FINE STRUCTURE IN THE SUNSPOT SPECTRUM – 2 TO 70 YEARS

ROBERT G. CURRIE
Magnetic Observatory of the CSIR, Hermanus, Cape Province, South Africa

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Abstract. Application of a new data adaptive approach to power spectrum estimation has yielded evidence for a double solar cycle line in the Zurich sunspot time history. There is significant power from 8 to 15 yr in the spectrum with the primary line at 11.1 yr and three attendant multiplets that may be significant. The first four harmonics of the solar cycle are detected too. Quite marginal evidence for a peak at ~ 65 yr in the spectrum is presented. These results closely correspond to those recently found in the geomagnetic spectrum.

1. Introduction

Chapman and Bartels (1940) assess earlier work and draw no conclusions as to a possible double solar cycle variation in Zurich sunspot numbers. Chernosky and Hagan (1958) and Chernosky (1966) consider the problem further and are mute on this point too.

Chernosky's evidence for a double solar cycle variation in terrestrial magnetic activity is good. Currie (1973) used H and Z component data from 49 observatories and detected fundamental lines in the geomagnetic spectrum at 21.4 ± 2.4 and 10.5 ± 0.47 yr for 20th-century data. He also found the first four harmonics of the 10.5-yr line and eight of the first nine harmonics of the 21.4-yr line. This breakthrough in obtaining quantitative estimates of the long period geomagnetic spectrum was made possible by a new data adaptive method of power spectrum estimation and prompted a reexamination of Zurich sunspot numbers using the same technique.

Two data adaptive methods, Maximum Entropy and Maximum Likelihood, are currently under development (Lacoss, 1971). In this paper we apply the Maximum Entropy Method, or MEM, originated by Burg (1968) as outlined by Ulrych (1972). A brief heuristic overview of the method is given in the Appendix.

2. Data and Analysis

The source time history is mean monthly Zurich relative sunspot numbers from 1749 to 1957 given by Chernosky and Hagan (1958). This record was analysed whole, and further analysed by segmenting it into two records over time spans 1749–1853 and 1854–1957. A 101-point low pass filter, with cutoff at 0.0833 on the unit dimensionless Nyquist scale, was applied and each record decimated by 12 to yield annual values. A 41-point and 21-point high pass filter were then applied to the whole and the two shorter records, respectively, prior to MEM processing. The amplitude response of the

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41-point filter is unity at 11 yr and falls to 0.6 at 22 yr; that of the 21-point operator is 0.6 and 0.2 at these respective periods.

Ulrych (private communication, 1972) has determined a preliminary optimum criterion for prediction error coefficients PEC of between 33% and 50% record length. MEM has been applied to a variety of geophysical data (unpublished) and we find empirically that the characteristics of the data themselves determine the optimum PEC. These investigations point up the necessity of computing every spectrum for more than one PEC value until a parameter is found which, after each PEC is computed, gives one an objective basis on which to decide whether or not to truncate MEM iterations. PEC in the present study varies from 26% to 52% of record length. After MEM processing spectral estimates were routinely multiplied by the inverse power frequency response of the filters to recolor the spectrum.

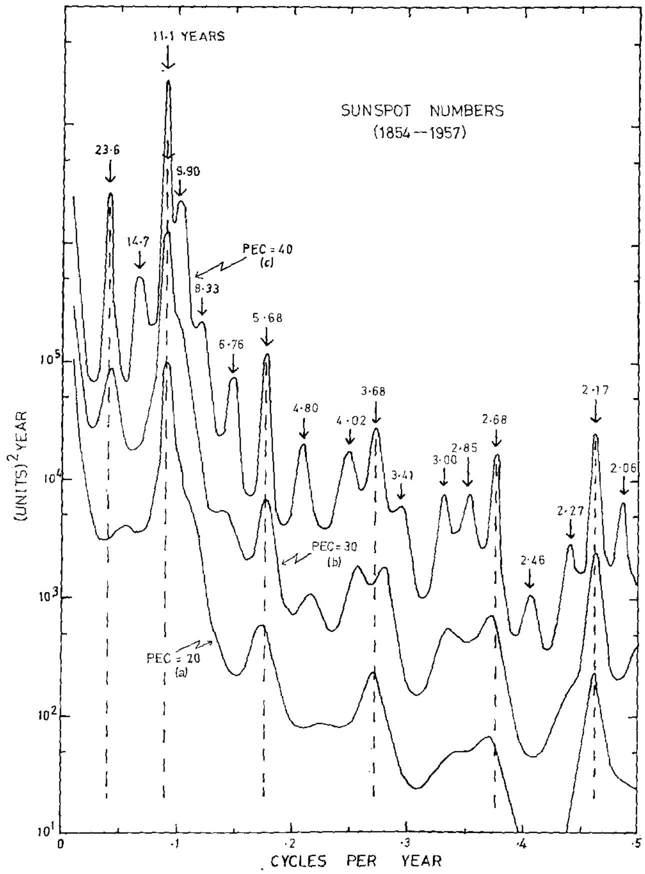

Fig. 1. Power spectra of sunspot numbers 1854–1957 where the double solar cycle line at 23.6 yr, the primary solar cycle line at 11.1 yr, and the primary harmonics are indicated by dashed vertical lines. The multiplet structure is discussed in the text. In this and Figure 2 the ordinate values apply to the (a) spectra and should be divided by 10 and 100 when applied to the (b) and (c) spectra, respectively.

3. Results

Figures 1 through 3 display MEM power spectra for sunspot numbers over the indicated time spans for three different PEC values. The lower spectra denoted by (a) in each figure are similar in respects to results obtained using the Blackman-Tukey method (for example, see Granger and Hatanaka, 1964). The nominal 11-yr line and first harmonic are resolved but in addition critical attention is drawn to peaks at higher frequencies. As PEC is increased MEM adapts or becomes better tuned to the frequency content of the data and begins sharpening all the broad peaks in curves (a). MEM further breaks these peaks into a fine multiplet structure and, at the low frequency end in Figures 1 and 3, detects a new line between 23 and 26 yr. In Figure 3 a line emerges at 66.6 yr but the two shorter records do not resolve it. Anderson (1939) assesses the history of the Zurich numbers noting that systematic observations were not begun until 1849 and that the earlier data possess a certain element of unreliability.

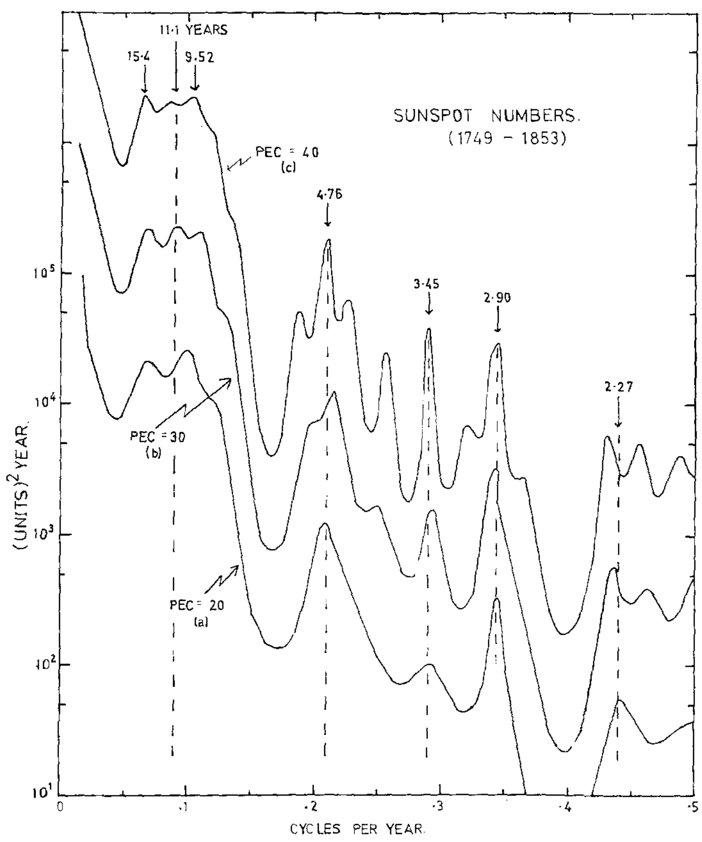

Fig. 2. Spectra of sunspot numbers 1749–1853 for three prediction error coefficient PEC values. The solar cycle band extends from 8 to 15 yr as in Figures 1 and 3. This and the previous Figure were computed from the 77 annual value data points available after low and high pass filtering operations.

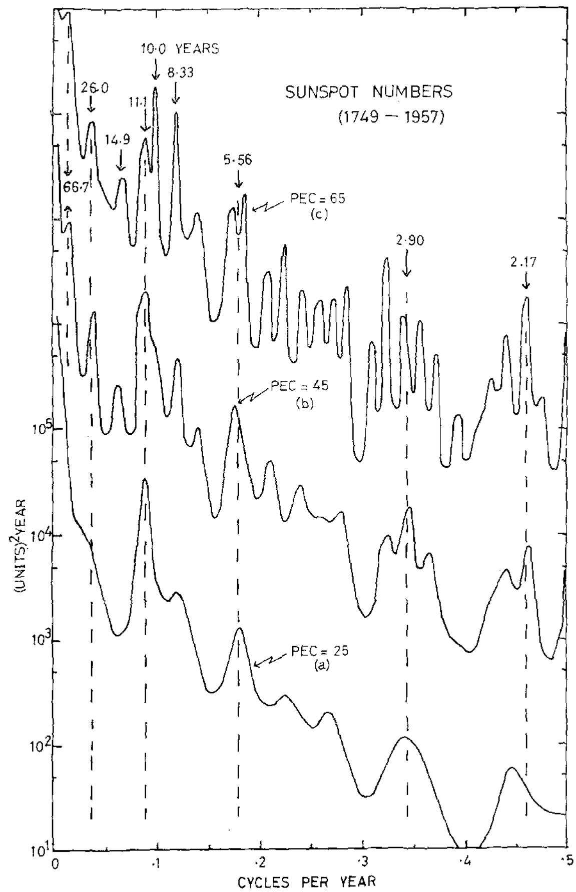

Fig. 3. Spectra of entire record of sunspot numbers 1749–1957. Most weight is given to the 1854–1957 record of Figure 1. Ordinate values apply to the (a) spectra and should be divided by 100 and 1000 when applied to the (b) and (c) spectra, respectively.

Thus, although all three figures are discussed below, most weight is given to the 1854–1957 record in Figure 1.

In Figure 1 dashed vertical lines point to six periods noted at the top. Increasing PEC clearly resolves a line at 23.6 yr in (b) and (c) that would not have been detected by classical methods for several reasons (see Appendix). As PEC increases to curve (c) in Figure 1 all the peaks in curve (a) are broken into multiplet structures. For the nominal solar cycle most of the power is still at 11.1 yr but there is significant power between 8 and 15 years with weak lines at 8.33, 9.90, and 14.7 yr.

Currie (1973) plots observed lines in spectra from 49 magnetic observatories against frequency, and finds they cluster at 21.4, 10.5, 7.10, 5.15, 4.35, 3.71, 3.39, 2.74, 2.38, and 2.15 yr. These periods correspond to the fundamental solar and double solar cycle lines and a series of harmonics associated with either or both. The great redundancy inherent in having records from 49 observatories available for analyses makes this evidence compelling. For sunspot numbers there is only one time history and the only redundancy possible is breaking the history into two segments as we have done; both are acceptable methods for increasing the confidence one has in the significance of lines that appear in spectra. A third method is the use of confidence limits based on statistical considerations which are currently under development for the MEM technique (Ulrych, private communication, 1972). We judge the six peaks in Figure 1 scored by dashed vertical lines as significant for three reasons: (1) similar lines appear in one or more of the other two spectra; (2) the signal to noise ratio is high and the lines generally dominant in curves (a–c); and (3) similar lines are well established in the geomagnetic spectrum (Currie, 1973). The lines are interpreted as the double solar cycle line (23.6 yr), the solar cycle line (11.1 yr), and its first four harmonics (5.7, 3.7, 2.7, and 2.2 yr). The percentage difference from 11.1 of the four harmonics averages 2%.

With only one time history, no positive interpretation of the remaining lines in Figure 1 is possible. Some could be harmonics of the double solar cycle line (as found in the geomagnetic spectrum) or the solar cycle multiplets (14.7, 9.90, 8.33 yr); some most probably have no significance. It is noteworthy that the multiplets of the nominal 11.1-yr solar cycle line in Figure 1 occur also in Figure 3, but at higher frequencies corresponding peaks in Figures 1 and 3 do not match well. Excluding in Figure 1 those peaks below eight years scored by dashed vertical lines, the period of every remaining peak is shown and can be fit into harmonic sequences of the solar cycle multiplets with a mean difference of 5%. While striking, no significance can be attached to this with only one time history available.

Figure 2 displays a broad band flat maximum between 8 and 15 yr. There is no line at 23.6 yr as in the previous figure but as noted earlier less confidence can be placed in the accuracy of this earlier record of the Zurich sunspot history. At higher frequencies dashed vertical lines point to peaks that correspond to harmonics of the solar cycle.

Figure 3 presents spectra for the entire 1749–1957 record. The multiplet line structure between 8 and 15 yr is again evidenced and a line at 26 yr detected. A small peak emerges at 66.7 yr. It is of doubtful significance as the response of the filter cuts unit amplitudes to 0.08 at this period. Figure 3 reflects the characteristics of both the shorter records and thus the results vis-à-vis Figure 1 are degraded. Only three harmonics of the nominal solar cycle are evident in curve (a) and one of these is no longer the dominant line on going to the (c) multiplet structure.

4. Discussion

In two of three spectra a line is detected between 23–26 yr and in Figure 1 the estimated signal to noise amplitude ratio is 5–6. Considering the variability in period of the sunspot cycle, these peaks are interpreted as the sunspot analogue of the basic or double sunspot solar cycle, where polarities of spots on either side of the Sun's equator reverse approximately every eleven years. This finding complements much better evidence for such a line in purely terrestrial data (Chernosky, 1966; Currie, 1973).

Fraser-Smith (1972) finds a line in the sunspot spectrum at 44 yr and nothing significant near 22 yr: our results are exactly converse.

It is reasonable to assume that whatever physical mechanism channels the 11-yr variation in sunspot numbers into magnetic activity on the Earth also holds for the nominal 22-yr variation. On short time scales the response of the magnetosphere to interplanetary conditions has two interaction modes (Hirshberg and Colburn, 1969). In one, magnetic activity correlates well with the dynamic pressure of the solar wind. In the other, it correlates well with the southward component of interplanetary magnetic fields. Either some such mode of interaction holds for time scales many orders of magnitude longer, or else these cycles are modulated by the observed short time scale variations. According to Ness (1971) no significant solar cycle changes in magnetospheric structure have been observed. However, Wilcox and Scherrer (1972) have inferred a 20-yr variation in predominant polarity (toward or away from the Sun) of the interplanetary field which they associate with the nominal 22-yr solar-magnetic cycle.

Chapman and Bartels (1940) state that individual solar cycles vary in length from 10 to 13 yr with a mean of 11.1 yr, while Menzel (1949) gives a range of 7.5 to 16 yr: presumably such estimates were based on visual examination of graphs. From Figures 1–3 we conclude that there is significant power in an 8- to 15-yr band and, weighting Figure 1 most heavily, the primary line in this band occurs at 11.1 yr; thus the range given by Menzel is favoured. Figures 1 and 3 suggest that peaks at 14.8, 10.0, and 8.3 yr may be significant. For 20th-century geomagnetic data Currie (1973) found that the mean solar cycle length is 10.5 ± 0.47 yr and significantly less than the nominal 11.1-yr period. Toman (1966) shows that from 1940–46 it averaged 10 yr and Chernosky (1966) concludes that the six cycles between 1901–64 have averaged less than 11 yr; all these results are consistent.

Figures 1–3 indicate that in the sunspot spectrum, as in the geomagnetic spectrum, harmonics of the nominal solar cycle line occur at approximately 5.5, 3.6, 2.8, and 2.2 yr. In the geomagnetic spectrum there are numerous harmonics of the nominal 22-yr line and such harmonics would be expected in the sunspot spectrum too. In the former, analyses of records from nearly 50 observatories were necessary before the evidence became compelling, whereas for the latter such redundancy is not available. All we can say is that some of the peaks below 8 yr may be harmonics of either the double solar cycle or the solar cycle multiplets. Harmonics of neither fundamental period have physical significance but their detection is important. For example Shapiro and Ward (1962) reported a peak in the sunspot spectrum at 25.7 month which we would interpret as the 4th harmonic of the solar cycle. Suggestions attributing this as the excitation mechanism for quasi-biennial stratospheric wind oscillations (see also Wescott, 1964) are untenable in our opinion.

A weak peak at roughly 65 yr appears in the spectrum of Figure 3. Fairly good evidence for a line near 60 yr in the geomagnetic spectrum has been found (Currie, 1973) and a relationship between velocity variations in westward drift of the geomagnetic field and fluctuations in speed of rotation of the Earth has also been noted in several studies (see Vestine and Kahle, 1968). The period of these latter fluctuations is roughly sixty years, and this period plays a prominent role in recent theoretical hydromagnetic studies of the Earth's core (Braginskiy, 1970a, 1970b). Although quite marginal, the sunspot evidence cannot be entirely dismissed and the excitation mechanism for such variations in the geomagnetic field is still open.

5. Summary

A line between 23 and 26 yr in the Zurich sunspot numbers is detected and, given the known variability of the nominal 11.1-yr solar cycle, is interpreted as the sunspot analogue of the 22-yr solar-magnetic-cycle. Much better evidence for a similar line is found in purely terrestrial data (Chernosky, 1966; Currie, 1973).

There is significant power from 8 to 15 yr in the sunspot spectrum. The primary line is at 11.1 yr with three attendent multiplets which may be significant. Harmonics of the solar cycle at approximately 5.5, 3.6, 2.8, and 2.2-yr are judged significant.

Quite marginal evidence for a peak at roughly 65 yr is found in the sunspot spectrum. Much better evidence for a similar line is found in the geomagnetic spectrum (Currie, 1973). Overall, there is quite a close correspondence between lines found in the sunspot and geomagnetic spectrum. Our results are substantially at variance with sunspot-magnetic index spectra published by Fraser-Smith (1972). For example we find no evidence for lines at 36–44 yr or 16 yr in either the geomagnetic or sunspot spectrum while Fraser-Smith reports significant lines at both periods in both sunspot and magnetic index spectra. We note that as yet no excitation mechanism for either period is known to exist on the Sun.

Acknowledgements

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Appendix

Most traditional computational methods for estimating the power spectrum are based on the approach of Blackman and Tukey (1959) or various versions of the Fast Fourier Transform. (For the most versatile and fast version see Singleton, 1969.) In the former the lagged autocorrelation function is estimated from the data, the function multiplied by a 'window', and the result Fourier transformed to obtain the spectrum; in the latter the squared magnitude of the FFT of the time series is computed and the magnitude squared is smoothed in the time or frequency domain to get a stable estimate of the spectrum. Following proper procedure, the end products of both methods are equivalent although many other variants are seen in practice. The advantage of the

FFT approach is principally a large saving of computer time both in computing the spectrum and filtering operations by Fast Convolution (see Stockham, 1969).

For a short time series (say of order 50 to 100 data points) the conventional approaches have severe limitations. Some authorities (for example Bendat and Piersol, 1966) recommend computation of the autocorrelation function with lags up to only 10% of record length although in practice lags up to 30% are often feasible. For 30%, with 50 data points, one could compute only 15 estimates of the spectrum. If the data were annual values and one expected a peak at 11 yr the lagged autocorrelation approach clearly cannot resolve the peak. Efforts along this line for geomagnetic data were tried repeatedly in the 60's and all failed (Eckhardt *et al.*, 1963; Currie, 1966; Banks and Bullard, 1966). Even the FFT approach without any smoothing would yield less than 25 estimates. A further additional limitation is the use of 'window' functions to gain stability. All 'windows' devised are independent of the data analyzed and broaden any spectral peaks present. Thus, the estimated spectrum tends to be the convolution of the window function and the true spectrum where the window does not depend upon the true spectrum. This is strikingly illustrated in Figure 4 of Lacoss' (1971) paper where, in contrast, MEM yields the true spectrum almost identically. For short time series, windows cannot be found which sufficiently resolve low frequency components of interest in the data. Instead, the low frequency portion of the spectrum becomes hopelessly smeared. The 'geomagnetic spectrum' from 4 to 33 yr presented by Currie (1968) is an excellent example of such smearing. While this 'spectrum' has proven of considerable value it bears little resemblance to the true spectrum in this period range (see Currie, 1973).

Two new nonlinear methods are now available for spectrum estimation: Maximum Likelihood Method (MLM) and Maximum Entropy Method (MEM). These methods have no fixed windows associated with them; rather, when power is being estimated at one frequency they adjust themselves to be least disturbed by power at other frequencies. In other words they are 'data adaptive', a concept with wide implications (Robinson, 1967). Both methods have been compared with the conventional approaches and found to show considerable promise (Lacoss, 1971). We will discuss briefly the Maximum Entropy Method.

The MEM expression for the power spectrum is

$$P(f) = \frac{(P_{M+1})\,\Delta}{\left|1 + \sum_{k=1}^{M} A_{Mk} e^{-2\pi i f k \Delta}\right|^2} \tag{1}$$

which, subject to a set of constraints, is derived by maximizing the expression

$$\int_{-u}^{u} \ln P(2\pi f \Delta)\, \mathrm{d}f\,, \tag{2}$$

where $u = 1/2\Delta$ is the Nyquist limit, Δ the sampling interval, and f the frequency. In (1), P_{M+1} is the updated mean output power of the $(M+1)$-point prediction-error filter whose first coefficient A_{11} is set equal to unity (for a discussion of prediction-error filters see Peacock and Treitel, 1969). The remaining coefficients are obtained in a iterative manner by combining the data with a matrix equation (see Ulrych, 1972). The iterative algorithm is due originally to Levinson (1948; see Appendix in Wiener, 1966).

By changing the symbolic meaning of the function and variables, those familiar with elements of information theory and statistical mechanics will recognize expression (2): in information theory it is the mathematical definition of information; in statistical mechanics it is the definition of entropy. The radically different approach initiated by Burg (1968) is thus based on maximizing two fundamental concepts in physics and present day communication theory. It is therefore not surprising that, although facets of MEM are still under development and need clarifying, the method is inherently superior to traditional approaches.

As noted earlier, with 50 annual data points, less than 25 estimates of the spectrum can be computed by conventional methods. For the same data, after computing the A_{Mk} coefficients and P_{M+1}, one can via Equation (1) compute as many estimates as one wishes. Thus, aside from ameliorating the 'window' difficulty one can obtain excellent resolution at the lowest frequencies. For example, with only 31 annual geomagnetic data values, Currie (1973) has detected the solar cycle; this could not be done by conventional methods. For the sunspot spectrum traditional approaches succeeded in resolving a line at 25.7 month (Shapiro and Ward, 1962) which they interpreted as a new fundamental cycle in the sunspot spectrum. With MEM it has been possible to elucidate more completely the nature of the spectrum beyond 2 yr and reinterpret this line as the 4th harmonic of the solar cycle.

Our high regard for MEM is based on approximately 2000 spectra computed and examined the past few months. As practical experience is gained certain empirical evidence arises. For much of our data we find that PEC must be roughly 50% record length to detect very low frequency line components. With such a PEC value too many lines at higher frequencies are thrown up, *i.e.* we believe MEM starts adapting to noise fluctuations. One solution is to decimate the time series and investigate portions of the spectrum in turn. Another is to compute the high frequency spectrum (say 0.25 u to u) for PEC = 33% and the rest for PEC = 50% using a variable resolution for the two segments. Echoing Lacoss (1971), when the length of available data is limited the MEM or MLM techniques should be seriously considered. MEM has also been applied to 'long' time series (Currie, unpublished). The series is broken into many (20 to 60) 'short' segments and the significance of a line is assessed on the basis of how often it occurs in all the spectra. This is a recognized procedure in spectrum analysis generally and reflects our basic distrust of statistical confidence limits applied to a single spectrum.

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