I non-degenerate and C_{hijk} vanishes asymptotically.

The motion of test particle in the model (2.18) is governed by the equation of geodesics, given by

$$\frac{d^2x}{ds^2} + \frac{M}{2(Mt+N)}\left(\frac{dx}{ds}\right)\left(\frac{dt}{ds}\right) = 0, \tag{3.11}$$

$$\frac{d^2y}{ds^2} + \left\{\frac{M + P\, e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}}{2(Mt+N)}\right\}\frac{dy}{ds}\frac{dt}{ds} = 0, \tag{3.12}$$

$$\frac{d^2z}{ds^2} + \left\{\frac{M - P\, e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}}{2(Mt+N)}\right\}\frac{dz}{ds}\frac{dt}{ds} = 0, \tag{3.13}$$

$$\frac{d^2t}{ds^2} + \frac{M}{2(Mt+N)}\left\{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dt}{ds}\right)^2\right\} + + \frac{v}{L^2}\left\{\frac{M + P\, e^{-32/3\,\pi\eta L(Mt+N)^{3/2}}}{2(Mt+N)}\right\}\left(\frac{dy}{ds}\right)^2 + + \frac{1}{vL^2}\left\{\frac{M - P\ {}^{-32/3\,\pi\eta L(Mt+N)^{3/2}}}{2(Mt+N)}\right\}\left(\frac{dz}{ds}\right)^2 = 0. \tag{3.14}$$

In the above S is the arclength along the geodesic.

If a particle is initially at rest that is if

$$\frac{dx}{dS} = \frac{dy}{dS} = \frac{dz}{dS} = 0 . \tag{3.15}$$

Then, we get

$$\frac{dt}{dS} = k(Mt + N)^{-1/2} , \tag{3.16}$$

where k is the constant of integration.

From the equation of geodesics, we find that for all such particles, the components of spatial acceleration would vanish: namely,

$$\frac{d^2x}{dS^2} = \frac{d^2y}{dS^2} = \frac{d^2z}{dS^2} = 0 \tag{3.17}$$

and the redshift for model (2.18) is given by

$$\frac{\lambda + \delta\lambda}{\lambda} = \frac{\xi_1(t) + U_z}{\xi_2(t)} \frac{[L^2(Mt + N)]^{1/2}}{[L^2(Mt + N) - U^2]^{1/2}} . \tag{3.18}$$

The pressure and density in the absence of viscosity are given by

$$8\pi p = 8\pi\varepsilon = \frac{[3M^2 - P^2]}{4L^2(Mt + N)^3} . \tag{3.19}$$

The relatively conditions

(i) $\varepsilon + p > 0$,

(ii) $\varepsilon + 3p > 0$,

lead to

$$3M^2 > P^2 . \tag{3.20}$$

The non-vanishing components of conformal curvature tensor in the absence of viscosity, are given by

$$C_{12}^{12} = C_{34}^{34} = \frac{1}{24L^2(Mt + N)^{3/2}} [- 6MP - 2P^2] , \tag{3.21}$$

$$C_{13}^{13} = C_{24}^{24} = \frac{1}{24L^2(Mt + N)^{3/2}} [6MP - 2P^2] , \tag{3.22}$$

$$C_{14}^{14} = C_{23}^{23} = \frac{4P^2}{24L^2(Mt + N)^{3/2}} . \tag{3.23}$$

In the absence of viscosity, the model is non-degenerate Petrov type-I and C_{hijk} vanishes asymptotically.

Case (*ii*). When $\eta/\theta = l$ (constant)

I.e.,

$$\eta = \frac{l}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right). \tag{3.24}$$

From Equations (2.4) and (3.24), we have

$$\eta = \frac{l}{LB(B/C)^K}\left[(K+2)\frac{B_4}{B} + (1-K)\frac{C_4}{C}\right]. \tag{3.25}$$

Equations (2.5) and (3.25) lead to

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = -16\pi l\left[(K+2)\frac{B_4}{B} + (1-K)\frac{C_4}{C}\right]\left[\frac{B_4}{B} - \frac{C_4}{C}\right]. \tag{3.26}$$

From Equations (2.6) and (3.25), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} = \frac{16\pi l(2K+1)}{3} \times \left[(K+2)\frac{B_4}{B} + (1-K)\frac{C_4}{C}\right]\left(\frac{B_4}{B} - \frac{C_4}{C}\right). \tag{3.27}$$

If we put $BC = \mu$ and $B/C = \nu$ in (3.26) and (3.27), we have

$$\frac{\left(\frac{\mu\nu_4}{\nu}\right)_4}{\mu} = -16\pi l\left[\frac{3}{2}\frac{\mu_4}{\mu} + \frac{2K+1}{2}\frac{\nu_4}{\nu}\right]\frac{\nu_4}{\nu}, \tag{3.28}$$

$$\frac{\mu_{44}}{\mu} = 16\pi l\left[\frac{2K+1}{3}\right]\left[\frac{3}{2}\frac{\mu_4}{\mu} + \frac{2K+1}{2}\frac{\nu_4}{\nu}\right]\frac{\nu_4}{\nu}. \tag{3.29}$$

Equations (3.28) and (3.29) lead to

$$\frac{\nu_4}{\nu} = -\frac{3}{2K+1}\frac{\mu_4}{\mu} + \frac{M}{\mu}, \tag{3.30}$$

M being a constant of integration.

From Equations (3.29) and (3.30), we have

$$\mu\mu_{44} = \alpha l\left[1 - \frac{\beta}{\alpha}\mu_4\right], \tag{3.31}$$

where

$$\alpha = \frac{8\pi M^2(2K+1)^2}{3} \quad \text{and} \quad \beta = 8\pi M(2K+1)\,.$$

If we insert $\mu_4 = f(\mu)$ in (3.31), we have

$$\mu^{\alpha l} = N e^{-(\alpha/\beta)f}\left(1 - \frac{\beta}{\alpha} f\right)^{-\alpha^2/\beta^2}. \tag{3.32}$$

Let us assume $N = e^l$, therefore, Equation (3.32) leads to

$$\mu = [1 - \beta/\alpha f)]^{-\alpha/\beta^2 l} \exp\left[-\frac{f}{\beta l} + \frac{1}{\alpha}\right]. \tag{3.33}$$

From Equations (3.30), (3.31), and (3.33), we have

$$v = \gamma e^{Mf/\alpha l}; \tag{3.34}$$

γ being constant of integration.

After suitable transformation of coordinates, the metric reduces to the form

$$\begin{aligned} \mathrm{d}S^2 = {} & L^2\gamma^{2K+1}\exp\left[\frac{2T}{\beta l} + \frac{1}{\alpha}\right]\left[1 - \frac{\beta}{\alpha}T\right]^{-\alpha/\beta^2 l} \times \\ & \times\left(\mathrm{d}X^2 - \frac{\mathrm{d}T^2}{(TT')^2} + \exp\left[\frac{MT}{l\alpha} - \frac{T}{l\beta}\right]\left[1 - \frac{\beta T}{\alpha}\right]^{\alpha/\beta^2 l}\mathrm{d}Y^2\right) + \\ & + \exp\left[-\left(\frac{MT}{l\alpha} + \frac{T}{l\beta}\right)\right][1 - \beta/\alpha T]^{-\alpha/\beta^2 l}\,\mathrm{d}Z^2\,, \end{aligned} \tag{3.35}$$

where

$$TT' = \alpha l\left[1 - \frac{\beta}{\alpha}T\right]^{1 + (\alpha/\beta^2 l)} \exp\left[\frac{T}{\beta l} - \frac{1}{\alpha}\right]$$

and

$$f = T\,. \tag{3.36}$$

In the absence of viscosity metric (3.35) reduces to

$$\begin{aligned} \mathrm{d}S^2 = {} & L^2\gamma^{2K+1}\exp\left[\frac{3\tau}{\beta} + \frac{1}{\alpha}\right]\left(\mathrm{d}X^2 - \frac{\mathrm{d}\tau^2}{\alpha^2 e^{-2/\alpha}}\right) + \\ & + e^{M\tau/\alpha}\,\mathrm{d}Y^2 + e^{-M\tau/\alpha}\,\mathrm{d}Z^2\,. \end{aligned} \tag{3.37}$$

4. Some Physical and Geometrical Features

The pressure and density for the model (3.35) are given by

$$8\pi p = 8\pi\varepsilon = \frac{[1 - \beta/\alpha T]^{3\alpha/\beta^2 l}}{L^2\gamma^{2K+1}\, e^{1/\alpha}}\left[\frac{3}{2}\frac{\alpha}{\beta}T - \frac{3}{4}T^2 - \frac{M^2}{4}\left(1 - \frac{\beta}{\alpha}T\right)^2\right]. \tag{4.1}$$

The model (3.35) has to satisfy the reality conditions

(i) $\varepsilon + p > 0$,

(ii) $\varepsilon + 3p > 0$,

which lead to

$$\frac{3}{2}\frac{\alpha}{\beta}T > \frac{3}{4}T^2 + \frac{M^2}{4}\left(1 - \frac{\beta}{\alpha}T\right)^2. \tag{4.2}$$

The scalar of expansion θ is given by

$$\theta = \frac{3\alpha}{2e^{1/2\alpha}\beta L^{2K+1/2}}\left[1 - \frac{\beta}{\alpha}T\right]^{3/2\alpha/\beta^2 l}. \tag{4.3}$$

The relation ω is identically zero and the shear is given by

$$\sigma^2 = \left[\frac{3}{2}\frac{\alpha^2}{\beta^2} + \frac{3}{2}T^2 - \frac{3\alpha}{\beta}T + \frac{M^2}{2}\left(1 - \frac{\beta}{\alpha}T\right)^2\right]\left[1 - \frac{\beta}{\alpha}T\right]^{3\alpha/\beta^2 l}, \tag{4.4}$$

the non-vanishing components of the conformal curvature tensor are

$$C_{12}^{12} = C_{34}^{34} = \frac{(1 - \beta/\alpha T)^{3\alpha/\beta^2 l}}{6e^{1/\alpha}L^2\gamma^{2K+1}}\left[-\frac{3}{2}\alpha l - \frac{3}{2}Ml\beta - \frac{M^2}{2} - \frac{9}{2}\frac{M\alpha}{\beta} + \right.$$
$$+ T\left(\frac{3}{2}\alpha\beta - \frac{3}{2}\frac{\alpha}{\beta} + \frac{3}{2}\frac{Ml\beta^2}{\alpha} + \frac{M^2\beta}{\alpha} + \frac{15}{2}M\right) +$$
$$\left. + T^2\left(\frac{3}{2} - \frac{3M\beta}{\alpha} - \frac{M^2\beta^2}{2\alpha^2}\right)\right], \tag{4.5}$$

$$C_{13}^{13} = C_{24}^{24} = \frac{(1 - \beta/\alpha T)^{3\alpha/\beta^2 l}}{6e^{1/\alpha}L^2\gamma^{2K+1}}\left[-\frac{3}{2}\alpha l + \frac{3}{2}Ml\beta - \frac{M^2}{2} + \frac{9}{2}\frac{M\alpha}{\beta} + \right.$$
$$+ T\left(\frac{3}{2}\alpha\beta - \frac{3}{2}\frac{\alpha}{\beta} - \frac{3}{2}\frac{Ml\beta^2}{\alpha} + \frac{M^2\beta}{\alpha} - \frac{15}{2}M\right) +$$
$$\left. + T^2\left(\frac{3}{2} + \frac{3M\beta}{\alpha} - \frac{M^2\beta^2}{2\alpha^2}\right)\right], \tag{4.6}$$

$$C_{14}^{14} = C_{23}^{23} = \frac{(1 - \beta/\alpha T)^{3\alpha/\beta^2 l}}{6e^{1/\alpha}L^2\gamma^{2K+1}}\left[3\alpha l + M^2 + T\left(-3l\beta - \frac{2M^2\beta}{\alpha} + \frac{3\alpha}{\beta}\right) + \right.$$

$$\left. + T^2\left(-3 + \frac{M^2\beta^2}{\alpha^2}\right)\right]. \tag{4.7}$$

In general, the model represents Petrov type-I non-degenerate, C_{hijk} vanishes asymptotically if $3\alpha/\beta^2 l + 2 < 0$. Since $\lim_{T\to\infty}$, $\sigma/\theta \neq 0$. Hence, the model does not approach isotropy for large value of T.

The motion of test particle in the model (3.35) is governed by the equation of geodesics, given by

$$\frac{d^2X}{dS^2} + \left(\frac{3\alpha}{\beta} - 2T\right)\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} e^{T/\beta l}\left(\frac{dX}{dS}\right)\left(\frac{dT}{dS}\right) = 0\,, \tag{4.8}$$

$$\frac{d^2Y}{dS^2} + \frac{\left[T + M\left(1 - \frac{\beta}{\alpha}T\right)\right]}{2}\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} e^{T/\beta l}\left(\frac{dY}{dS}\right)\left(\frac{dT}{dS}\right) = 0\,, \tag{4.9}$$

$$\frac{d^2Z}{dS^2} + \frac{\left[T - M\left(1 - \frac{\beta}{\alpha}T\right)\right]}{2}\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} e^{T/\beta l}\left(\frac{dZ}{dS}\right)\left(\frac{dT}{dS}\right) = 0\,, \tag{4.10}$$

$$\frac{d^2T}{dS^2} + \left(\frac{3\alpha}{\beta} - 2T\right)\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} e^{T/\beta l}\left\{\left(\frac{dX}{dS}\right)^2 + \left(\frac{dT}{dS}\right)^2\right] +$$

$$+ \frac{\left[T + M\left(1 - \frac{\beta}{\alpha}T\right)\right]}{2L^2 e^{1/\alpha}\gamma^{2K+1}}\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} \exp\left[\frac{T}{l}\left(\frac{M}{\alpha} - \frac{2}{\beta}\right)\right]\left(\frac{dY}{dS}\right)^2 +$$

$$+ \frac{\left[T - M\left(1 - \frac{\beta}{\alpha}T\right)\right]}{2L^2 e^{1/\alpha}\gamma^{2K+1}}\left[1 - \frac{\beta}{\alpha}T\right]^{\alpha/\beta^2 l} \exp\left[\frac{T}{l}\left(-\frac{M}{\alpha} - \frac{2}{\beta}\right)\right]\left(\frac{dZ}{dS}\right)^2 = 0\,. \tag{4.11}$$

In the above S is the arc length along the geodesic. If a particle is initially at rest, i.e.,

$$\frac{dX}{dS} = \frac{dY}{dS} = \frac{dZ}{dS} = 0\,. \tag{4.12}$$

Hence,

$$\frac{\mathrm{d}T}{\mathrm{d}S} = \exp \int \left(2T - \frac{3\alpha}{\beta}\right)\left(1 - \frac{\beta}{\alpha}T\right)^{\alpha/\beta^2 l} e^{T/\beta l}\,\mathrm{d}T. \tag{4.13}$$

From the equation of geodesics, we find that for all such particles, the components of spatial acceleration would vanish: namely,

$$\frac{\mathrm{d}^2X}{\mathrm{d}S^2} = 0 = \frac{\mathrm{d}^2Y}{\mathrm{d}S^2} = \frac{\mathrm{d}^2Z}{\mathrm{d}S^2}.$$

In the absence of viscosity, the shear σ is given by

$$\sigma^2 = \left[\frac{3}{2}\frac{\alpha^2}{\beta^2} + \frac{M^2}{2}\right]e^{-3\tau/\beta}.$$

The non-vanishing components of conformal curvature tensor in absence of viscosity are given by

$$C_{12}^{12} = \frac{e^{-3\tau/\beta}}{6e^{1/\alpha}L^2\gamma^{2K+1}}\left[-\frac{M^2}{2} - \frac{9}{2}\frac{M\alpha}{\beta}\right],$$

$$C_{13}^{13} = \frac{e^{-3\tau/\beta}}{6e^{1/\alpha}L^2\gamma^{2K+1}}\left[-\frac{M^2}{2} + \frac{9}{2}\frac{M\alpha}{\beta}\right],$$

$$C_{14}^{14} = \frac{e^{-3\tau/\beta}}{6e^{1/\alpha}L^2\gamma^{2K+1}}[M^2].$$

In absence of viscosity, the model represents expanding, shearing, and non-rotating universe in general. For large values of T, the expansion in the model stops and the space-time is non-denegerate Petrov type-I. For large values of T, the space-time is conformly flat. Since $\lim_{T\to\infty}$, $\sigma/\theta \neq 0$. Hence, the model does not approach isotropy for large values of T also.

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