BIANCHI TYPE-II AND III MODELS IN SELF-CREATION COSMOLOGY

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Abstract. Spatially-homogeneous and anisotropic Bianchi type-II and III cosmological models are obtained in Barber's second self-creation theory of gravitation both in vacuum and in the presence of perfect fluid with pressure equal to energy density. Some properties of the models are discussed.

1. Introduction

Barber (1982) proposed two self-creation cosmologies by modifying the Brans and Dicke (1961) theory and general relativity. These modified theories create the Universe out of self-contained gravitational and matter fields. Recently, Brans (1987) has pointed out that Barber's first theory is not only in disagreement with experiment, but is actually inconsistent. Barber's second theory is a modification of general relativity to a variable G-theory. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples to the trace of the energy-momentum tensor. Hence, the field equations in Barber's second theory are

$$R_{ij} - \tfrac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} \tag{1}$$

and

$$\Box\phi = \frac{8\pi}{3}\lambda T, \tag{2}$$

where λ is a coupling constant to be determined from experiments. The measurements of the deflection of light restricts the value of the coupling to $|\lambda| < 10^{-1}$. In the limit $\lambda \to 0$ this theory approaches the standard general relativity theory in every respect. Barber (1982) and Soleng (1987) have discussed the F–R–W models while Reddy and Venkateswarlu (1989) have studied the Bianchi type-VI_0 cosmological model in Barber's second theory of gravitation.

In this paper we present Bianchi type-II and III cosmological solutions in Barber's second theory of gravitation both in vacuum and in the presence of perfect fluid with pressure equal to energy density. The models represent Bianchi type-II and Bianchi type-III vacuum and anisotropic Zel'dovich universes in self-creation cosmology.

2. Bianchi Type-II Models in Self-Creation Cosmology

We consider the non-flat Bianchi type-II metric (cf. Lorenz, 1980), in the form

$$dS^2 = -dt^2 + R^2 dx^2 + S^2 dy^2 + 2S^2 x\, dy\, dz + (S^2x^2 + R^2)\, dz^2 , \tag{3}$$

where R and S are functions of t only.

The energy-momentum tensor T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p) V_i V_j + p g_{ij} , \tag{4}$$

together with

$$g_{ij} V^i V^j = -1 ; \tag{5}$$

where V^i is the four-velocity vector of the fluid and p and ρ are the proper pressure and energy density, respectively. By the use of co-moving coordinates the field equations (1) and (2) for the metric (3) can be written as

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{1}{4}\frac{S^2}{R^4} = -8\pi\phi^{-1} p , \tag{6}$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{3}{4}\frac{S^2}{R^4} = -8\pi\phi^{-1} p , \tag{7}$$

$$\frac{R_4^2}{R^2} + \frac{2R_4 S_4}{RS} - \frac{S^2}{4R^4} = 8\pi\phi^{-1}\rho , \tag{8}$$

$$\phi_{44} + \frac{2\phi_4 R_4}{R} + \frac{\phi_4 S_4}{S} = \frac{8\pi\lambda}{3}(\rho - 3p) . \tag{9}$$

Again using correspondence to general relativity defining equivalent densities and pressures as

$$\rho_{\text{eq}} = \rho/\phi , \tag{10}$$

$$p_{\text{eq}} = p/\phi , \tag{11}$$

we can write the energy-conservation equation of general relativity (cf. Soleng, 1987) in the form

$$\left(\frac{\rho}{\phi}\right)_4 + \left(\frac{\rho}{\phi} + \frac{p}{\phi}\right)\frac{(R^2S)_4}{R^2S} = 0 , \tag{12}$$

where the subscript 4 denotes ordinary differentiation with respect to t.

2.1. Vacuum model

When $\rho = p = 0$, the field equations (6)–(12) reduce to vacuum case.

If we add Equations (6) and (8) we can write

$$[R(RS)_4]_4 = 0 . \tag{13}$$

By introducing a new variable T in Equation (13) such that

$$dt = R^2 S\, dT . \tag{14}$$

We can write Equation (13) as

$$[\log RS]'' = 0 , \tag{15}$$

where a dash stands for differentiation with respect to T.

The solution of Equation (15) is

$$RS = \exp(aT + b) , \tag{16}$$

where a and b are constants of integration.

From Equation (9) we get

$$\phi = k_1 T + k_2 , \tag{17}$$

Equation (16) allows us to write

$$\left(\frac{R_4}{R}\right)^2 + \frac{2R_4 S_4}{RS} + \left(\frac{S_4}{S}\right)^2 = \frac{a^2}{(R^2 S)^2} . \tag{18}$$

From Equations (8) and (18) we have

$$\left(\frac{S_4}{S}\right)^2 + \frac{1}{4}\frac{S^2}{R^4} = \frac{a^2}{(R^2 S)^2} ; \tag{19}$$

and by introducing T defined by (14) we finally have

$$(S^2)' = \pm S^2 [4a^2 - S^4]^{1/2} \tag{20}$$

with $S^4 \leq 4a^2$. Here the equality sign corresponds to the maximum value of S.

The solution of Equation (20) is

$$S^2 = 2a \operatorname{sech}[2a(T + m)] , \tag{21}$$

where m is a constant of integration.

By use of Equations (16) and (22) we obtain

$$R^2 = (2a)^{-1} \cosh[2a(T + m)] \exp[2(aT + b)] . \tag{22}$$

Thus the general solution of the field equations in vacuum can be written as

$$\begin{aligned} R^2 &= (2a)^{-1} \cosh[2a(T + m)] \exp[2(aT + b)] , \\ S^2 &= 2a \operatorname{sech}[2a(T + m)] , \end{aligned} \tag{23}$$

with $\phi = k_1 T + k_2$.

The corresponding metric of the solution can now be written in the form

$$\begin{aligned} dS^2 &= (2a)^{-1} \cosh[2a(T+m)] \exp[2(aT+b)] \times \\ &\quad \times \{-\exp[2(aT+b)]\, dT^2 + dx^2 + dz^2\} + \\ &\quad + 2a \,\mathrm{sech}[2a(T+m)]\, [dy + x\, dz]^2 \end{aligned} \tag{24}$$

with the scalar field given by Equation (17).

2.2. Zel'dovich Universe

When $\rho = p$, the field equations (6)–(12) reduce to stiff-fluid case and yield an exact solution given by

$$S^2 = A \,\mathrm{sech}[A(T+m)], \tag{25}$$

$$R^2 = A^{-1} \exp[2(aT+b)] \cosh[A(T+m)], \tag{26}$$

$$\phi = \cos\left[\sqrt{\frac{16\pi\lambda C_1}{3}}\, T + C_2\right], \tag{27}$$

where $A = 2(a^2 + 8\pi C_1)^{1/2}$, and a, b, C_1, C_2, and m are constants of integration.

Thus we get the following model for Zel'dovich universe in Barber's second self-creation theory

$$\begin{aligned} dS^2 &= A^{-1} \cosh[A(T+m)] \exp[2(aT+b)] \{-\exp[2(aT+b)]\, dT^2 + \\ &\quad + dx^2 + dz^2\} + A \,\mathrm{sech}[A(T+m)]\, [dy + x\, dz]^2 . \end{aligned} \tag{28}$$

The pressure p and the energy density ρ in the model (28) are given by

$$p = \rho = AC_1 \cos\left(\sqrt{\frac{16\pi\lambda C_1}{3}}\, T + C_2\right) \mathrm{sech}[A(T+m)] \exp[-4(aT+b)] . \tag{29}$$

For the reality of p and ρ, it is necessary that $C_1 > 0$. Here the pressure, density, and the scalar field are not singular at $T = 0$.

The spatial volume V of the model (28) is given by

$$V = A^{-1} \cosh[A(T+m)] \exp 4(aT+b), \tag{30}$$

which shows the expansion of the Universe with the time.

3. Bianchi Type-III Models in Self-Creation Cosmology

We consider the Bianchi type-III line element in the form

$$dS^2 = -dt^2 + e^{2\alpha} dx^2 + e^{2(\beta + x)} dy^2 + e^{2\gamma} dz^2 , \tag{31}$$

where α, β, γ are functions of cosmic time t. The introduction of a new time variable

T defined by

$$\mathrm{d}T = \frac{1}{e^{\alpha+\beta+\gamma}}\,\mathrm{d}t$$

transforms the metric (31) into

$$\mathrm{d}S^2 = -e^{2(\alpha+\beta+\gamma)}\,\mathrm{d}T^2 + e^{2\alpha}\,\mathrm{d}x^2 + e^{2(\beta+x)}\,\mathrm{d}y^2 + e^{2\gamma}\,\mathrm{d}z^2 . \tag{32}$$

The energy-momentum tensor is T_{ij} for perfect fluid distribution is given by Equations (4) and (5). By use of the co-moving coordinates the field equations (1) and (2) for the metric (32) can be written as

$$\beta_{44} + \gamma_{44} - \alpha_4\gamma_4 - \beta_4\gamma_4 - \alpha_4\beta_4 = -8\pi\phi^{-1}p\,e^{2(\alpha+\beta+\gamma)} , \tag{33}$$

$$\alpha_{44} + \gamma_{44} - \alpha_4\beta_4 - \beta_4\gamma_4 - \alpha_4\gamma_4 = -8\pi\phi^{-1}p\,e^{2(\alpha+\beta+\gamma)} , \tag{34}$$

$$\alpha_{44} + \beta_{44} - \alpha_4\beta_4 - \beta_4\gamma_4 - \alpha_4\gamma_4 - e^{2(\beta+\gamma)} = -8\pi\phi^{-1}p\,e^{2(\alpha+\beta+\gamma)} , \tag{35}$$

$$\alpha_4\beta_4 + \beta_4\gamma_4 + \alpha_4\gamma_4 - e^{2(\beta+\gamma)} = 8\pi\phi^{-1}\rho\,e^{2(\alpha+\beta+\gamma)} , \tag{36}$$

$$\alpha_4 - \beta_4 = 0 , \tag{37}$$

$$\phi_{44} = \frac{8\pi\lambda}{3}\,(\rho - 3p)\,e^{2(\alpha+\beta+\gamma)} . \tag{38}$$

From Equation (37) we have

$$\beta_4 = \alpha_4 , \tag{39}$$

which implies

$$\beta = \alpha + C ; \tag{40}$$

where C being the constant of integration.

By use of Equations (39) and (40) the field equations (33)–(38) reduce to

$$\alpha_{44} + \gamma_{44} - 2\alpha_4\gamma_4 - \alpha_4^2 = -8\pi\phi^{-1}p\,e^{2(2\alpha+C+\gamma)} , \tag{41}$$

$$2\alpha_{44} - \alpha_4^2 - 2\alpha_4\gamma_4 - e^{2(\alpha+C+\gamma)} = -8\pi\phi^{-1}p\,e^{2(2\alpha+C+\gamma)} , \tag{42}$$

$$\alpha_4^2 + 2\alpha_4\gamma_4 - e^{2(\alpha+C+\gamma)} = 8\pi\phi^{-1}\rho\,e^{2(2\alpha+C+\gamma)} , \tag{43}$$

$$\phi_{44} = \frac{8\pi\lambda}{3}\,(\rho - 3p)\,e^{2(2\alpha+C+\gamma)} . \tag{44}$$

By use of the correspondence to general relativity and defining equivalent densities and pressures as

$$\rho_{\mathrm{eq}} = \rho/\phi , \tag{45}$$

$$p_{\mathrm{eq}} = p/\phi . \tag{46}$$

We can write the energy-conservation equation of general relativity (cf. Soleng, 1987) in the form

$$\left(\frac{\rho}{\phi}\right)_4 + \left(\frac{\rho}{\phi} + \frac{p}{\phi}\right)(2\alpha_4 + \gamma_4) = 0\,, \tag{47}$$

where the subscript 4 denotes ordinary differentiation with respect to T.

3.1. Vacuum model

When $\rho = p = 0$, the field equations (41)–(44) and (47) reduce to vacuum case which admit the exact solution

$$\alpha = -\log(1 - C_1\, e^{2a_1T})\,, \tag{48}$$

$$\gamma = a_1 T + b_1\,, \tag{49}$$

$$\phi = aT + b\,; \tag{50}$$

where a, b, a_1, b_1, and C_1 are constants of integration. The corresponding vacuum model can now be written in the form

$$\begin{aligned} \mathrm{d}S^2 = & -(1 - C_1\, e^{2a_1T})^{-4} \exp[2C + 2a_1T + 2b_1]\, \mathrm{d}T^2 + \\ & + (1 - C_1\, e^{2a_1T})^{-2}[\mathrm{d}x^2 + \exp(2C + 2x)\, \mathrm{d}y^2] + \\ & + \exp[2(a_1T + b_1)]\, \mathrm{d}z^2\,, \end{aligned} \tag{51}$$

with the scalar field given by Equation (50).

Equation (51) describes the Bianchi type-III vacuum universe in Barber's second self-creation theory of gravitation. It may be observed that the model has no initial singularity.

3.2. Zel'dovich universe

When $\rho = p$, the field equations (41)–(47) reduce to the stiff-fluid case and yield an exact solution given by

$$e^{\alpha} = \frac{\exp\left[\left(\frac{Q - 4a}{4}\right)(T + k)\right]}{\left\{1 - \exp\left[\frac{Q}{2}\,(T + k)\right]\right\}}\,, \tag{52}$$

$$e^{\gamma} = \exp[aT + b]\,, \tag{53}$$

$$\phi = \cos\left(\sqrt{\frac{16\pi\lambda m}{3}}\; T + n\right); \tag{54}$$

where a, b, m, n, and k are constants and Q is given by

$$Q = \sqrt{16a^2 + 128\pi m}\,.$$

Thus we get the following model for Zel'dovich universe in Barber's second self-creation theory

$$\mathrm{d}S^2 = -\frac{\exp[(Q-4a)(T+k)+2(aT+b+C)]}{\left[1-\exp\frac{Q}{2}(T+k)\right]^4}\,\mathrm{d}T^2 + \\ + \frac{\exp\left[\left(\frac{Q-4a}{2}\right)(T+k)\right]}{\left\{1-\exp\left[\frac{Q}{2}(T+k)\right]\right\}^2}\,[\mathrm{d}x^2+\exp(2C+2x)\,\mathrm{d}y^2] + \\ + \exp[2(aT+b)]\,\mathrm{d}z^2\,, \qquad (55)$$

with the scalar field given by Equation (54).

The pressure p and the energy density ρ in the model (55) is given by

$$p=\rho=m\cos\left(\sqrt{\frac{16\pi\lambda m}{3}}\,T+n\right)\left[1-\exp\frac{Q}{2}(T+k)\right]^4\times \\ \times\exp[(4a-Q)(T+k)-2(aT+b+C)]\,,$$

where the pressure, density, and the scalar field are not singular at $T=0$. The volume element in the model (55) is

$$(-g)^{1/2}=\frac{\exp[(Q-4a)(T+k)+2(aT+b+C)+x]}{\left[1-\exp\frac{Q}{2}(T+k)\right]^4}\,, \qquad (56)$$

which shows the expansion of the Universe with time.

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