A DOUBLE LAYER REVIEW*

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Abstract. A review is given of the main results on electrostatic double layers (sometimes called 'space charge layers' or 'sheaths') obtained from theory and laboratory and space experiments up to the spring of 1977.

The paper begins with a definition of double layers in terms of potential drop, electric field, and charge separation. Then a review is made of the theoretical results obtained so far. This covers, among other things, necessary criteria for existence and stability, quantitative estimates of charge separation and thickness, and some probable cause of DL-formation in terms of an instability.

Next, experimental results obtained in the laboratory are compared with the theoretical results. Due to recent progress in experimental technique, the interior of a double layer can now be studied in much more detail than was possible before.

By means of barium jets and satellite probes, double layers have now been found at the altitudes that were previously predicted theoretically. The general potential distribution above the auroral zone, suggested by inverted V-events and electric field reversals, is corroborated.

1. Introduction

Elementary electrodynamics tells us that charged particles can never be accellerated by magnetic fields. Electric fields are required, either electrostatic or induced by time-varying magnetic fields.

Since energetic electrons and ions are often observed in plasmas, it is important to understand the mechanisms that can create and maintain electric fields in plasmas. Several such mechanisms have been discussed by Block and Fälthammar (1976). One of these is the double layer mechanism, which is of electrostatic nature, since in a double layer the electric field is sandwiched between two opposite space charge layers. If a magnetic field **B** is present, it may in principle be arbitrarily directed, but the simpler case with $\mathbf{E}\|\mathbf{B}$ is most commonly discussed.

Our understanding of double layers is far from complete, in spite of the fact that they have been studied in laboratories for several decades. However, progress has been made in recent years, both theoretically and experimentally and through observations in space. The present paper represents an attempt to summarize the state of the art, with special emphasis on properties of importance in space plasmas.

2. Definition of a Double Layer

The meaning of the term 'double layer' seems sometimes to have been confused. In this paper, as in the author's previous publications on the subject, a double layer

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is defined as consisting of two equal but oppositely charged, essentially parallel but not necessarily plane, space charge layers (see Figure 1). The potential, electric field and space charge density vary qualitatively within the layer, as shown in Figure 2. If the potential would not vary monotonically through the entire layer but would contain a few maxima and minima (as, for example, in Figure 3), we may still call it a double layer (provided the conditions below are fulfilled) although, strictly speaking, it consists of more than one double layer.

The following three conditions must be fulfilled:

(i) The potential drop ϕ_0 through the layer must obey the relation

$$|\phi_0| \gtrsim kT/e, \tag{1}$$

where T is the temperature of the coldest plasma bordering the layer, and k and e have their usual meanings.

(ii) The electric field is much stronger inside the double layer than outside, so the integrated positive and negative charges nearly cancel each other.

(iii) Quasi-neutrality is locally violated in both space charge layers.

In addition, a typical but not strictly necessary condition is that the collisional mean free path is much longer than the double layer thickness. Experimental as well as theoretical evidence indicate that as long as collisions play an appreciable role a double layer will not be formed.

It may be pointed out, in order to prevent some common misunderstandings, that the definition neither includes some particular type of instability or wave, causing or maintaining the layer, nor some minimum current density through the layer.

In the remainder of the paper double layers will be abbreviated as DL.

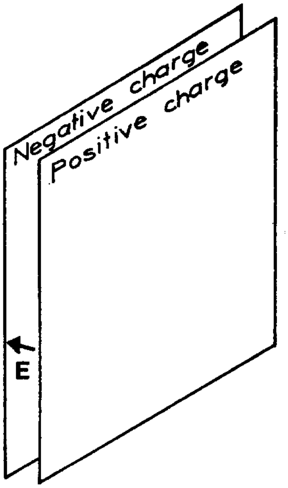

Fig. 1. Schematic picture of a double layer.

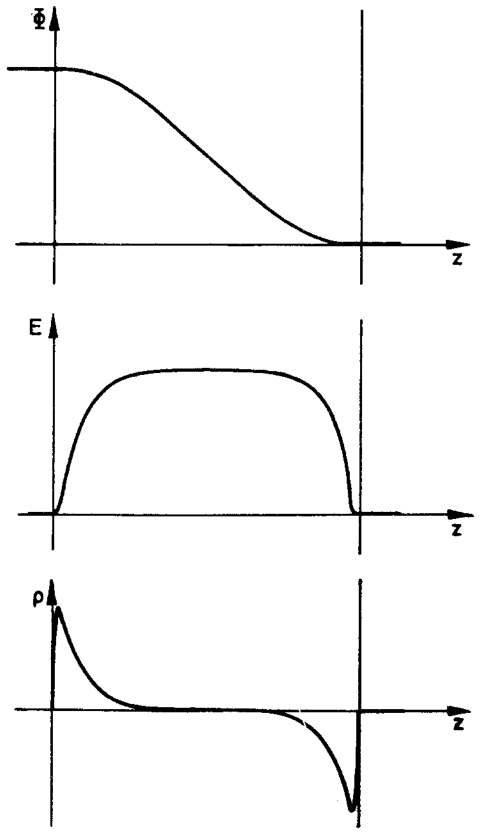

Fig. 2. Potential, electric field and space charge distributions through a double layer.

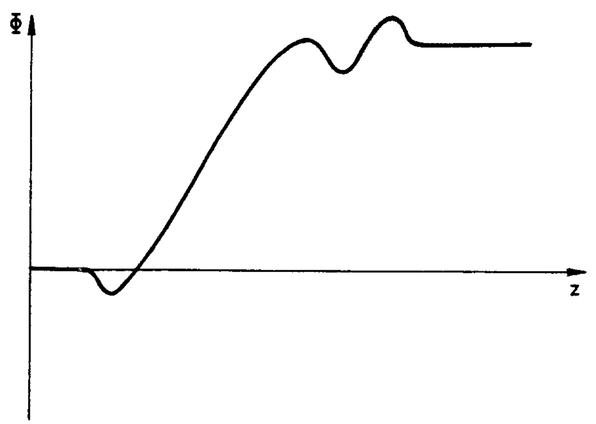

Fig. 3. Even space charge layers with this potential distribution may be called a double layer, although it, strictly speaking, consists of five double layers.

3. Theoretical Preliminaries

Bernstein, Greene and Kruskal (1957) proved the existence of an unlimited class of solutions to the Vlasov equations (BGK solutions), containing stationary potential structures with the above conditions (i)–(iii) fulfilled. They demonstrated that essentially arbitrary one-dimensional potential distributions can be constructed, if a suitable number of particles trapped in potential troughs are added.

The particles associated with the DL potential variation of Figure 2 may be conveniently divided into four classes: free and trapped (or reflected) electrons and ions, as shown in Figure 4. In principle, three of these suffice to maintain the DL. For example, assume that the free electron and ion number densities are equal on the low potential side. Due to the acceleration in the DL the corresponding free electron density is much less than the free ion density on the high potential side. The difference must be made up of a suitable number of reflected electrons. However, in practice a DL with only one type of trapped particles is exceptional, so usually all four classes of particles are required.

The first self-consistent DL solution was constructed by Langmuir (1929) who used only cold particles (delta-function distributions). We know now that this solution

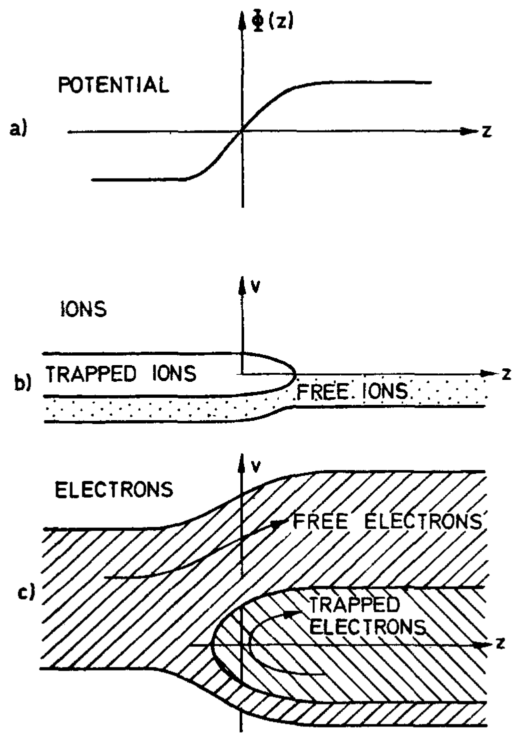

Fig. 4. (a) Potential distribution of a double layer. (b) Phase space for trapped and free ions. (c) Phase space for trapped and free electrons.

is unstable. Block (1972) used fluid theory with finite temperatures to construct a solution that was asymptotically self-consistent for $e\phi_0 \gg kT$ (strong DL). Knorr and Goertz (1974) showed that, given three of the four particle velocity distributions (water-bags in their case) and the potential variation (a hyperbolic tangent approximating the DL), the fourth particle velocity distribution can be uniquely determined, and was in their case found to be nowhere negative in velocity space. They, thus, gave an example of a BGK solution, demonstrating BGK's assertion that essentially arbitrary potential distributions can be constructed if the right number of trapped particles are added.

Thus, it is obvious that one necessary condition for the existence of a DL is that trapped electron and ion populations can be maintained. This can be done in at least three different ways; namely, by

(a) electrostatic potential troughs;
(b) magnetic mirrors;
(c) collisions, causing backscattering.

These will be exemplified in Sections 5 and 6 on experiments and space applications.

In closing their paper, BGK noted that 'whether such waves can exist in an actual plasma will depend on factors ignored in this paper, as in most previous works, namely inter-particle collisions, and the stability of the solutions against various kinds of perturbations'.

The general DL stability problem has not yet been solved, but some necessary stability and existence criteria have been derived or at least been made plausible. Furthermore, stable and oscillating DL have been produced in laboratory plasmas and strong evidence for DL in the low magnetosphere above the auroral zones now exists, as will be discussed below.

Before entering into a more detailed description of DL properties, the following nomenclature is introduced.

The word 'temperature' will be used as a general term for kinetic energy spread around a mean energy for some class of particles, even if the energy distribution is not strictly thermal. That is, for *any* one-dimensional velocity distribution $f(v)$ we have

$$\int_{-\infty}^{+\infty} f(v) m v^2 \, \mathrm{d}v = nm\langle v^2 \rangle = nmu_D^2 + nkT, \tag{2}$$

where u_D is the drift velocity. Equation (2) thus serves as definition of the 'temperature' T.

In terms of condition (1) for the definition of a DL, we say that a DL is strong if $e\phi_0 \gg kT_{fe}$, where T_{fe} is the free electron temperature. If $e\phi_0/kT_{fe} < 10$, say, the DL is weak (cf. Equation (1)).

4. Theoretical Results

In Section 2 three distinguishing properties of DL were introduced. The first ($e\phi_0 \gtrsim kT$) has already been discussed to some extent and does not need any further elaboration. The second, on essentially vanishing net charge of a DL, can be derived from the momentum balance and Poisson's equation. It leads, under certain conditions which are often fulfilled, to a relation between ion and electron flux through the layer. The third, on violation of quasi-neutrality, can be discussed quantitatively to some extent, if the plasma instability causing DL formation can be identified.

4.1. Vanishing net charge and the Langmuir condition

The space charge separation in a DL is maintained by a balance between inertia and electrostatic forces. This balance is determined by the Poisson and momentum equations. If index α denotes population (free or trapped, electron or ion), the one-dimensional collisionless, stationary Boltzmann equation for each population is

$$u \frac{\partial f_\alpha}{\partial z} + \frac{q_\alpha E}{m_\alpha} \frac{\partial f_\alpha}{\partial u} = 0. \tag{3}$$

The momentum equation is obtained by multiplication with $m_\alpha u$ and integration over velocity space. By virtue of (2) we get

$$\frac{\partial}{\partial z} (n_\alpha k T_\alpha + n_\alpha m_\alpha u_{D\alpha}^2) = n_\alpha q_\alpha E. \tag{4}$$

If this is inserted in Poisson's equation

$$\varepsilon_0 \, \partial E / \partial z = \sum_\alpha n_\alpha q_\alpha , \tag{5}$$

multiplied by E and integrated with respect to z, a first integral is obtained in the form

$$\sum_\alpha (n_\alpha k T_\alpha + n_\alpha m_\alpha u_{D\alpha}^2) - \tfrac{1}{2}\varepsilon_0 E^2 = \text{constant}. \tag{6}$$

This pressure balance equation was derived from fluid theory by Block (1972).

Evidently, if the potential varies monotonically with z, there are four nkT terms but only two drift energy terms, since $u_D = 0$ for trapped particles.

If the DL has no net charge, E is the same on both sides, so comparing (6) on both sides we may leave out the electrostatic pressure. If, in addition, the DL is strong, the ram pressures of the free particles after acceleration (or before retardation if the drift is backwards through the DL) will dominate, by definition. Hence,

$$(n_e m_e u_{De}^2)_+ \approx (n_i m_i u_{Di}^2)_- , \tag{7}$$

where index + means the value on the positive potential side of the DL, and correspondingly for −. Since both species have been accelerated through the same DL potential, their energies are about equal, so that

$$\frac{n_e u_{De}}{n_i u_{Di}} = (m_i / m_e)^{1/2} , \tag{8}$$

which is called the Langmuir condition (Langmuir, 1929; Block, 1972). In order to maintain zero net charge in the DL, the flux of electrons through the layer must be much larger than the ion flux since each electron spends a shorter time in the DL.

The Langmuir condition implies, of course, that there is a non-vanishing current through the layer, but it must be stressed that it is only valid for strong layers. In principle, the current can be made arbitrarily small. Assume, e.g., that about half of the free electrons would have their velocities reversed. Each electron would contribute exactly the same amount of charge as without the reversal, but u_{De} would be much smaller and the free electron 'temperature' would increase correspondingly. Thus, the DL would not be strong. It is clear that a DL can, at least in principle, exist without any net current. Experimental evidence supporting this conclusion will be discussed below. However, in most practical cases the Langmuir condition holds surprisingly well even for rather weak layers, since usually the number of retarded free electrons in the DL is quite small.

4.2. The Bohm Criterion and a Critical Current

Consider a strong DL such that almost all free particles are accelerated in the layer. At the negative boundary free electrons are entering the layer with drift velocity u_{De}. Suppose the trapped ions are Maxwellian with temperature T_{ti}. Since the layer is strong, free ions and trapped electrons have negligible densities at the negative boundary. Put the zero potential level at this boundary. The free electrons and trapped ions have equal densities there, but just inside the layer there is negative space charge (cf. Figure 2). Both the ion and electron densities are decreasing with increasing potential, i.e. towards the interior of the layer so that

$$n_i \sim \exp(-e\phi/kT_{ti}), \tag{9}$$

$$n_e \sim u_{De}^{-1} = (u_{De0}^2 + 2e\phi/m_e)^{-1/2}, \tag{10}$$

where $u_{De0} = u_{De}$ at $\phi = 0$.

To maintain a negative charge, n_i must decrease faster with increasing potential than n_e. From (9) and (10) it is easily seen that this requires $m_e u_{De0}^2 > kT_i$. However, we have here neglected the influence of the free electron temperature T_{fe}. Using fluid theory, Block (1972) found the minimum drift velocity of the electrons when entering the layer to be given by

$$m_e u_{De0}^2 = k(\gamma T_{fe} + T_{ti}), \tag{11}$$

where γ is the adiabatic constant for the free electrons. However, fluid theory may not give the correct result. Laboratory and computer experiments (Goertz and Joyce, 1975) indicate that

$$m_e u_{De0}^2 > 2kT_{fe} \tag{12}$$

may be sufficient to maintain a DL. Note, however, that in laboratory experiments $T_i \ll T_e$, normally, and both in the laboratory and in the computer experiments referred to, the velocity distributions are far from Maxwellian.

Conditions like (11) and (12) are called Bohm criteria, since a similar criterion valid for wall sheaths was derived by Bohm (1949). An identical criterion (with electrons and ions interchanged) is valid at the positive boundary of the DL.

The Bohm condition (12) implies a critical current density

$$j_c = ne\sqrt{2kT_e/m_e}, \tag{13}$$

below which a DL cannot be maintained. It should, however, again be pointed out that this is valid only for strong layers. Since an arbitrary number of the free electron velocities may be reversed without changing the charge balance, no such critical current density exists for weak layers. If the numbers of forward and backward moving free electrons are comparable, the DL is weak since then the free electron thermal energy is comparable to $e\phi_0$.

In fact, j_c represents *the* current density through a strong double layer, since it is also the maximum current density that the neighbouring plasma can deliver. Thus, if the current source tends to drive a higher current, either the DL will increase its potential drop or the cross-section of the current channel will increase in the plasma, if possible.

It is perhaps also conceivable that the DL will be pushed towards a region with higher density or temperature since on the positive side of the layer the current can be carried by the superthermal electron beam accelerated in the layer. This has been suggested by Smith and Goertz (1977). In any case, the DL and the transition from subthermal to superthermal free electrons must coincide if the layer is strong.

4.3. A PLASMA INSTABILITY AS THE CAUSE OF DL FORMATION

Since the critical current j_c is also the maximum current that a normal plasma can carry, it must represent a plasma instability limit. Several current driven instabilities are known with different threshold currents. The ion-acoustic and ion-gyro instabilities (in case there is a longitudinal magnetic field) have lower thresholds than given by (13). They cannot cause DL, but only some noise or turbulence, which at least in many cases is found to have little effect on the plasma properties (see Section 5). However, the Buneman instability has the right threshold, and it can therefore be suspected to be responsible for DL formation, in particular since it is of electrostatic nature, as is the DL. That does not mean, of course, that electrostatic waves must necessarily be observed in the vicinity of, or at an existing DL. They should be there during the formation, but once the DL is established there is no need for them. In fact, the DL probably represents the end result of a process initiated by some instability.

Assume that a spectrum of electrostatic waves is excited. Although Equation (6) is based on the time-independent Vlasov equation, it suggests that the electric field energy density, averaged over the period of the unstable waves, may act as a pressure on the particles. Indeed that is the case, as is well known in plasma wave theory. The force is then given by the gradient of the electrostatic energy density. Smith and Goertz (1977) propose that this plays an important role in DL formation, since this 'ponderomotive force' acts differently on different particle species. Due to their smaller mass, the electrons are more affected. According to Smith and Goertz (1977) the force acting on species α is

$$F_\alpha \propto -\frac{\partial}{\partial z}\langle\tfrac{1}{2}\varepsilon_0 E^2\rangle\frac{1}{m_\alpha}, \tag{14}$$

where the brackets indicate averaging over a wave period. Furthermore, they show that an arbitrarily steep gradient develops at the instability onset position. At this point charge separation occurs so that a DL may be formed.

The BGK theory (and applications of it – see, e.g., Montgomery and Joyce, 1969) shows that there exist equilibria of DL, characterized by a balance between inertia and electrostatic forces, so no instability waves are required to maintain them, provided they are reasonably stable. However, the particle beams produced upon acceleration of the free particles through the DL may, of course, interact with the plasma, causing wave turbulence, which must not necessarily be of electrostatic nature. What kind of waves are produced is, of course, determined by the particle velocity distributions, the presence of magnetic fields and perhaps other conditions, e.g. geometry.

4.4. Quantitative estimate of charge separation

The third condition defining a DL (that quasi-neutrality is violated), can now be cast in a more quantitative form. We have seen that the Bohm criterion (13) suggests that the Buneman instability be responsible for the formation of a DL. According to Smith (1977), we can then argue as follows.

The growth constant γ for fast growing Buneman unstable waves is of the order of

$$\gamma \sim \omega_{pi}(v_g/u_{De}), \tag{15}$$

where ω_{pi} is the ion plasma frequency and v_g is the group velocity of the disturbance. To obtain a measure of the relative deviation from charge neutrality, the Poisson equation may be used. To this end, we need characteristic length and potential scales. The length is found from (15)

$$\lambda = u_{De}/\omega_{pi}, \tag{16}$$

and the potential should be

$$\varphi = kT_e/e; \tag{17}$$

so that the Poisson equation yields

$$|\varrho| \sim \varepsilon_0 \frac{\varphi}{\lambda^2} = ne \frac{m_e}{m_i} \tag{18}$$

if the Bohm criterion (12) is used. Hence, it is seen that the relative deviation from quasi-neutrality is of the order of the electron to ion mass ratio.

This result can be used as an interpretational test of experimental results. If an electric field change ΔE is observed within a distance L in a plasma with electron density n, and if

$$\frac{\varepsilon_0 \, \Delta E}{Len} = \frac{\Delta n}{n} \geqslant \frac{m_e}{m_i}, \tag{19}$$

it may almost certainly be concluded that there is a double layer. Equation (19) is derived from $\varepsilon_0 \operatorname{div} \mathbf{E} = \varrho$, and Δn is the average deviation from exact charge neutrality.

Taking advantage of the BGK result that essentially any potential structure may be constructed, assume that a DL is made up of the rectangular charge distribution shown in Figure 5. The thickness of the layer is L_D, the potential drop is ϕ_0 and the average electric field is $\langle E \rangle$. For this special case the following relations hold:

$$L_D = \beta^{-1} \left(\frac{4e\phi_0}{kT_e} \right)^{1/2} \lambda_D = \nu L_D, \tag{20}$$

$$\langle E \rangle = \phi_0 / L_D = \phi_0 / \nu \lambda_D, \tag{21}$$

where

$$\lambda_D^2 = \frac{\varepsilon_0 k T_e}{ne^2} = (\text{Debye length})^2, \tag{22}$$

and

$$\beta^2 = \Delta n / n. \tag{23}$$

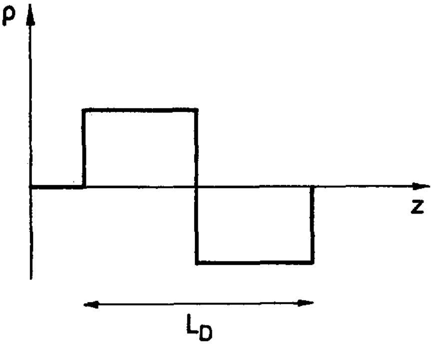

Fig. 5. Idealized space charge distribution model, assumed for estimation of double layer thickness (Equation (20)) and average electric field (Equation (21)).

Thus, β^2 is a measure of the average deviation from charge neutrality. Taking $\beta^2 \sim m_e/m_i$ as given by (18) we see that the thickness L_D of a DL is at least of the order of 50 Debye lengths.

The above model is based on the assumption of a certain *shape* of the charge or potential distribution, independent of the potential ϕ_0. This, of course, means that the corresponding particle distributions must depend on ϕ_0. Another alternative is to assume that the particle distributions are independent of ϕ_0. Then the potential distribution will depend on ϕ_0. This has been further elaborated by Shawhan *et al.* (1977). The expression (20) for L_D is then different, but the value of $\nu = L_D/\lambda_D$ is still of the same order of magnitude for typical DL observed in nature or in the laboratory, although the dependence on ϕ_0 is not the same.

It may be concluded that the thickness of a DL is generally large compared to the Debye length, but small compared to space plasmas and most laboratory plasmas. This will be further substantiated in the sections on experiments and space observations.

4.5. Stability

As has been already pointed out, the general stability problem has not yet been solved. However, three theoretical results of importance have been obtained.

Knorr and Goertz (1974) constructed a self-consistent DL solution to the Vlasov equation and proved that the plasmas on both sides of the layer were Penrose-stable at all points for sufficiently large free ion energies. Their proof has certain limitations, however. First, it is only valid for the particular velocity distributions, chosen. Second, the analysis is not complete since it does not cover modes due to the inhomogeneity.

Wahlberg (1977) studied electrostatic waves propagating in a plasma in the presence of an electric field. He found that perturbations excited in the drifting electron population gain energy if the electrons are decelerated by the field, but they lose energy if the electrons are accelerated. Referring to DL he states that 'a configuration of this kind could have a tendency to stabilize an otherwise unstable situation'. This can, of course, be true only if a majority of the free electrons are accelerated in the layer. A DL where a majority are decelerated would be more unstable and such layers, to the author's knowledge, have not been observed. Only layers with about half of the electrons moving 'backwards' have been observed.

Smith (1977) has suggested that the number of charged particles within a Debye sphere, N_D, is of importance for the stability. The thermal fluctuation amplitude of the charge of a Debye sphere is about equal to one electron charge, i.e. the relative violation of charge neutrality is N_D^{-1}. It should therefore be expected that stability at least requires that

$$N_D^{-1} < m_e/m_i, \tag{24}$$

i.e. the random charge fluctuations should be less than the charges constituting the DL.

There is some experimental support for this suggestion, as will be seen in the following sections.

In a magnetized plasma with a transverse extent much larger than the diameter of the current channel containing a DL, the equipotential surfaces in the DL must turn parallel to the magnetic field on both sides of the current channel as shown in Figure 6. The associated **E** × **B** drift of the plasma represents an inertia that must contribute to the stability of the DL. Obviously, a disturbance cannot remove the DL at once. The decay can only occur when the corresponding **E** × **B** drift has been stopped. The most natural way to do that must be to let the DL move along the magnetic field, probably with the Alfvén velocity, in the upward direction in Figure 6.

The mathematical models by Swift (1975, 1976) of DL (shocks) with arbitrary angle between the electric and magnetic fields are, of course, very important for understanding the equipotential structure shown in Figure 6.

In addition, the Bohm criteria and Langmuir condition must be regarded as necessary for the stable existence of a DL with the mentioned exceptions. Furthermore, it must be noted that if the Langmuir condition cannot be fulfilled in the laboratory or earth-fixed frame (or whatever frame is applicable), the DL may move with a suitable velocity, such that *in the frame of the DL* the condition is fulfilled.

4.6. Velocity Distributions

In conclusion, it must be pointed out that some of the above results depend on the detailed velocity distributions of all types of particles. That is most obviously the

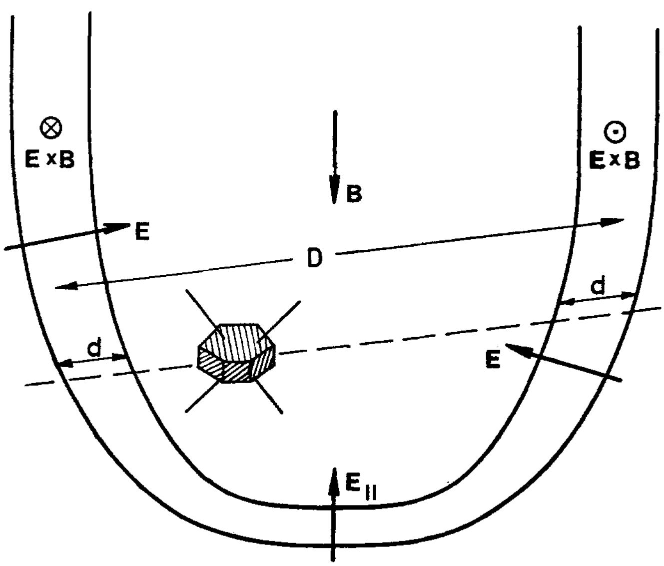

Fig. 6. Qualitative drawing of equipotential surfaces in a magnetized plasma, e.g. the topside ionosphere. The satellite S3-3 (Mozer *et al.*, 1977) flew through, what appeared to be 'pairs' of double layers, with thickness $d \sim$ 3–10 km and separation $D \sim$ 30 km.

case for the Bohm criterion. The Langmuir condition is, of course, accurately valid for strong layers, but its accuracy at weaker layers depends on the velocity distributions. A few very energetic electrons which are retarded in the DL, will not greatly affect the Langmuir condition but may contribute significantly to the 'temperature'.

The stability of the DL may depend strongly on the velocity distributions. However, that is a problem which has not been seriously attacked so far. On the other hand, the stability of the surrounding plasmas has been studied for special distributions. It is known that for Maxwellian distributions (displaced by the drift velocity) the ion-acoustic and ion-gyro electrostatic instabilities have onset drift velocities lower than that required to fulfill the Bohm criterion so no DL can be formed through these instabilities. A natural question is then: Could not the turbulence excited by these instabilities destabilize a DL formed at the appropriate (higher) drift velocity?

Experimental evidence shows that the answer is *no*, at least in many cases. Theoretically, this may eventually be understood, either in terms of non-linear effects limiting the fluctuations to harmless levels, or in terms of non-Maxwellian distributions, for which the onset drift velocity for the mentioned instabilities is raised to higher values. For example, Jentsch (1976) has shown that this is the case if the maximum of the electron distribution is flattened, making it more 'waterbag-like'. At the same time the ion distribution should be Maxwellian. This is understandable, since the inverse electron Landau damping is lowered due to the flatness of the electron distribution, whereas the ion Landau damping is still present. Thus, a higher drift velocity is required to balance the ion damping, as stated by Jentsch.

Persson (1963) has shown that if the pitch angle distributions for electrons and ions are unequal in a magnetic mirror-field, quasi-neutrality can only be satisfied if there is a finite electric field parallel to the magnetic field. He also suggested that a necessary condition for quasi-neutrality everywhere in a magnetic mirror-field is that there is no field-aligned current (Persson, private communication). In other words, if there is a net field-aligned current along a magnetic mirror-field, quasi-neutrality must be violated, at least at one point. A DL will probably be formed there. This is demonstrated for nearly isotropic velocity distributions by Lennartsson (1977). 'Nearly isotropic' means constant in pitch-angle space, except for some pitch-angle intervals where the distribution vanishes.

4.7. Summary of Theoretical Results

In the following sections experiments and space observations will be compared with the theoretical results presented above. To facilitate this comparison a summary of the results is given below.

A. *BGK equilibria*. The existence of essentially arbitrary, self-consistent, one-dimensional potential structures and waves is demonstrated (Bernstein *et al.*, 1957). A DL may be regarded as a BGK equilibrium with a potential structure as shown in Figure 2.

B. Trapped or at least *reflected particle populations* are required to maintain a DL. This is an inherent property of all BGK equilibria and all DL theories.

C. *Pressure balance* (Equation (6)). The sum of all kinetic pressures minus the electrostatic pressure is a constant.

D. *The Langmuir condition* (Equation (8)) states that *the ratio of ion to electron flux* through a strong layer is inversely proportional to the mass ratios (Langmuir, 1929).

E. *The Bohm criterion* for strong layers (Bohm, 1949). The existence of a DL requires a *minimum electron drift velocity* about equal to the electron thermal velocity V_{eT}.

F. A DL is *current-limiting* since the upstream plasma cannot carry more than the random electron thermal current density env_{eT}.

G. *The cause of DL formation* must be a *plasma instability*, probably the Buneman instability. Smith and Goertz (1977) show that an extremely steep *gradient of wave energy density* is produced by the instability. They propose that the corresponding *ponderomotive force* is an important factor in the formation of a DL.

Once the layer is formed no instability is needed to maintain it but secondary instabilities of other types may be excited by the beams from the layer.

H. *The relative charge separation* in a DL (Equation (18)) is of the order of the electron to ion mass ratio (Smith, 1977).

I. *The DL thickness* is at least of the order of $\sqrt{m_i/m_e}$ times the Debye length of the neighbouring plasma (Equations (20)–(23)).

J. *The stability* of a DL is not understood but it is found that

(a) the plasmas on both sides are Penrose-stable under certain conditions (Knorr and Goertz, 1974),

(b) perturbations in the free electrons lose energy when the electrons are accelerated in a DL (Wahlberg, 1977),

(c) random charge fluctuations may destabilize a DL if the number of electrons in a Debye sphere is less than the ion to electron mass ratio (Smith, 1977).

K. Swift (1975, 1976) has modelled self-consistently DL with arbitrary angle between the magnetic and electric fields.

5. Experiments

In this section experimental results, related to the theoretical results discussed in Section 4, will be discussed. A large number of experiments cannot, for lack of space, be mentioned here. Those mentioned are not necessarily the best, but they are best known to the author.

The capital letters of the subheadings below refer to the same in Section 4.7.

A. *Potential structure.* In many experiments a potential jump $\phi_0 > kT_{ef}/e$ was observed, but the internal structure of the layer could not be studied, due to a minute layer thickness (Torvén and Babic, 1976). However, Quon and Wong (1976) produced a DL in a plasma with a fairly large Debye length (~1 mm) thus obtaining a DL thickness of a few cm. They found a monotonic potential variation quite similar to that of Figure 2.

B. *Reflected particles* can be supplied in three different ways, as stated in Section 3.

(a) Electrostatic potential troughs are an essential part of BGK equilibria. Combining several opposing DL, it is often possible to achieve the necessary self-consistency, although the whole structure may have to move with respect to the plasma.

In an ordinary gaseous discharge the potential distribution is typically as shown in Figure 7. The hump at B can provide one end of an ion trough if a DL is formed in the positive column, say at D.

Quon and Wong (1976) observed that no DL could be formed without a positive bias on a grid on the low potential side of the DL.

(b) Backscattered electrons are typically produced by collisions in the positive column of a gaseous discharge. Thus, the reflected electrons are automatically provided. In Quon and Wong's experiment beam–plasma interaction thermalized the beam in a downstream region 5–10 cm beyond the DL. This produced hot electrons, some of which were moving backwards for subsequent reflection by the DL.

(c) Magnetic mirrors can, of course, also reflect particles. Evidence for such reflections facilitating DL formation has been obtained in the ionosphere. This will be discussed in Section 6.

C. *The pressure balance*, to the author's knowledge, has not been verified in any experiments so far. Electron velocity distributions have been measured in some experiments (see, e.g., Andersson, 1976) but it seems as if very few have been interested in the ions, and yet the ion beam ram pressure dominates on the low potential side and cannot be neglected in the pressure balance.

Torvén and Babic (1976) observed the wall flux of neutral particles and ions at both sides of the DL. However, these fluxes depend on the pressures perpendicular to the current, and the pressure balance equation, Equation (6), accounts for the 'parallel' pressures only. Nevertheless, Torvén and Babic found that the position of the DL could be controlled by adjusting the total (neutral plus ion) particle fluxes to the wall on the cathode and anode sides of the DL, as measured at fixed positions about one metre apart, i.e. far from the DL. These wall fluxes could be adjusted independently through gas reservoir temperatures at the cathode and anode, respectively.

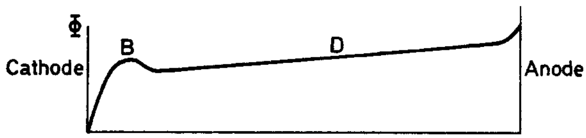

Fig. 7. Typical potential distribution in a gaseous discharge tube with no double layer. If a layer develops at D, ions may be trapped between it and the potential hump at B.

Assuming isotropic pressures and no gradients (except at the DL) between the wall-flux-meters, it would in principle be possible to obtain all terms in the pressure balance equation, including the ion ram pressure, which could be inferred from the Langmuir condition. However, such assumptions are probably not justified. More direct measurements are required.

D. *The Langmuir condition* has not been verified either, for the same reason. Andersson *et al.* (1969) and Quon and Wong (1976) measured the electron beam energy and found good agreement with the observed DL potential drop. Knowing the discharge current they could, of course, easily calculate the electron flux. Andersson *et al.* (1969) obtained an upper limit for the ion flux by equating it with the neutral gas flux from the cathode gas reservoir. This neutral flux can be partly absorbed in the walls, can partly return to the cathode region as ions through the DL, and can partly return without being ionized. In steady state (reached after several minutes) no wall absorption can occur. The upper limit for the ion current corresponds to the assumption that all neutral particles were ionized. This upper limit exceeded the ion current obtained from the Langmuir condition by a factor of 2.

E. *The Bohm criterion* for a minimum electron drift on the upstream side has been verified by almost all experimenters (see, e.g., Torvén and Babic, 1975; Quon and Wong, 1976).

F. *The current-limiting* property of a DL has been demonstrated by Torvén and Babic (1975, 1976). After the formation of a DL the current can only be increased by about 30–50% due to electron heating and somewhat increased electron density (see Figure 8). Attempts to increase the current beyond that point result in a drastic increase of the DL potential and collapse of the discharge.

G. Evidence for an *electrostatic instability* as the cause for DL formation is found in several experiments. Current driven ion-acoustic instabilities were observed by Quon and Wong (1976) and by Torvén and Babic (1976) when the current was

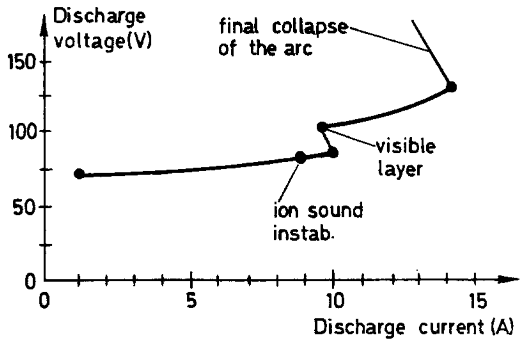

Fig. 8. Voltage–current characteristic of an arc discharge according to Torvén and Babic (1976). At *b* ion-acoustic instabilities are observed. At *c–d* the Bohm criterion is fulfilled so a double layer can form. The ultimate current limitation is reached at *f*.

somewhat less than the critical current for DL formation. The latter observed positive potential peaks propagating with the ion-sound speed. The Buneman instability, which was assumed to be responsible for DL formation (Section 4.3), is closely related to the ion-acoustic instability, but of course it cannot be observed below its threshold current density. It is, thus, not unexpected to find ion-acoustic instabilities as precursors for the Buneman instability. However, neither Quon and Wong nor Torvén and Babic observed ion-acoustic waves when the electron drift velocity was less than about one-third of the electron thermal velocity, which may be surprising since the ion-acoustic threshold drift is much lower. A related observation may be, that in an ordinary discharge without DL, the current density is about proportional to the electron number density, so the electron drift velocity (which is about one-third of the electron thermal velocity) is almost independent of current density over a few orders of magnitude (Klarfeld, 1938; Torvén, 1965). This means that even if the current density is more than two orders of magnitude below the critical DL current density, the electron drift is as high as one-third of the critical drift velocity, and yet no sign of ion-acoustic waves is seen. If such waves are really present their amplitude must be so low that they do not influence the plasma conductivity.

H and I. *The relative charge separation* can be easily calculated from the measured potential drop, *DL thickness*, and electron density, using Equations (18)–(23). Table I gives some characteristic data for three different experiments and some observations in space. It is seen that typically the relative charge separation ϱ/en is about an order of magnitude larger than m_e/m_i. This is in reasonable agreement with the dimensional analysis in Section 4.4.

An additional support to the charge separation estimate in Section 4.4 is the result of the numerical simulation experiment by Goertz and Joyce (1975). They used an ion to electron mass ratio of only 16, which resulted in a relative charge separation of the order of 0.1, as expected from Equation (18).

J. *DL stability* has been demonstrated in the laboratory under certain, rather general conditions. It is therefore encouraging that the limited theoretical progress made so far points in this direction.

The experimental evidence available shows that some DL are stable and some are explosive in the sense that the potential drop grows to extremely high values, thus quenching the discharge plasma. The decisive factor appears to be not some local plasma property, but the circuit inductance (Torvén and Babic, 1976). The DL is explosive if the circuit is sufficiently inductive to maintain the current against the rising DL voltage. This, of course, requires that the DL is not so unstable that it disappears during the explosion, i.e. during a few circuit time constants L/R. It should be noted, however, that this time constant decreases with increasing DL voltage, since the layer is dissipative. When the magnetic energy of the circuit has been dissipated the current must disappear and the discharge is extinguished.

If the circuit is less inductive, the rising DL voltage can cause a sufficient current decrease to quench the layer before it has become too strong, since the Bohm criterion

TABLE I

Double layer and plasma properties in some experiments

Authors:	Goertz and Joyce (1975)	Torvén and Babic (1975)	Andersson (1976)	Quon and Wong (1976)	Mozer *et al.* (1977)
Experimental conditions:	Numerical one-dimensional computer simulation	Lab. discharge Constant tube cross-section	Lab. discharge. Step in tube cross-section	Lab. discharge between two independent plasma sources	Satellite probe at 2000–8000 km altitude
Ions:	Very light	Mercury	Mercury	Argon	H^+ and O^+
m_i/m_e:	16	3.7×10^5	3.7×10^5	7×10^4	$(2–30) \times 10^3$
n_e (m^{-3}):	—	2×10^{17}	$(2–30) \times 10^{16}$	10^{14}	$(1–50) \times 10^6$
T_e (eV):	—	10	2	4	~ 100 (?)
λ_D (m):	—	5×10^{-5}	$< 10^{-4}$	10^{-3}	$\gtrsim 10$
L_D (m):	$\sim 50\lambda_D$	$\sim 3 \times 10^{-3}$	$\sim 3 \times 10^{-3}$	$\sim 3 \times 10^{-2}$	$10^3–10^4$
E ($V\,m^{-1}$):	$\sim kT_e/e\lambda_D$	$\sim 10^4$	$\sim 10^4$	500	0.1–1
ϕ_0 (V):	$\gtrsim 100kT_e/e$	30	15–20	15	$10^3–10^4$
ϱ/en_e:	0.1	10^{-3}	10^{-3}	10^{-2}	$\lesssim 10^{-2}$
N_D:	~ 100 per λ_D	$\sim 10^5$	$\gtrsim 3 \times 10^4$	10^6	$> 10^8$
Stability:	Stable	Chopping	Stable, due to step in tube cross section	Stable	Unknown

Code: n_e, T_e = electron density and temperature; λ_D = Debye length;
L_D, E, ϕ_0 = double layer thickness, typical electric field, potential drop;
ϱ = space charge density; N_D = number of electrons in Debye sphere.

(or equivalently the Buneman instability) threshold is no longer exceeded. A complete extinction of the discharge does not then occur, but rather some fluctuating current. When the layer has disappeared the effective resistance is decreased so the current can increase again until the DL reappears. This is called current chopping (Torvén and Babic, 1976).

Alternatively, a stationary level with a completely stable DL may be reached. This is particularly the case if a longitudinal gradient is present, e.g. in density or temperature. Such a gradient can be maintained by a longitudinal magnetic field with varying strength (mirror-field) as is the case in the upper ionosphere, or by a varying cross-section of the discharge tube, which is the case in the experiment by Andersson *et al.* (1969). A weak DL can then often exist even at currents far below the critical current given by Equation (13) if the gradient is sufficiently steep. Apparently two plasmas with different properties cannot match together without a potential jump. This is probably related to the finding by Lennartsson (1977) that a finite current along a magnetic mirror-field makes it difficult to find a velocity distribution such that quasi-neutrality can be maintained everywhere.

Smith's suggestion (Smith, 1977) that stability requires

$$N_D > m_i/m_e$$

(cf. Equation (24)), where N_D is the number of electrons within a Debye sphere, is to some extent corroborated by experiments. This is shown by Table I for a few experiments. A systematic study of DL with regard to this stability criterion would be very desirable.

6. Observations in Space

Several observations indicate the existence of electric fields parallel to the magnetic field in the upper ionosphere and the lower magnetosphere above the auroral zones. However, only experimental observations directly indicating DL will be mentioned here.

6.1. Strong density gradients in the topside ionosphere

Extremely strong density gradients have frequently been observed in the upper ionosphere. It is very suggestive to associate such steep gradients with strong potential drops. In fact, the DL theory can accommodate such gradients more easily than any other known theory.

The first observations of these density gradients were made with topside sounders. Calvert (1966) saw horizontal density variations of more than 60% per km in the topside F-layer. Hagg (1967) observed densities as low as 8–100 electrons per cm^3 in a great number of small regions at 1500–3000 km altitude over the auroral zone with horizontal density gradients of over 30% per km. Herzberg and Nelms (1969) observed sudden vertical density steps at about 200–300 km altitude.

Mozer *et al.* (1977) measured the plasma density with instrumentation onboard the S3-3 satellite at 2000–8000 km altitude. They found steep horizontal density gradients over the auroral zone of the same order of magnitude as those found by Calvert and Hagg.

Although an interpretation of steep gradients in terms of DL must be uncertain, perhaps controversial, it seems difficult to find any mechanism that could explain them except a DL, which must not necessarily be strong. On the other hand, we have seen that a DL can certainly exist without being associated with any net density jump whatsoever. This is well demonstrated by the observations of Mozer *et al.* (1977). In the only pass across the auroral zone, displayed in that paper, the two steepest density gradients are associated with strong electric fields ($\geqslant 100$ mV m^{-1}), but such strong fields are also observed in a few cases when the gradients are fairly weak.

6.2. Measurements of pitch-angle distributions

Using an unusually good resolution ($\sim 0.5°$) in pitch angle, Albert and Lindstrom (1970) found evidence indicating the existence of at least three DL in the F-layer at auroral latitudes. They observed the pitch-angle spectra of ~ 10 keV electrons with pitch angles Ψ in the range $75° < \Psi < 105°$. These spectra were peaked at certain angles, which varied systematically with altitude and energy in a way consistent with the interpretation that these particles were trapped between the magnetic mirror below, and at least three DL above the rocket. The altitudes of the layers could be estimated, and were found to be about 250, 270 and 280 km, with potential drops of 80, 160 and 160 V, respectively.

Although these observations were rather uncertain, as indicated by the rather large error bars in the diagrams, Albert's and Lindstrom's analysis of their data illuminates extremely well the interaction of a DL with energetic particles, and it is therefore recommended for a detailed study.

6.3. Barium jet experiments

During the last decade the barium cloud technique has been developed to a standard tool for measurements of perpendicular electric fields. More recently, the shaped charge method has enabled experimentalists to inject barium jets along the magnetic field to very high altitudes. If such a jet encounters a DL, one could expect four different effects which could all occur at the same time:

(i) The DL is weakened, perhaps even annihilated, due to the increase of the plasma density when the jet arrives.

(ii) The positive ions are accelerated, decelerated or even reflected, depending on the direction and strength of the DL potential drop.

(iii) The perpendicular drift of the Ba jet above the layer may differ from that below. This is due to the fact that a parallel potential drop which varies with a perpendicular coordinate allows the perpendicular electric field to change with the parallel coordinate even if curl $\mathbf{E} = 0$. Thus

$$\text{curl}\,\mathbf{E} = \text{curl}\,\mathbf{E}_{\parallel} + \text{curl}\,\mathbf{E}_{\perp} = 0,$$

which means that, if z is the parallel coordinate, we may for example have

$$\frac{\partial E_z}{\partial x} = \frac{\partial E_x}{\partial z} \neq 0.$$

(iv) If the jet hits the layer at a position where the DL electric field is nearly perpendicular to the magnetic field, a shear in the perpendicular velocity may occur as indicated in Figure 9. That part of the jet which first encounters the DL is peeled off and **E** × **B** drifts in the strong DL field.

Three different barium jet experiments have to date indicated the existence of DL at high altitudes above the auroral zone. Wescott *et al.* (1976) observed a 40 mV m^{-1} transverse electric field above (but not below) 5500 km. The jet was split in three streaks drifting away with different velocities. The Ba^+-ions also gained a parallel energy of at least 34 eV at this altitude. The authors found no other explanation to the behaviour than a DL. Evidently, at least some layers can survive a Ba jet attack.

Haerendel *et al.* (1976) observed an upward acceleration of Ba^+-ions through 190 eV at 2500 km during an experiment at Søndre Strømfjord, Greenland, on 17 December, 1974. On a second release at the same place on 11 January, 1975, the Ba^+-ions gained 7.4 ± 3 keV of parallel energy at 7500 km altitude. Although these results were consistent with a DL, a more distributed electric field, over an altitude range of perhaps a few hundred km, is not excluded. The authors do not want to conclude the existence of DL in these instances.

In Table II a summary of these measurements is given.

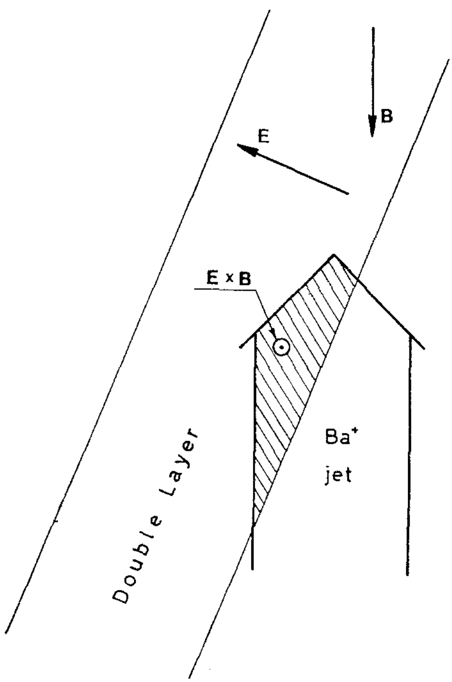

Fig. 9. A barium jet hitting an oblique double layer is peeled off due to fast **E** × **B** drift in the layer.

TABLE II

Observations of large, high-altitude auroral electric fields

Experiments	Observing situation	Altitude (km)	Associated currents	Observed		$\int E_{\parallel}\,dh$ (V)
				$E_{\perp}$ (mV m^{-1})	$E_{\parallel}$ (mV m^{-1})	
A. Satellites						
S3-3, Summer 1976 Vector Electric Fields (Mozer *et al.*, 1977)	Local dusk, 76° mag. lat.; (L = 7–20). Several opposing electric field pairs (~30 km transverse scale)	7600 (2000–8000)	$>10^{-6}$ A m^{-2}	400–800	200–800	Not reported
B. Rocket Barium Jets						
Poker Flat, Alaska 4 December 1973 Skylab Beta (Wescott *et al.*, 1976)	Local dusk, 67°3 invariant lat. Auroral omega band on same fieldline. 800 γ substorm	$\gtrsim$5500	0.2 A m^{-1} upward sheet, oriented EW, south of Ba^+	40	Not measured	Not reported
Søndre Strømfjord, 17 December 1974 (Haerendel *et al.*, 1976)	0930 MLT, 75° mag. lat. (polar cusp). Magnetically quiet. Only diffuse red aurora	$\gtrsim$2500	Not reported	Not reported	Not reported	190
Søndre Strømfjord, 11 January 1975 (Haerendel *et al.*, 1976)	0930 MLT, 75° mag. lat. (polar cusp). Over green auroral arcs	$\gtrsim$7500	Not reported	140 at 260 km	Not reported	7400

6.4. Electric Probe Measurements of DL Fields

The most clearcut evidence for the existence of DL above the auroral zones has been obtained from the S3-3 satellite, as reported by Mozer *et al.* (1977). They measured all three components of the DC electric field and the AC spectrum up to about 100 kHz. Magnetic field, plasma density and particle measurements were also made on the same satellite. The particle measurements enable a check of charging effects on the electric field probes, so as to exclude false signals due to that reason.

It was found that both parallel and perpendicular electric fields in the range of 100–1000 mV m^{-1} frequently occur at $L = 7$–20 at all altitudes (2000–8000 km) and local times (05–23 hours) reached by the satellite. They are positively correlated with magnetic activity and occur in regions where the magnetometer indicates Birkeland currents in excess of 10^{-6} A m^{-2}. The strong fields are confined in regions, which are a few km thick. Such regions often appear in pairs, where the perpendicular fields are directed towards each other, in agreement with the potential distribution shown in Figure 6.

The thickness of the strong *E*-field regions is a few km (*d* in Figure 6) and the strength of the field is several 100 mV m^{-1}, which requires a net charge density corresponding to about 10^4 electrons per m^3. Since the total electron density is of the order of 1–50 cm^{-3}, the relative deviation from charge neutrality is of the order of one electron per 100–5000, in excellent agreement with the expected value for a DL (cf. Table I). Furthermore, the AC noise level is at least two or three orders of magnitude too low to account for the anomalous resistivity consistent with the measured DC electric field (Shawhan *et al.*, 1977).

The number of electrons in a Debye-sphere, N_D, is sufficiently large ($\geqslant 10^8$) to satisfy Smith's necessary (but not sufficient) requirement for stability even if the temperature is as low as 1 eV and the density is the highest measured by Mozer *et al.*

Since no other known mechanism for production of parallel electric fields can account for the strong fields measured by Mozer *et al.* (see Block and Fälthammar, 1976; Shawhan *et al.*, 1977) it can be almost certainly concluded that they must be due to DL unless, for some now unknown reason, something should turn out to be wrong with the measurements.

7. Comparison With Earlier Results

Both the barium and satellite double probe results described in the previous section agree well with earlier ideas and observations. The potential distribution shown schematically in Figure 6 was suggested by Block (1969, 1972, 1975). Carlqvist and Boström (1970) found that it could explain shear motions of auroral forms seen in all-sky camera movies. Measurements of perpendicular electric fields on the Injun 5 satellite led Gurnett (1972) to propose the same kind of potential distribution. A detailed comparison between these electric field reversals and the inverted V precipitation events observed on the same satellite by Frank was found by Gurnett and Frank (1973) to be consistent with this picture.

Block (1972, 1975) has discussed the most likely DL altitudes. If a model of exospheric plasma properties as function of altitude can be constructed, it is an easy matter to determine where the threshold for the Buneman instability is most easily reached for a Birkeland current confined in a geomagnetic flux tube. Almost any reasonable model gives a minimum threshold somewhere between one half and two or three Earth radii. Therefore, it is not surprising that double layers have now been found in that altitude range.

The self-consistent models by Swift (1975, 1976) of oblique double layers predict a thickness of the order of 10 ion-gyro radii. This has also been confirmed by the satellite measurements, although Swift's assumption of very large scale lengths for the parallel electric field has not been borne out by the observations.

The results of the recent barium-shaped charge and S3-3 satellite double probe experiments have thus nicely confirmed and extended several earlier predictions and observations.

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