A ROLE OF COSMIC RAYS IN GENERATION OF RADIO AND OPTICAL RADIATION BY PLASMA MECHANISMS

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Abstract. The radiation of ultrarelativistic particles is examined in a quasi-uniform magnetic field superimposed by a wide spectrum of magnetic, electric, and electron density inhomogeneities created in a turbulent plasma. The radiation spectrum from a particle of a given energy is shown to acquire a high-frequency power-law tail with the same spectral index as the index ν of small-scale turbulence. For a power-law spectrum of ultrarelativistic electrons, $dN(\mathscr{E})/d\mathscr{E} \sim \mathscr{E}^{-\xi}$, with a cut-off at some energy $\mathscr{E}_{\max}$, the radiation spectrum consists of a few power-law ranges; the radiation intensity may suffer jumps at frequencies which separate these ranges.

In the high-frequency range the spectral index ν is determined by small-scale magnetic and electric fields. At intermediate frequencies the main contribution comes from the synchrotron radiation in a large-scale field; the radiation spectrum has an index $\alpha = (\xi - 1)/2$. The same index may be produced by large-scale Langmuir waves. At lower frequencies the radiation spectrum increases owing to the transition radiation caused by electron density fluctuations; in this case the spectral index is equal to $\xi + 1 - \nu$.

The possibility of diagnostics of high-frequency cosmic plasma turbulence from radiation of high-energy particles is discussed. It is shown that the proposed theory may explain some features in the spectra of several cosmic objects.

1. Introduction

The synchrotron radiation from ultrarelativistic electrons plays an important role in experiments with laboratory plasma (see, e.g., Bekefi, 1966) and also as a source of cosmic radioemission (Ginzburg and Syrovatskii, 1964). In the presence of a plasma turbulence an electron simultaneously interact with a more or less uniform regular magnetic field and random electric and magnetic fields of various scales produced by turbulent plasma pulsations. The radiation is also affected by plasma electrons and by inhomogeneities of electron number density.

The emission of electromagnetic waves due to the interaction of highly energetic particles with different modes of plasma turbulence has already been examined by a number of authors (see, e.g., Kaplan and Tsytovich, 1969; Bel'kov *et al.*, 1980; Ginzburg and Tsytovich, 1984, and references quoted therein). However, to our knowledge, these authors do not take into account a joint effect of regular magnetic field and turbulent pulsations. The only exception is the work of Tamoykin (1978) who analysed the transition radiation from ultrarelativistic particles due to electron-number-density inhomogeneities in a uniform magnetic field. Nevertheless, for realistic conditions the radiation spectra are usually formed under the joint action of a large-scale magnetic field and small-scale turbulent fields. A study of spectral and polarization properties of the radiation is important not only for clarifying the effect of the turbulence on the synchrotron emission but also for solving an inverse problem: how to investigate the properties of the turbulence using observational data on highly energetic particles. This

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second aspect is of special importance in astrophysics since nowadays direct methods to examine the turbulence in highly ionized hot regions are still absent.

In this paper we consider the formation of the radiation from an ensemble of ultrarelativistic particles in a magnetic field in the presence of turbulent pulsation. In Section 2 the radiation intensity is expressed through the distribution function of radiating particles, and the kinetic equation for the distribution function is derived. The latter equation is accurate enough to describe the behaviour of an ultrarelativistic particle over time-scales important for the problem under discussion. In Section 3 we obtain a solution to the kinetic equation and find the non-stationary distribution function of the radiating particle with account for a large-scale magnetic field, small-scale electric, and magnetic fields, and the density effect.

The emission spectrum produced by a separate ultrarelativistic particle is analysed in detail in Section 4. It is shown that for a small-scale turbulence with a minimum scale $l_{\min} \ll mc^2/eB$ (B being the large-scale magnetic field) the emission spectrum extends into the frequency range $\omega \gg \omega_c = \omega_B \gamma^2$, up to $\omega_{\max} \approx (c/l_{\min})\gamma^2$, where $\omega_B = eB/mc$, $\gamma = \mathscr{E}/mc^2$. In this range the spectrum is power-law, $I_\omega \propto \omega^{-\nu}$, where ν is the spectral index of the turbulence. At $\omega \lesssim \omega_c$ the spectrum is determined by a joint effect of regular and random fields, its shape being expressed via some integrals over single variable which depend on several parameters and are simplified in some limiting cases. We evaluate the spectrum numerically for various relations between the regular and random field components.

If $\omega \lesssim \omega_p \gamma$, the transition radiation due to plasma inhomogeneities becomes important. The role of the transition radiation is analysed in Section 5. The main effect is produced by the radiation concerned with fluctuations of electron number density, provided $\omega_B \ll \omega_p$ (which is often the case in astrophysics). Here ω_p is plasma frequency. That is why the transition radiation in the presence of developed MHD turbulence will be determined mainly by magneto-acoustic waves and shock fronts but not by Alfvén waves. The spectrum of the transition radiation in a low-frequency range is power-law, $I_\omega \propto \omega^{\nu-2}$.

In Section 6 we study the formation of the radiation spectra in radiosources with account for a large-scale magnetic field, small-scale random fields and the transition emission. If the energy distribution of radiating particles is power-law (with a cut-off at some energy $\mathscr{E}_{\max}$), the emission spectrum consists of several power-law parts whose slopes, boundaries and relative intensities may provide information on the turbulence in a radiating object.

In Section 7 we propose an interpretation of some available observational data on the ground of the developed theory with allowance for the radiation polarization. Section 8 presents a summary and main conclusions.

2. Radiation Intensity from a Relativistic Particle in Random Fields

The intensity of the particle emission in a magnetic field with random inhomogeneities in the presence of a turbulent plasma may be evaluated by the method developed by

Migdal (1954) to study the bremsstrahlung from ultrarelativistic particles in a medium. A Fourier component of the radiation field generated by one particle is given by

$$\mathbf{B}_{\mathbf{n},\,\omega} = \frac{ie}{2\pi c^2 R}\, e^{ikR} \int_{-\infty}^{+\infty} [\mathbf{k}, \mathbf{v}(t)]\, e^{i[\omega t - \mathbf{k}\mathbf{r}(t)]}\, \mathrm{d}t\,, \tag{1}$$

where e, $\mathbf{r}(t)$, and $\mathbf{v}(t)$ are the particle charge, radius-vector, and velocity, respectively; $\mathbf{n}$ is a unit vector along a line of sight, and $\mathbf{k}$ is the wave-vector of the radiating electromagnetic wave in a medium. Hereafter we adopt the condition

$$\omega_p \gg \omega_B \tag{2}$$

(ω_p and ω_B being the electron plasma- and electron-cyclotron frequencies, respectively), and will be interested in radiation frequencies $\omega \gg \omega_p$. Under these conditions, one can neglect the plasma gyrotropy and use a scalar dielectric function $\varepsilon(\omega) = 1 - (\omega_p/\omega)^2$, so that $\mathbf{k} = (\omega/c)\sqrt{\varepsilon}\,\mathbf{n}$. If in Equation (1) we keep the current produced by the emitting particle itself, we neglect the effects of the transition emission and transition scattering due to plasma inhomogeneities. These effects become important at sufficiently low frequencies $\omega \lesssim \omega_p \gamma$, and will be analysed in Sections 5–7.

An energy $\mathscr{E}_{\mathbf{n},\,\omega}$ radiated by the particle in a direction of $\mathbf{n}$ at a frequency ω is found as a flux of Pojnting vector within a unit solid angle. With the aid of (1) it may be written as

$$\mathscr{E}_{\mathbf{n},\,\omega} = \frac{e^2\omega^2}{2\pi^2 c^3} \operatorname*{Re}_{T\to\infty} \int_{-T}^{T} \mathrm{d}t \int_{0}^{\infty} \mathrm{d}\tau\, e^{i\omega\tau} \langle e^{-i\mathbf{k}[\mathbf{r}(t+\tau)-\mathbf{r}(t)]}\, [\mathbf{n}, \mathbf{v}(t+\tau)] \times$$
$$\times\, [\mathbf{n}, \mathbf{v}(t)] \rangle\,. \tag{3}$$

Brackets $\langle \cdots \rangle$ denote averaging over possible particle trajectories which are random owing to the presence of turbulent fields. The averaging may be expressed as

$$\langle e^{-i\mathbf{k}\mathbf{r}(t+\tau)+i\mathbf{k}\mathbf{r}(t)}\, [\mathbf{n}, \mathbf{v}(t+\tau)]\, [\mathbf{n}, \mathbf{v}(t)] \rangle = \int \mathrm{d}^3 v\, \mathrm{d}^3 v'\, \mathrm{d}^3 r\, \mathrm{d}^3 r' \times$$
$$\times\, e^{-i\mathbf{k}(\mathbf{r}'-\mathbf{r})}\, [\mathbf{n}, \mathbf{v}']\, [\mathbf{n}, \mathbf{v}] F(\mathbf{r}, \mathbf{v}, t) W(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}', \tau)\,, \tag{4}$$

where W is the probability that a particle in a state $(\mathbf{r}, \mathbf{v})$ at a moment t will appear in a state $(\mathbf{r}', \mathbf{v}')$ at a moment $t + \tau$. If the random field is statistically uniform and stationary, W depends only on the differences $\mathbf{r}' - \mathbf{r}$ and $t' - t = \tau$ but not on $\mathbf{r}$ and t. Also note that W obeys the initial condition

$$W(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; 0) = \delta(\mathbf{r} - \mathbf{r}')\, \delta(\mathbf{v} - \mathbf{v}')\,. \tag{5}$$

Furthermore, in Equation (4) $F(\mathbf{r}, \mathbf{v}, t)$ is the familiar particle distribution function. Let us adopt the initial condition

$$F(\mathbf{r}, \mathbf{v}, 0) = \delta(\mathbf{r} - \mathbf{r}_0')\, \delta(\mathbf{v} - \mathbf{v}_0)\,. \tag{6}$$

Let the particle move in the uniform magnetic field $\mathbf{B}_0$ superimposed by random magnetic and electric fields, $\mathbf{B}(\mathbf{r}, t)$ and $\mathbf{E}(\mathbf{r}, t)$, defined by corresponding correlation tensors. The probability W may be found from the kinetic equation averaged over fluctuations of random fields. The averaging procedure is well-known from the theory of particle scattering by random electromagnetic fields (Toptygin, 1985). However, in the case under discussion the kinetic equation should be derived with better accuracy because the conditional probability W must describe the particle motion over time-scales of the order of the time required to emit photons of given wavelengths. This typical time τ depends on radiation frequency ω and may appear to be shorter than the correlation time of small-scale fields. As for the standard scattering problems, they commonly require time-scales much larger than the correlation times.

For correct description of the particle emission at $\omega \gg \omega_p$, one needs to keep the time-integral in the collisional term of the kinetic equation (Toptygin, 1985)

$$\frac{\partial W}{\partial t} + \mathbf{v}\,\frac{\partial W}{\partial \mathbf{r}} - (\boldsymbol{\Omega}\,\mathcal{O})W + e\mathbf{E}\,\frac{\partial W}{\partial \mathbf{p}} =$$

$$= \int_0^\infty \mathrm{d}\tau \left\{ \left(\frac{ec}{\mathscr{E}}\right)^2 \mathcal{O}_\alpha T_{\alpha\beta}(\Delta\mathbf{r}(\tau), \tau)\,\mathcal{O}_\beta + \right.$$

$$+ e^2\,\frac{\partial}{\partial p_\alpha}\,K_{\alpha\beta}(\Delta\mathbf{r}(\tau), \tau)\,\frac{\partial}{\partial p_\beta} - \frac{e^2 c}{\mathscr{E}}\left(\mathcal{O}_\beta S_{\alpha\beta}\,\frac{\partial}{\partial p_\alpha} + \right.$$

$$\left.\left. + \frac{\partial}{\partial p_\alpha}\,S_{\alpha\beta}\,\mathcal{O}_\beta\right)\right\} W(\mathbf{r} - \Delta\mathbf{r}(\tau), \mathbf{p} - \Delta\mathbf{p}(\tau), t - \tau)\,. \tag{7}$$

In this case

$$\mathcal{O} = \left[\mathbf{v}, \frac{\partial}{\partial \mathbf{v}}\right],$$

$$\begin{aligned} T_{\alpha\beta}(\mathbf{r}, t) &= \langle B^{st}_\alpha(\mathbf{r}_1, t_1) B^{st}_\beta(\mathbf{r}_2, t_2)\rangle\,, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2\,, \\ K_{\alpha\beta}(\mathbf{r}, t) &= \langle E^{st}_\alpha(\mathbf{r}_1, t_1) E^{st}_\beta(\mathbf{r}_2, t_2)\rangle\,, \quad t = t_1 - t_2\,, \\ S_{\alpha\beta}(\mathbf{r}, t) &= \langle E^{st}_\alpha(\mathbf{r}_1, t_1) B^{st}_\beta(\mathbf{r}_2, t_2)\rangle \end{aligned} \tag{8}$$

are the correlation tensors of small-scale components of random fields. Under small-scale we mean those fields which contain harmonics with wave-numbers $k \geq k_*$, where the critical value k_* obeys the inequality

$$R \gg k_*^{-1} \gg R/\gamma\,, \tag{9}$$

where R is one of two Larmor radii: either $R_\perp = cp_\perp/e\,|\mathbf{B}_0 + \tilde{\mathbf{B}}|$ which corresponds to a particle moving with given pitch-angle in a large-scale field of definite direction; or $R_{st} = cp/e\,\sqrt{(\langle B^2_{st}\rangle)}$ that is a typical gyroradius in random fields. In this case $\tilde{B}$ denotes the large-scale component of random fields which contain harmonics $k < k_*$; $p_\perp = p \sin\vartheta$ is the transverse particle momentum (with respect to the large-scale field $\mathbf{B}_0 + \tilde{\mathbf{B}}$). If $\sin\vartheta\, |\mathbf{B}_0 + \tilde{\mathbf{B}}| > \sqrt{(\langle B_{st}^2 \rangle)}$ in Equation (9), one should set $R = R_\perp$, and in opposite case one should set $R = R_{st}$.

Equation (7) is simplified at $B_{st} \gg E_{st}$ (e.g., for MHD turbulence) or at $E_{st} \gg B_{st}$ (Langmuir turbulence). Under these conditions one may keep only one term, containing either $T_{\alpha\beta}$ or $K_{\alpha\beta}$, on the right-hand side of Equation (7). For an MHD turbulence, the phase velocities of waves are much smaller than the particle velocities. Hence, one can treat magnetic inhomogeneities as static ones, and neglect the second argument of $T_{\alpha\beta}$:

$$T_{\alpha\beta}(\Delta\mathbf{r}(\tau), \tau) = \int T_{\alpha\beta}(\mathbf{k})\, e^{i\mathbf{k}\Delta\mathbf{r}(\tau)}\, \mathrm{d}^3 k$$
$$= \tfrac{1}{3} \langle B_{st}^2 \rangle \{\psi(\Delta r)\, \delta_{\alpha\beta} + \psi_1(\Delta r) \Delta r_\alpha \Delta r_\beta / \Delta r^2\}\,. \tag{10}$$

The latter equality corresponds to the case of the static isotropic turbulence, and ψ_1 is expressed via ψ (see Toptygin, 1985).

Furthermore, variation of the particle velocity and momentum along the correlation length of random field is small, and

$$\Delta\mathbf{p}(\tau) = 0, \qquad \Delta\mathbf{r}(\tau) = \mathbf{v}\tau\,. \tag{11}$$

Finally, since $v_\beta \mathcal{O}_\beta = 0$, for MHD turbulence Equation (7) may be written as

$$\frac{\partial W}{\partial t} + \mathbf{v}\,\frac{\partial W}{\partial \mathbf{r}} - \mathbf{\Omega}\,\mathcal{O}\,W = \frac{1}{3}\,\langle B_{st}^2 \rangle \left(\frac{ec}{\mathscr{E}}\right)^2 \mathcal{O}^2 \int_0^\infty \mathrm{d}\tau\, \psi(v\tau) W(\mathbf{r} - \mathbf{v}\tau, \mathbf{p}, t - \tau)\,. \tag{12}$$

For a Langmuir turbulence, the phase velocities of sufficiently large-scale harmonics may exceed the velocity of light: namely, $v_{ph}^l \approx \omega_p / k \gtrsim c$ at $l \gtrsim 2\pi c/\omega_p$. This does not allow one to omit the second argument in the correlator $K_{\alpha\beta}$. In the isotropic case this correlator takes the form

$$K_{\alpha\beta}(\mathbf{v}\tau, \tau) = \int |E|^2_{\mathbf{k}} (k_\alpha k_\beta / k^2)\, e^{i(\mathbf{kv} - \omega_{\mathbf{k}})\tau}\, \mathrm{d}^3 k$$
$$= \tfrac{1}{3} \langle E_{st}^2 \rangle \left\{\vartheta(v\tau, \tau)\, \delta_{\alpha\beta} + \vartheta_1(v\tau, \tau) v_\alpha v_\beta / v^2\right\}. \tag{13}$$

The correlator $K_{\alpha\beta}$ is real, if $\omega_{-\mathbf{k}} = -\omega_{\mathbf{k}}$; in the case of a Langmuir turbulence without account for thermal corrections one has $\omega_{\mathbf{k}} = \omega_p = (4\pi N e^2/m_e)^{1/2}$. The kinetic equation may be simplified because radiating particles are ultra-relativistic. In this case the operator $(\partial/\partial p_\alpha) K_{\alpha\beta} (\partial/\partial p_\beta)$ may be replaced by $(1/3p^2) \langle E_{st}^2 \rangle\, \mathcal{O}^2 \vartheta(v\tau, \tau)$. The relative error introduced by this replacement does not exceed γ^{-2}, that is the ratio of the 'transverse' mass of the relativistic particle to its 'longitudinal' mass. Finally, we obtain that Equation (12) remains valid for a Langmuir turbulence, if one replaces $\langle B_{st}^2 \rangle \to \langle E_{st}^2 \rangle$, $\psi(v\tau) \to \vartheta(v\tau, \tau)$.

3. Solution of the Kinetic Equation

To find the conditional probability W from Equation (12) let us carry out Fourier transformations with respect to coordinates and time. In this case, according to (3) and (4), frequency ω and wave-vector $\mathbf{k}$ correspond to a radiating wave

$$-i(\omega - \mathbf{kv}) W_{\mathbf{k},\omega} - \mathbf{\Omega}\, \mathcal{O}\, W_{\mathbf{k},\omega} = \mathcal{O}^2 q(\omega, \theta) W_{\mathbf{k},\omega}\,, \tag{14}$$

where

$$q(\omega, \theta) = \frac{1}{3}\, \langle B_{st}^2 \rangle \left(\frac{ec}{\mathscr{E}}\right)^2 \int_0^\infty \psi(v\tau)\, e^{i(\omega - \mathbf{kv})\tau}\, \mathrm{d}\tau\,. \tag{15}$$

The exponent in the latter integral depends on an angle θ between $\mathbf{k}$ and $\mathbf{v}$. Hence, we have the second-order differential Equation (14) whose coefficients are complicated functions of independent variable θ. This equation cannot be solved analytically. To avoid this difficulty we approximate $q(\omega, \theta)$ by an expression which is independent of θ. Let us make use of the fact that relativistic particles emit radiation into a narrow cone with a half-cone angle $\theta_0 \sim \gamma^{-1}$ and replace θ^2 by some quantity $(a-1)\gamma^{-2}$, with $a - 1 \approx 1$. The value of a may be determined, for instance, from the requirement that at high frequencies, where the perturbation expansion (the method of equivalent photons, see Appendix) is valid, the present method yields the same result as the perturbation expansion. For power-law spectra of random fields, this gives

$$a = [\tfrac{4}{3}(\nu + 2)]^{1/\nu}\,.$$

Then, at small θ we have

$$\omega - \mathbf{kv} = \frac{\omega}{2}\, [\gamma^{-2} + \theta^2 + \omega_p^2/\omega^2] = \frac{\omega}{2}\, (a\gamma^{-2} + \omega_p^2/\omega^2)\,, \tag{16}$$

where the dielectric function $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ is used.

Let $q(\omega)$ denote $q(\omega, \theta)$ at $\theta = \sqrt{(a-1)}\gamma^{-1}$. We consider now instead of Equation (14), different equations

$$\frac{\partial W_{\mathbf{k}}(\tau)}{\partial \tau} + (i\mathbf{kv} - \mathbf{\Omega}\, \mathcal{O}) W_{\mathbf{k}}(\tau) = q(\omega)\, \mathcal{O}^2 W_{\mathbf{k}}(\tau)\,. \tag{17}$$

The effective scattering rate $q(\omega)$ is a complex function of ω. The quantity $W_{\mathbf{k},\omega}$ to be found is the Fourier transform of $W_{\mathbf{k}}(\tau)$ with respect to τ, taken at a given frequency ω.

Let us simplify Equation (17) by means of several substitutions. The substitution

$$W_{\mathbf{k}}(\tau) = \exp(\mathbf{\Omega}\, \mathcal{O}\, \tau) f_{\mathbf{k}}(\theta, \tau) \tag{18}$$

reduces (17) to the equation

$$\frac{\partial f_{\mathbf{k}}}{\partial \tau} + i\mathbf{kv}(\tau) f_{\mathbf{k}} = q(\omega)\, \mathcal{O}^2 f_{\mathbf{k}}\,, \tag{19}$$

where

$$\mathbf{v}(\tau) = \exp(-\boldsymbol{\Omega}\,\mathcal{O}\,\tau)\mathbf{v}\exp(\boldsymbol{\Omega}\,\mathcal{O}\,\tau) = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}\cos\Omega\tau + [\mathbf{b}, \mathbf{v}_{\perp}]\sin\Omega\tau \quad (20)$$

is the particle velocity in a uniform magnetic field; $\mathbf{b}$ is a unit vector along the large-scale field $\mathbf{B}_0 + \tilde{\mathbf{B}}$. Substituting

$$f_{\mathbf{k}}(\theta, \tau) = v^{-2}\,\delta(v - v_0)\exp\left[-i\,\frac{\omega v}{c}\,(1 - \omega_p^2/2\omega^2)\tau\right]u(\boldsymbol{\theta}_0, \boldsymbol{\theta}, \tau) \quad (21)$$

and expanding the coefficients of the equation up to second-order terms over small angles

$$\boldsymbol{\theta} = \frac{\mathbf{v}}{v} - \mathbf{n}(1 - \theta^2/2),\ \boldsymbol{\theta}_0 = \frac{\mathbf{v}_0}{v_0} - \mathbf{n}(1 - \theta_0^2/2),\ \boldsymbol{\Omega}\tau \quad (22)$$

we obtain the final equation for $u(\boldsymbol{\theta}_0, \boldsymbol{\theta}, \tau)$,

$$\frac{\partial u}{\partial \tau} - \frac{i\omega}{2}\,(\boldsymbol{\theta} - [\mathbf{n}, \boldsymbol{\Omega}]\tau)^2 u = q(\omega)\Delta_\theta u\,. \quad (23)$$

In this case $\Delta_\theta = \partial^2/\partial\theta_x^2 + \partial^2/\partial\theta_y^2$ is the Laplace operator with respect to $\boldsymbol{\theta}$, $\boldsymbol{\theta}_0$ is an initial-angle $\boldsymbol{\theta}$. According to (18), (21), and (5),

$$u(\boldsymbol{\theta}_0, \boldsymbol{\theta}, 0) = \delta(\boldsymbol{\theta}_0 - \boldsymbol{\theta})\,. \quad (24)$$

A solution to Equation (23) may be sought in the form

$$u = \exp\{\alpha(\boldsymbol{\theta} - [\mathbf{n}, \boldsymbol{\Omega}]\tau)^2 + \beta\boldsymbol{\theta}\boldsymbol{\theta}_0 + \xi\theta_0^2 + \delta\boldsymbol{\theta}[\mathbf{n}, \boldsymbol{\Omega}] + \varepsilon\boldsymbol{\theta}_0[\mathbf{n}, \boldsymbol{\Omega}] + \gamma\}\,, \quad (25)$$

where α, β, ξ, δ, ε, and γ are functions of τ. Substituting this solution into (23) and separating coefficients at different powers of $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_0$, we come to the set of ordinary nonlinear equations,

$$\begin{aligned}
&\dot{\alpha} - \tfrac{1}{2}i\omega = 4q\alpha^2\,, && \dot{\delta} - 2\alpha = 4q\alpha\delta\,, \\
&\dot{\beta} = 4q\alpha\beta\,, && \dot{\varepsilon} = -4q\alpha\beta\tau + 2q\beta\delta\,, \\
&\dot{\xi} = q\beta^2\,, && \dot{\gamma} = q(4\alpha + \Omega_\perp^2\delta^2) - 2\alpha\Omega_\perp^2\tau(1 + 2q\delta)\,.
\end{aligned} \quad (26)$$

In this case $\Omega_\perp^2 = [\mathbf{n}, \boldsymbol{\Omega}]^2$ is the squared component of the gyrofrequency transverse to line-of-sight. These equations may be solved successively, beginning from the first one. The integration constants are chosen in such a way to satisfy the initial condition (24). Then we have

$$\begin{aligned}
&\alpha(\tau) = -x\coth z\tau\,, \qquad \beta(\tau) = 2x\sinh^{-1}z\tau\,, \qquad \xi(\tau) = -x\coth z\tau\,, \\
&\delta(\tau) = -1/2q\,, \qquad \varepsilon(\tau) = -2x\tau\sinh^{-1}z\tau + 1/2q\,, \\
&\gamma(\tau) = -\ln(\pi\sinh z\tau/x) + \Omega_\perp^2\tau/4q\,,
\end{aligned} \quad (27)$$

where

$$x = (1 - i)(\omega/16q)^{1/2}, \qquad z = (1 - i)(\omega q)^{1/2}. \tag{28}$$

Use of Equations (18), (21) and evaluation of $W_k(\theta, \tau)$ from $u(\boldsymbol{\theta}_0, \boldsymbol{\theta}, \tau)$ (in this case the action of the operator $\exp(\boldsymbol{\Omega}\,\mathcal{O}\,\tau)$ reduces to the replacement $\boldsymbol{\theta} \to \boldsymbol{\theta} + [\mathbf{n}, \boldsymbol{\Omega}]\tau$), we obtain the distribution function in the form

$$W_{\mathbf{k}}(\boldsymbol{\theta}, \tau) = v^{-2}\,\delta(v - v_0)\exp\left[-i\,\frac{\omega v}{c}\,(1 - \omega_p^2/2\omega^2)\tau\right] w(\boldsymbol{\theta}_0, \boldsymbol{\theta}, \tau),$$

where

$$w(\boldsymbol{\theta}_0, \boldsymbol{\theta}, \tau) = \frac{x}{\pi \sinh z\tau} \exp\left\{-x^2(\theta^2 + \theta_0^2)\coth z\tau + 2x\boldsymbol{\theta}\boldsymbol{\theta}_0 \sinh^{-1} z\tau - \right.$$
$$\left. - \frac{1}{2q}\,(\boldsymbol{\theta} - \boldsymbol{\theta}_0)[\mathbf{n}, \boldsymbol{\Omega}] - \frac{1}{4q}\,\Omega_\perp^2\,\tau\right\}. \tag{29}$$

In particular, this distribution function describes the particle motion in a uniform magnetic field. To analyse such a case one should switch off the effect of magnetic inhomogeneities ($q \to 0$). For this to be done one should expand $\coth(z\tau)$ and $\sinh^{-1}(z\tau)$ in Equation (29) into the Taylor series, keeping terms $\sim (z\tau)$.

4. The Spectrum of Intrinsic Radiation of Ultrarelativistic Particles

Since we are interested in statistically uniform and stationary fields, the radiation energy (which may generally be calculated from Equation (3)) appears to be proportional to time. To find an energy radiated per unit time (i.e., the radiation intensity $I_{\mathbf{n},\,\omega}$) one must divide (3) by the total radiation time $2T$. This is equivalent to omitting the integration over $\mathrm{d}t$. Let us integrate the radiation intensity over directions $\mathbf{v}_0$. Finally, making use of (29), we come to the expression

$$I_\omega = \frac{e^2\omega^2}{2\pi^2 c}\,\mathrm{Re}\int_0^\infty \mathrm{d}\tau \exp\left[\frac{i\omega\tau}{2\gamma^2}\left(1 + \frac{\omega_p^2\gamma^2}{\omega^2}\right)\right]\int \mathrm{d}^2\theta\,\mathrm{d}^2\theta'\,(\boldsymbol{\theta}\boldsymbol{\theta}')w(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau), \tag{30}$$

where the dependence on an angle between $\mathbf{n}$ and the magnetic field is still remained. Note that $\int F(\mathbf{r}, \mathbf{v}, t)\mathrm{d}^3r\,\mathrm{d}v\,\mathrm{d}^2\theta_0 = 1$ due to the normalization of the distribution function.

While we integrate the right-hand side of Equation (30), w may be conveniently replaced by $w(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau) - w^0(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau)$, where

$$w^0(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau) = \delta(\boldsymbol{\theta} - \boldsymbol{\theta}')\exp(i\omega\theta^2\tau/2). \tag{31}$$

is the free-particle distribution function which does not contribute into the radiation.

After this replacement the result of integration can be represented in the form

$$\int \mathrm{d}^2\theta' (\boldsymbol{\theta\theta}') \left[w(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau) - w^0(\boldsymbol{\theta}, \boldsymbol{\theta}', \tau) \right]$$

$$= -\frac{2i}{\omega} \frac{\mathrm{d}}{\mathrm{d}\tau} \left[\exp \left\{ -\theta^2 x \tanh z\tau - \frac{1}{2q} \boldsymbol{\theta}[\mathbf{n}, \boldsymbol{\Omega}] (1 - \cosh^{-1} z\tau) - \right. \right.$$

$$\left. \left. - \frac{\Omega_\perp^2}{4q} \left(\tau - \frac{\tanh z\tau}{4qx} \right) \right\} - \exp(i\omega\theta^2 \tau/2) \right] - \left[\frac{\boldsymbol{\theta}[\mathbf{n}, \boldsymbol{\Omega}]}{4qx} \frac{\sinh z\tau}{\cosh^2 z\tau} + \right.$$

$$\left. + \frac{i\Omega_\perp^2}{2q\omega} \tanh^2 z\tau \right] \exp \left\{ -\theta^2 x \tanh z\tau - \frac{\boldsymbol{\theta}[\mathbf{n}, \boldsymbol{\Omega}]}{2q} (1 - \cosh^{-1} z\tau) - \right.$$

$$\left. - \frac{\Omega_\perp^2}{4q} \left(\tau - \frac{\tanh z\tau}{4qx} \right) \right\}. \tag{32}$$

Integration of the first term over τ by parts, with omitting rapidly oscillating exponents, and subsequent integration over θ in accordance with (30) enable us to represent I_ω as the integral over single variable τ,

$$I_\omega = -\frac{e^2\omega^2}{2\pi c \gamma^2} \left(1 + \frac{\omega_p^2 \gamma^2}{\omega^2} \right) \mathrm{Re} \int_0^\infty \mathrm{d}\tau \exp \left[\frac{i\omega\tau}{2\gamma^2} \left(1 + \frac{\omega_p^2 \gamma^2}{\omega^2} \right) \right] \times$$

$$\times \left[\frac{\coth z\tau}{x} \exp \left\{ \frac{\Omega_\perp^2}{8q^2 x} (\coth z\tau - \sinh^{-1} z\tau) - \frac{\Omega_\perp^2 \tau}{4q} \right\} - \frac{2i}{\omega\tau} \right] +$$

$$+ \frac{e^2\omega^2}{4\pi c} \mathrm{Re} \frac{i\Omega_\perp^2}{q\omega x} \int_0^\infty \mathrm{d}\tau \frac{1 - \cosh z\tau}{\sinh z\tau} \exp \left\{ \frac{i\omega\tau}{2\gamma^2} \left(1 + \frac{\omega_p^2 \gamma^2}{\omega^2} \right) - \right.$$

$$\left. - \frac{\Omega_\perp^2 \tau}{4q} + \frac{\Omega_\perp^2}{8q^2 x} (\coth z\tau - \sinh^{-1} z\tau) \right\}. \tag{33}$$

To separate real and imaginary parts of (33) one should take into account that x and z are expressed through the complex parameter

$$q(\omega) = q'(\omega) + iq''(\omega) = |q(\omega)| \, e^{i\vartheta}. \tag{34}$$

Let us perform integration over the complex variable $t = z\tau$. The integration contour in the t-plane represents a ray inclined at an angle $(\vartheta/2) - (\pi/4)$ to the axis $t' = \mathrm{Re}\, t$. The integrand has no peculiarities in the sector between this ray and the t'-axis. Hence, we may turn the integration contour to the t'-axis. As a result, we obtain the spectral radiation power in the form

$$I_\omega = \frac{8e^2 q'(\omega)}{3\pi c}\,\gamma^2\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right)^{-1}\Phi_1(s_1, s_2, r) + $$

$$+\frac{e^2\omega}{4\pi c\gamma^2}\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right)\Phi_2(s_1, s_2, r)\,, \tag{35}$$

where Φ_1 and Φ_2 are given by the integrals

$$\Phi_1 = \frac{6\,|s|^4}{s_1 s_2}\,\mathrm{Im}\int_0^\infty \mathrm{d}t\,\exp(-2st)\left\{\coth t\,\times\right.$$

$$\left.\times\,\exp\left[-2rs^3\left(\coth t - \sinh^{-1} t - \frac{t}{2}\right)\right] - \frac{1}{t}\right\},$$

$$\Phi_2 = 2r\,|s|^2\,\mathrm{Re}\int_0^\infty \mathrm{d}t\,\frac{\cosh t - 1}{\sinh t}\,\times \tag{36}$$

$$\times\,\exp\left[-2st - 2rs^3\left(\coth t - \sinh^{-1} t - \frac{t}{2}\right) - i\vartheta\right],$$

depending on dimensionless parameters s_1, s_2, r,

$$s = s_1 - is_2 = \frac{\exp(-i\pi/4 - i\vartheta/2)}{4\sqrt{2}\,\gamma^2}\left(\frac{\omega}{|q(\omega)|}\right)^{1/2}\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right);$$

$$r = 32\gamma^6\left(\frac{\Omega_\perp}{\omega}\right)^2\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right)^{-3}. \tag{37}$$

The functions Φ_1 and Φ_2 generalize the function

$$\Phi(s) = 24s^2\int_0^\infty \exp(-2st)\,\sin(2st)\,(\coth t - t^{-1})\,\mathrm{d}t\,,$$

introduced by Migdal (1954) when analysing the bremsstrahlung in a medium, to the case of regular magnetic field and random turbulent fields. On the other hand, these functions generalize the function

$$\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^\infty K_{5/3}(\eta)\,\mathrm{d}\eta\ ,$$

which describes the radiation in a uniform magnetic field to the case when small-scale electric and magnetic fluctuations are present.

The parameter s depends on the particle scattering rate $q(\omega)$. Let us calculate $q(\omega)$ for the correlator of the form

$$\psi(v\tau) = \frac{2^{(3-\nu)/2}}{\Gamma(\nu/2 - \frac{1}{2})} (\omega_* \tau)^{(\nu-1)/2} K_{(\nu-1)/2}(\omega_* \tau) . \tag{38}$$

This choice of the correlator corresponds to a power-law dependence of the energy-density of the small-scale fields on wave-number, $P(k) \propto k^{-\nu}$ at $k \gg k_* = \omega_*/c$ (Toptygin, 1985). According (15) and (16),

$$\begin{aligned} q(\omega) &= \tfrac{1}{3} \langle B_{st}^2 \rangle (ec/\mathscr{E})^2 \int_0^\infty \psi(v\tau)\, e^{i\alpha\tau}\, \mathrm{d}\tau = \\ &= \frac{\sqrt{\pi}\,\Gamma(\nu/2)}{3\Gamma(\nu/2 - \frac{1}{2})} \frac{\omega_{st}^2 \omega_*^{\nu-1}}{\gamma^2(\alpha^2 + \omega_*^2)^{\nu/2}} + \\ &\quad + i\, \frac{(\nu - 1)\omega_{st}^2 \alpha}{3\gamma^2 \omega_*^2} F\left(\frac{\nu+1}{2}, 1, \frac{3}{2}; -\frac{\alpha^2}{\omega_*^2}\right), \end{aligned} \tag{39}$$

where $\omega_{st}^2 = e^2 \langle B_{st}^2 \rangle m^{-2} c^{-2}$ is the squared non-relativistic frequency of the radiating particle in a random field; α is given by (16), i.e., $\alpha = (\omega/2)(a\gamma^{-2} + \omega_p^2/\omega^2)$ and $F(a, b, c; z)$ is a hypergeometric function. Its value is very sensitive to the ratio

$$\beta = (\omega/2\omega_*) \left(\frac{a}{\gamma^2} + \frac{\omega_p^2}{\omega^2}\right),$$

which reaches minimum, $\beta_{\min} = \omega_p a^{1/2}/\omega_* \gamma$, at $\omega = \omega_p \gamma a^{-1/2}$. If $\beta_{\min} \gg 1$, which takes place at $L_* \gg 2\pi R a^{-1/2}(\omega_B/\omega_p)$ and is compatible with (9) in accordance with (2), the Fourier transform of (38) is noticeably simplified at all frequencies,

$$q(\omega) = \frac{\sqrt{\pi}\, 2^\nu \Gamma(\nu/2)}{3\Gamma(\nu/2 - \frac{1}{2})} \frac{\omega_{st}^2 \omega_*^{\nu-1}}{(a\gamma^{-2} + \omega_p^2/\omega^2)^\nu \omega^\nu \gamma^2} + i\, \frac{2\omega_{st}^2}{3\gamma^2 \omega(a\gamma^{-2} + \omega_p^2/\omega^2)} . \tag{40}$$

In this case $1 < \nu < 2$. At $\nu = 2$ we have

$$q(\omega) = \frac{4\omega_{st}^2 \omega_*}{3\gamma^2 \omega^2 (a\gamma^{-2} + \omega_p^2/\omega^2)^2} + i\, \frac{2\omega_{st}^2}{3\gamma^2 \omega(a\gamma^{-2} + \omega_p^2/\omega^2)} . \tag{41}$$

As seen from these expressions, an order-of-magnitude estimate is

$$\frac{q''}{q'} \approx \left(\frac{\omega}{\omega_*}\right)^{\nu-1} (a\gamma^{-2} + \omega_p^2/\omega^2)^{\nu-1} \approx \beta^{\nu-1} > 1 ; \tag{42}$$

and $q'/q'' \to 0$ with the growth of ω. Nevertheless, the main contribution into the radiation intensity comes from $q'(\omega)$, which determines I_ω at $\omega \gg \omega_p \gamma$ and $\omega \ll \omega_p \gamma$.

The integrals (36) are simplified in the following limiting cases:

(i) $|s| \ll 1$, $4r|s|^3 \ll 1$;

$$\Phi_1 \approx \frac{6|s|^4}{s_1} \frac{2 - r(3s_1^2 - s_2^2)}{s_1^2(2 - rs_1^3)^2 + s_2^2(2 + rs_2^2)^2 + 3s_1^2 s_2^2 |s|^2 r^2}, \tag{43}$$

$$\Phi_2 \approx 2rs^2 \frac{[2s_1 + r(3s_1 s_2^2 - s_1^3)] \cos\vartheta + [2s_2 - r(3s_1^2 s_2 - s_2^3) \sin\vartheta}{s_1^2(2 - rs_1^3)^2 + s_2^2(2 + rs_2^2)^2 + 3s_1^2 s_2^2 |s|^2 r^2}.$$

(ii) $|s| \ll 1$, $4r|s|^3 \gg 1$; or $|s| \gg 1$, $r \gg 1$;

$$\Phi_1 \approx -\frac{2|s|^4}{s_1 s_2} \operatorname{arctg} \frac{s_2(3s_1^2 - s_2^2)}{s_1(3s_2^2 - s_1^2)} - \frac{3\pi |s|^4}{2s_1 s_2}, \tag{44}$$

$$\Phi_2 \approx \frac{2^{4/3}\Gamma(2/3)}{3^{1/3}} \frac{r^{1/3}|s|^2}{[s_1^2(3s_2^2 - s_1^2)^2 + s_2^2(3s_1^2 - s_2^2)^2]^{1/3}} \times$$

$$\times \cos\left[\frac{2}{3} \operatorname{arctg} \frac{s_2(3s_1^2 - s_2^2)}{s_1(3s_2^2 - s_1^2)} + \vartheta\right].$$

(iii) $|s| \gg 1$, $4r|s|^3 \gg 1$, but $r \ll 1((r/32)^{1/2} \ll 1)$;

$$\Phi_1 \approx 1 - 2^{-5/4}\pi^{1/2}(3r^{1/4}|s|^4/s_1 s_2)\exp(-2^{7/2}3^{-1}r^{-1/2}),$$
$$\Phi_2 \approx 2^{3/4}\pi^{1/2}r^{1/4}\exp(-2^{7/2}3^{-1}r^{-1/2}). \tag{45}$$

Let us analyze the asymptotic dependences of the radiation intensity on frequency. First of all, we show that in the absence of random fields and plasma ($\omega_p = 0$) the above equations yield the well-known expressions (Ginzburg and Syrovatskii, 1964) for the spectral density of the synchrotron radiation in vacuum. The neglect of random fields corresponds to $|s| \gg 1$. At low frequencies, $\omega \ll \Omega_\perp \gamma^3$ and $\omega \ll \omega_* \gamma^2$, and at $\omega_p = 0$, according to (37) and (39), we have $r \gg 1$ and $q'(\omega) \gg q''(\omega)$, i.e., $s_1 \approx s_2$. In this case, Equations (44) reduce to

$$\Phi_1 \approx -8\pi s_1^2, \qquad \Phi_2 \approx 2^{1/3}3^{1/6}\Gamma(2/3)r^{1/3}.$$

Making use of (46), (37), and (35), we find that

$$I_\omega \approx \frac{3^{1/6}\Gamma(2/3)}{\pi} \frac{e^2}{c\gamma^2} (\Omega_\perp \gamma^3)^{2/3} \omega^{1/3}. \tag{47}$$

At high frequencies, $\omega \gg \Omega_\perp \gamma^3$, we have $r \ll 1$, although, again, $|s| \gg 1$. From (45) we get

$$I_\omega \approx \frac{e^2}{2\sqrt{\pi}\, c\gamma^2} (\Omega_\perp \gamma^3 \omega)^{1/2} \exp(-2\omega/3\Omega_\perp \gamma^3). \tag{48}$$

Equations (47) and (48) coincide with the familiar equation (Ginzburg and Syrovatskii, 1964) of the synchrotron radiation in a uniform magnetic field in the absence of a plasma.

Now let us consider a more general case when a particle is affected by both, regular (large-scale) and random (small-scale) fields.

1. *High frequencies* $\omega \gg \Omega_\perp \gamma^3$. In this case we obtain

$$I_\omega \approx \frac{2^{\nu+1}\Gamma(\nu/2)}{3\pi^{1/2}(\nu+2)\Gamma(\nu/2-\frac{1}{2})}\,\frac{e^2}{c}\,\omega_{st}^2\gamma^2(\omega_*\gamma^2)^{\nu-1}\omega^{-\nu}. \tag{49}$$

This expression is valid up to $\omega = \omega_{\max} = c\gamma^2/l_{\min}$, $l_{\min}$ being the minimal turbulence scale, while at $\omega > \omega_{\max}$ the spectrum has a cut-off. If the turbulence spectrum is not power-law, the radiation spectrum at $\omega < \omega_{\max}$ follows the shape of the turbulence spectrum,

$$I_\omega \approx \frac{8e^2 q'(\omega)}{3\pi c}\,\gamma^2, \tag{50}$$

where $q'(\omega)$ is the scattering rate proportional to the cosine-amplitude of the correlation function of random fields. In the absence of random inhomogeneities the radiation spectrum suffers an exponential fall in accordance with (48).

2. *Intermediate frequencies* $\omega_p\gamma \ll \omega \ll \Omega_\perp\gamma^3$. If the transverse (with respect to the line-of-sight) component of the large-scale field is sufficiently small, so that $\omega_{st} \gg \Omega_\perp\gamma$ and also $\omega_* > \omega_{st}$, then

$$I_\omega \approx \left[\frac{4\Gamma(\nu/2)}{3\pi^{3/2}\Gamma(\nu/2-\frac{1}{2})}\right]^{1/2}\frac{e^2\omega_{st}}{c}\left(\frac{\omega}{\omega_*\gamma^2}\right)^{1/2}. \tag{51}$$

If, however, the large-scale field is strong enough, and $\Omega_\perp\gamma \gg \omega_{st}$, the random field plays little role. In this case for frequencies in question one can use Equation (47) which yields $I_\omega \propto \omega^{1/3}$.

3. *Low frequencies*

$$\omega_p \ll \omega \ll \omega_p\gamma. \tag{52}$$

At these frequencies the radiation is strongly affected by the density effect (the term ω_p^2/ω^2). This effect is well-known in the theory of bremsstrahlung (Ter Mikaeljan, 1954) and magnetic bremsstrahlung (Tsytovich, 1951) in a regular magnetic field. In the latter case (at $\Omega_\perp\gamma \gg \omega_{st}$) one has

$$I_\omega \approx \frac{e^2}{2\sqrt{\pi}\,c}\,(\Omega_\perp\omega_p)^{1/2}\exp(-2\omega_p^3/3\Omega_\perp\omega^2). \tag{53}$$

However, at lower frequencies random fields, again, become dominant. This leads to a power-law (but not exponential) fall of the intensity with decreasing frequency,

$$I_\omega \approx \frac{2^{\nu+3}\Gamma(\nu/2)}{9\sqrt{\pi}\,\Gamma(\nu/2-\frac{1}{2})}\,\frac{e^2\omega_{st}^2}{c\gamma^2\omega_p}\left(\frac{\omega_*}{\omega_p}\right)^{\nu-1}\left(\frac{\omega}{\omega_p}\right)^{\nu+2}. \tag{54}$$

The radiation spectra from a separate particle for various relations between regular and random fields are shown in Figure 1.

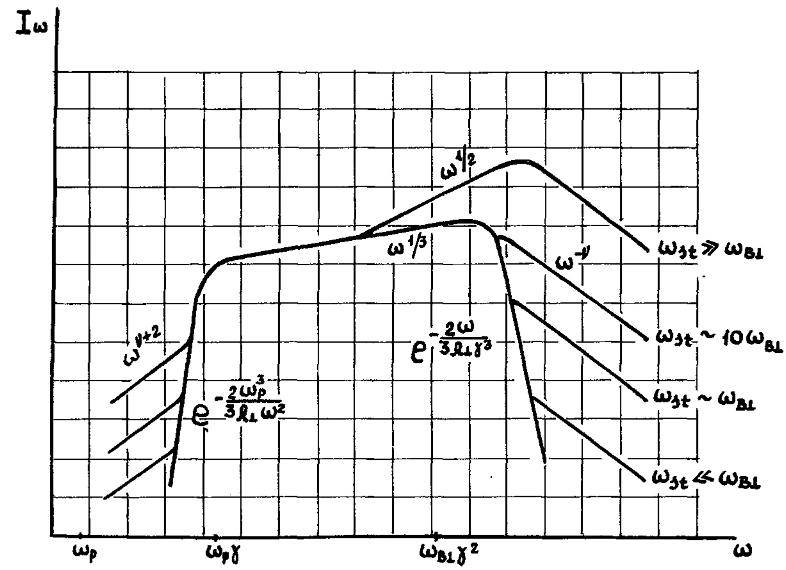

Fig. 1. The spectrum of intrinsic radiation from one relativistic electron for different ratios of random and regular magnetic fields. The inner contour-synchrotron radiation in absence of random magnetic field.

However, one should bear in mind that Equations (51)–(54) describe only one fraction of the total radiation from an ultrarelativistic particles. The other fraction is produced by the transition radiation (the transition scattering of electromagnetic field of a moving particle on plasma inhomogeneities) and should be evaluated by different methods.

5. Transition Radiation of Ultrarelativistic Particles in a Turbulent Plasma

The transition radiation was predicted theoretically by Ginzburg and Frank (1946) who showed that a particle, which moves with constant velocity and crosses a boundary between media with different dielectric functions, radiates electromagnetic waves. An analogous effect takes place when the particle moves through a randomly inhomogeneous medium (Ginzburg and Tsytovich, 1984), particularly, through a turbulent plasma. This phenomenon is of special interest for astrophysical applications, because the intensity of the transition radiation is independent of particle mass; hence, this radiation may be produced not only by electrons but also by nuclei. This gives a principal possibility to obtain direct information on the nucleon component of cosmic rays in distant objects.

The transition radiation should be taken into account also while investigating the radiation spectra from electrons in the low-frequency domain $\omega \lesssim \omega_p \gamma$, where the intrinsic radiation of particles is suppressed by the density effect. The transition radiation is influenced by the density effect, although in another way, and is not suppressed by this effect.

First of all, let us determine the main factors which affect the transition radiation in a magneto-active turbulent plasma. It is well-known (see e.g., Bekefi, 1966) that the polarization properties of the magneto-active plasma are described by the dielectric tensor whose components are expressed through the quantities

$$\varepsilon_{\pm} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_B)}, \qquad \varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2}, \tag{55}$$

where $\pm$ correspond to two types of transverse electromagnetic waves which may propagate along the magnetic field.

As follows from (55), fluctuations of $\varepsilon_{\alpha\beta}$ may arise either due to fluctuations of the electron number density δN or due to fluctuations δB of the magnetic field. In the first case at $\omega_B \ll \omega_p < \omega$ we have

$$\delta\varepsilon_{\pm} = -\frac{\delta N}{N}\left(\frac{\omega_p}{\omega}\right)^2, \tag{56}$$

whereas in the second case

$$\delta\varepsilon_{\pm} = \pm\frac{\delta B}{B}\frac{\omega_B}{\omega}\left(\frac{\omega_p}{\omega}\right)^2. \tag{57}$$

If $\delta N/N \approx \delta B/B$, the fluctuations of N lead to much stronger variations $\delta\varepsilon_{\alpha\beta}$ than the fluctuations of B, owing to the presence of a small factor ω_B/ω in Equation (57). The ratio of the radiation intensities produced by the fluctuations δB and δN will be proportional to $(\omega_B/\omega)^2$. Therefore, we may expect that in the presence of δN and δB the dominant contribution into the radiation will come from inhomogeneities of the electron number density; only in the absence of δN the transition radiation will be determined by magnetic inhomogeneities.

Now let calculate the intensity of the transition radiation of ultrarelativistic particles in a turbulent plasma. Owing to its nature, the transition radiation is the radiation of plasma electrons excited by a highly energetic particles. The effect is not concerned with the variation of the fast-particle velocity and, consequently, with the particle mass (at a given Lorentz-factor). The total radiation is equal to the sum of the intrinsic radiation of the ultrarelativistic particle and the radiation of plasma electrons (the transition radiation). This total radiation is determined by the current produced by the relativistic particle itself

$$\mathbf{j}^q(\mathbf{r}, t) = q\mathbf{v}(t)\,\delta(\mathbf{r} - \mathbf{r}(t)) \tag{58}$$

plus the current

$$\mathbf{j}^m(\mathbf{r}, t) = e \int \mathbf{v} f_m(\mathbf{r}, \mathbf{p}, t)\,\mathrm{d}^3p/(2\pi)^3\,, \tag{59}$$

induced by the plasma electrons under the action of the relativistic particle. In Equation (59) f_m denotes the component of the distribution fucntion of the plasma electrons which describes perturbations introduced by the fast particle (e is the electron charge, and q is the charge of the fast particle).

Calculating the electromagnetic field created by the total current $\mathbf{j}_{\text{tot}} = \mathbf{j}^q + \mathbf{j}^m$, and finding then the energy radiated by the relativistic particle, instead of (3) we derive the more general expression

$$\begin{aligned}\mathscr{E}_{\mathbf{n},\,\omega} = (2\pi)^6\,\frac{\omega^2}{c^3}\,\{\langle|[\mathbf{n}, \mathbf{j}^q_{\mathbf{k},\,\omega}]|^2\rangle + \langle|[\mathbf{n}, \mathbf{j}^m_{\mathbf{k},\,\omega}]|^2\rangle + \\ + \langle[\mathbf{n}, \mathbf{j}^q_{\mathbf{k},\,\omega}]\,[\mathbf{n}, \mathbf{j}^{m*}_{\mathbf{k},\,\omega}]\rangle + \langle[\mathbf{n}, \mathbf{j}^{q*}_{\mathbf{k},\,\omega}]\,[\mathbf{n}, \mathbf{j}^m_{\mathbf{k},\,\omega}]\rangle\}\,.\end{aligned} \tag{60}$$

The first term in curly brackets is expressed through the Fourier component of the current of the relativistic particle,

$$\mathbf{j}^q_{\mathbf{k},\,\omega} = q \int_{-\infty}^{+\infty} \mathbf{v}(t)\exp[-i\mathbf{k}\mathbf{r}(t) + i\omega t]\,\mathrm{d}t/(2\pi)^4\,, \tag{61}$$

and reduces easily to the form which corresponds to Equation (3). This term describes the radiation connected with velocity variations of the relativistic particle; it has been analysed in detail in preceding sections. The last two terms in (60) describe the interference of the intrinsic and transition radiations. One can easily see that for ultrarelativistic radiators moving in a magneto-active plasma the interference term is always small (here we omit a detailed proff of this statement).

To calculate the intensity of the transition radiation, which is described by the second term in Equation (60), let us evaluate the correction to the distribution function of the plasma electrons, $f^{(2)}_m$, produced by the plasma turbulence and the electromagnetic field of the relativistic particle. The plasma will be treated as cold and magneto-active. It will be assumed that the plasma is turbulent, with Alfvén and magneto-acoustic waves being present but high-frequency plasma waves being absent.

Solving the collisionless Boltzmann equation by iterations, we obtain the Fourier transform of the distribution function under study in the form

$$\begin{aligned}f^{m(2)}_{\mathbf{k},\,\omega} = \frac{e^2}{i(\omega - \mathbf{k}\mathbf{v})} \int \mathbf{F}_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}\,\frac{\partial}{\partial\mathbf{p}}\left\{\frac{\mathbf{F}_{\mathbf{k}',\,\omega'}}{i(\omega' - \mathbf{k}'\mathbf{v})}\,\frac{\partial}{\partial\mathbf{p}}\,f(\mathbf{p})\right\}\mathrm{d}\omega'\,d^3k' + \\ + \frac{e}{i(\omega - \mathbf{k}\mathbf{v})} \int \mathbf{F}_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}\,\frac{\partial}{\partial\mathbf{p}}\,\delta f_{\mathbf{k}',\,\omega'}\,\mathrm{d}\omega'\,\mathrm{d}^3k'\,.\end{aligned} \tag{62}$$

In this case,

$$\mathbf{F}_{\mathbf{k},\,\omega} = \mathbf{E}_{\mathbf{k},\,\omega} + [\mathbf{v}, \mathbf{B}_{\mathbf{k},\,\omega}]/c\,; \tag{63}$$

where $\mathbf{E}_{\mathbf{k},\,\omega}$ and $\mathbf{B}_{\mathbf{k},\,\omega}$ are, generally, sums of the fields of plasma mode (m) and relativistic particle (q),

$$\mathbf{E}_{\mathbf{k},\,\omega} = \mathbf{E}^m_{\mathbf{k},\,\omega} + \mathbf{E}^q_{\mathbf{k},\,\omega}\,, \qquad \mathbf{B}_{\mathbf{k},\,\omega} = \mathbf{B}^m_{\mathbf{k},\,\omega} + \mathbf{B}^q_{\mathbf{k},\,\omega}\,; \tag{64}$$

$f(\mathbf{p})$ is the distribution function of the unperturbed plasma (in the absence of MHD waves and the relativistic particle); $\delta f_{\mathbf{k},\,\omega}$ describes variations of the plasma density in magneto-acoustic waves, so that

$$\int f(\mathbf{p})\, \mathrm{d}^3p/(2\pi)^3 = N_0\,; \qquad \int \delta f_{\mathbf{k},\,\omega}(\mathbf{p})\, \mathrm{d}^3p/(2\pi)^3 = \delta N_{\mathbf{k},\,\omega}\,. \tag{65}$$

Note that $\delta f_{\mathbf{k},\,\omega} = 0$ for Alfvén waves.

Equation (62) does not include the large-scale field $\mathbf{B}_0$, because $\mathbf{B}_0$ enters the current density only through the combination $\omega \pm \omega_B \approx \omega$ (see Equations (55)) in which it may be neglected. Evaluating the Fourier transform of the current with the aid of (59), (62), and (65), we obtain

$$\mathbf{j}^{m(2)}_{\mathbf{k},\,\omega} = -\frac{e^3 N_0}{m^2 c\omega} \int \mathrm{d}^3k' \,\frac{\mathrm{d}\omega'}{\omega - \omega'}\, [\mathbf{E}^q_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}, \mathbf{B}^m_{\mathbf{k}',\,\omega'}] + $$
$$+ \frac{ie^2}{m\omega} \int \mathbf{E}^q_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}\, \delta N_{\mathbf{k}',\,\omega'}\, \mathrm{d}\omega'\, \mathrm{d}^3k'\,, \tag{66}$$

where the fluctuations of the electron number density should be expressed through amplitudes of magneto-acoustic waves (Landau and Lifshitz, 1960). While deriving (66), we have made some natural simplifications; namely, we have neglected the terms $\sim v_{Te}/c$ and u/c, v_{Te} and u being the thermal and hydrodynamical velocities of the plasma, respectively. In addition, in (64) we have omitted the field $\mathbf{E}^m_{\mathbf{k},\,\omega}$, because in a cold plasma the electric field of MHD waves is of vortex character (is not associated with fluctuations of the electron number density) and is smaller than the magnetic field $\mathbf{B}^m_{\mathbf{k},\,\omega}$. Under the conditions as formulated one may set $\mathbf{B}_{\mathbf{k},\,\omega} = \mathbf{B}^m_{\mathbf{k},\,\omega}$.

The first term in (66) describes the current associated with magnetic fluctuations in the absence of fluctuations of the electron number density. For MHD turbulence, this happens if only Alfvén waves are present. The transition radiation induced by this current was analysed by Bel'kov *et al.* (also see Ginzburg and Tsytovich, 1984). According to Bel'kov *et al.* (1980), the radiation intensity which corresponds to the first term in (66) (labelled by a) is of the form

$$I^a_\omega = \frac{2\pi q^2 e^2 \omega_p^4}{m^2 c^4 \omega^4} \int_{k'_{\min}}^{\infty} k' P(k') \Phi(k_{\min}/k')\, \mathrm{d}k'\,, \tag{67}$$

where

$$k'_{\min} = \frac{\omega}{2c}\,(\gamma^{-2} + \omega_p^2/\omega^2)\,, \qquad \Phi(z) = z - \ln z - 1\,. \tag{68}$$

Let us assume that the spectrum of magnetic inhomogeneities is a power-law

$$P(k) = \begin{cases} 0\,, & k < k_0\,, \\ A_\nu k^{-\nu-2}\,, & k \geq k_0\,, \end{cases} \qquad A_\nu = \frac{(\nu-1)k_0^{\nu-1}\,\langle B_{st}^2\rangle}{4\pi}\,. \tag{69}$$

Then from Equation (67) we obtain

$$I_\omega^a = \frac{2\pi A_\nu}{\nu^2(\nu+1)}\,\frac{e^2q^2}{(mc^2)^2}\left(\frac{\omega_p}{\omega}\right)^4\left(\frac{2c}{\omega}\right)^\nu(\gamma^{-2} + \omega_p^2/\omega^2)^{-\nu}\,. \tag{70}$$

However, in a plasma with developed MHD turbulence all modes are expected, including magneto-acoustic modes which lead to the radiation not described by Equation (70). According to the estimates provided by Equations (56), (57), the main contribution in this case comes from the radiation due to inhomogeneities of the electron number density. Keeping only the second term in (66), we present the radiation energy in the form

$$\mathscr{E}_{\mathbf{n},\,\omega}^{m} = \frac{e^4}{m^2c^3}\int [\mathbf{n}, \mathbf{E}^q_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}]\,\times$$
$$\times\;[\mathbf{n}, \mathbf{E}^{q*}_{\mathbf{k}-\mathbf{k}'',\,\omega-\omega''}]\,\langle \delta N_{\mathbf{k}',\,\omega'}\,\delta N^*_{\mathbf{k}'',\,\omega''}\rangle\,\mathrm{d}\omega'\,\mathrm{d}^3k'\,\mathrm{d}\omega''\,\mathrm{d}^3k''\,. \tag{71}$$

In this expression it is sufficient to substitute the transverse field of the ultrarelativistic particle calculated in the constant-velocity approximation

$$\mathbf{E}^q_{\mathbf{k},\,\omega} = -\,\frac{2iq(\mathbf{v} - \mathbf{k}(\mathbf{kv})/k^2)\,\delta(\omega - \mathbf{kv})}{(2\pi)^2\,\varepsilon(\omega)\,\omega(1 - c^2k^2/\omega^2\varepsilon(\omega))}\,, \tag{72}$$

because the transverse field component is larger than the longitudinal one.

Further, let us express the fluctuations of N via amplitudes of magneto-acoustic waves (Landau and Lifshitz, 1960),

$$\delta N_{\mathbf{k},\,\omega} = N_0(\mathbf{B}_0\mathbf{B}^m_{\mathbf{k},\,\omega})/B_0^2\,. \tag{73}$$

For an isotropic distribution of wave-vectors of magneto-acoustic waves in the random phase approximation we have

$$\begin{aligned} &\langle \delta N_{\mathbf{k},\,\omega}\,\delta N^*_{\mathbf{k}',\,\omega'}\rangle = \langle\,|\delta N|^2_{\omega,\,\mathbf{k}}\rangle\;\delta(\omega - \omega')\,\delta(\mathbf{k} - \mathbf{k}')\,, \\ &|\delta N|^2_{\omega,\,\mathbf{k}} = N_0^2(|B^m|^2_{\mathbf{k},\,\omega}/B_0^2)\,[\mathbf{k}, \mathbf{b}_0]^2/k^2\,, \end{aligned} \tag{74}$$

where $\mathbf{b}_0 = \mathbf{B}_0/B_0$. Moreover, the frequencies of magneto-acoustic waves may be set zero because they are much smaller than the frequencies of electromagnetic waves radiating by particles, i.e.,

$$|B|^2_{\mathbf{k},\,\omega} = P(k)\,\delta(\omega)\,. \tag{75}$$

Evaluating the radiation intensity, we have

$$I^m_{\mathbf{n},\,\omega} = \frac{e^2q^2\omega_p^4}{2\pi m^2c^5\omega_B^2}\int \mathrm{d}^3k'\,P(k')\,\frac{[\mathbf{b}_0,\mathbf{k}']^2\,[\mathbf{n},\mathbf{v}_*]^2\,\delta(\omega-(\mathbf{k}-\mathbf{k}')\mathbf{v})}{k'^2\omega^2[1-c^2(\mathbf{k}-\mathbf{k}')^2/\omega^2\varepsilon(\omega)]^2}\,, \tag{76}$$

where

$$\mathbf{v}_* = \mathbf{v} - \omega(\mathbf{k}-\mathbf{k}')/(\mathbf{k}-\mathbf{k}')^2\,.$$

While integrating (76) over orientations of $\mathbf{k}'$ with the aid of the delta-function, we shall assume that the angle between $\mathbf{k}'$ and $\mathbf{k}$ is equal to the angle between $\mathbf{k}'$ and $\mathbf{v}$ because the radiation is predominantly directed along $\mathbf{v}$. In addition, in the expression for $\mathbf{v}_*$ we may replace $(\mathbf{k}-\mathbf{k}')^2$ by k^2 since $k'^2 \ll k^2$ and $\mathbf{k}'$ is almost perpendicular to $\mathbf{k}$. Then, we obtain

$$I^m_{\mathbf{n},\,\omega} = \frac{e^2q^2\omega_p^4(1+\cos^2\theta_0)}{2m^2c^4\omega_B^2\omega^2}\int\limits_{k'_{\min}(\theta)}^{\infty} k'\,\mathrm{d}k'\,\frac{\theta^2P(k')}{(\theta^2+\gamma^{-2}+\omega_p^2/\omega^2)^2}\,. \tag{77}$$

Here we have expanded the integrand in terms of small angle θ between $\mathbf{n}$ and $\mathbf{v}$; θ_0 denotes an angle between $\mathbf{v}$ and $\mathbf{B}_0$; and

$$k'_{\min}(\theta) = (\omega/2c)\,(\theta^2+\gamma^{-2}+\omega_p^2/\omega^2)\,.$$

Let us integrate the radiation intensity (77) over all directions of $\mathbf{k}$. By interchanging the order of integration over $\mathrm{d}\theta$ and $\mathrm{d}k'$, we represent the spectral radiation density in the form

$$I^m_{\omega} = \frac{\pi e^2q^2\omega_p^4(1+\cos^2\theta_0)}{2m^2c^4\omega_B^2\omega^2}\int\limits_{k'_{\min}}^{\infty} P(k')k'\,\mathrm{d}k'\int\limits_0^{\theta^2_{\max}}\frac{\theta^2\,\mathrm{d}\theta^2}{(\theta^2+\gamma^{-2}+\omega_p^2/\omega^2)^2}\,, \tag{78}$$

where $k'_{\min}$ is given by Equation (68), and

$$\theta^2_{\max} = (2ck'/\omega)\,(1-k'_{\min}/k')\,. \tag{79}$$

Actually, for MHD waves $k' \ll \omega/c$, i.e., $\theta^2_{\max} \ll 1$ which confirms the above assumption on the smallness of θ.

Integrating over $\mathrm{d}\theta^2$ yields

$$I^m_{\omega} = \frac{\pi e^2q^2\omega_p^4(1+\cos^2\theta_0)}{2m^2c^4\omega_B^2\omega^2}\int\limits_{k'_{\min}}^{\infty} k'P(k')\Phi(k'_{\min}/k')\,\mathrm{d}k'\,, \tag{80}$$

where $\Phi(z)$ is given by (68). Evaluating (80) with the same spectrum of magnetic inhomogeneities as has been used for obtaining (70) and averaging over directions of large-scale magnetic field B_0, we get

$$I_\omega^m = \frac{2^{\nu-1}(\nu-1)e^2q^2\omega_p^4\langle B_{st}^2\rangle}{3\,v^2(\nu+1)m^2c^3\omega_B^2\omega^3}\left(\frac{\omega_0}{\omega}\right)^{\nu-1}(\gamma^{-2}+\omega_p^2/\omega^2)^{-\nu}. \tag{81}$$

Comparing the contributions of Alfvén and magneto-acoustic waves into the intensity of the transition radiation we have

$$I_\omega^m/I_\omega^a = \tfrac{1}{3}(\omega_p/\omega_B)^2(\omega/\omega_p)^2 \gg 1\,. \tag{82}$$

This ratio agrees with the semi-quantitative estimate performed above from comparison between (56) and (57).

It should be noted that Equation (81) is valid for describing the transition radiation from a relativistic particle in the presence of an ensemble of weak shocks. In this case one should put $\nu = 2$. At $\omega \ll \omega_p\gamma$ the spectral intensity is constant, while at $\omega \gg \omega_p\gamma$ it depends on frequency as ω^{-4}.

Since for relativistic particles the interference between the intrinsic and transition radiations is small, the full radiation intensity equals the sum of the intensities calculated in Sections 4 and 5.

6. The Effect of Plasma Turbulence on Spectra of Synchrotron Radiation from Cosmic Objects

The presence of highly energetic radiating electrons in astrophysical objects is possible only in the case of persistent electron acceleration; otherwise the synchrotron losses will lead to a rapid electron deceleration. According to the current point of view, the most effective mechanisms of electron acceleration to ultrarelativistic energies are those associated with large-scale MHD turbulence (including shocks) and related small-scale turbulence which contains various MHD and plasma modes (Axford, 1981; Toptygin, 1980; Galeev, 1984). Hence, the presence of strong enough fluctuations of electromagnetic fields and plasma density is necessary for the existence of relativistic radiating particles in radiosources. An analysis of the synchrotron spectra should be carried out with account for the effects of turbulent plasma.

A theoretical study of stochastic acceleration mechanisms (Berezinsky *et al.*, 1984; Toptygin, 1985) and observational data (Ginzburg and Syrovatskii, 1964; Berezinsky *et al.*, 1984; Toptygin, 1985) reveals that in many astrophysical objects the energy spectrum of radiating particles is quite well fitted by power-law in a wide energy range. To evaluate the radiation from an ensemble of relativistic electrons one needs to integrate the above expressions (35), (81) for the spectra of separate particles with the distribution function which corresponds to an ensemble as a whole.

Following Ginzburg and Syrovatskii (1964) let us take the electron spectrum in the form

$$\mathrm{d}N_e = K_e \gamma^{-\xi} \mathrm{d}\gamma, \quad \gamma_1 \leq \gamma \leq \gamma_2 . \tag{83}$$

Then, the radiation from the electron ensemble is expressed as

$$P(\omega) = \int_{\gamma_1}^{\gamma_2} I_\omega(\gamma)\, \mathrm{d}N_e(\gamma) . \tag{84}$$

For the intrinsic particle radiation, $I_\omega(\gamma)$ is determined by Equation (35). It is clear that at arbitrary values of the parameters corresponding double integrals may be evaluated only by numerical methods. However, it is possible to obtain asymptotic expressions for $P(\omega)$ from the asymptotics of I_ω in certain frequency intervals.

1. *High frequencies* at which the radiation of particles with $\gamma_1 \leq \gamma \leq \gamma_2$ is determined by stochastic fields (Equation (49)). These are the frequencies $\omega \gg \omega_* = \omega_{B\perp} \gamma_2^2 \Lambda$, where Λ is a logarithmic factor determined by the relative level of the turbulence and magnetic field,

$$\Lambda = \frac{3}{2} \ln \left[\frac{3^{\nu+3/2}(\nu+2)\Gamma(\nu/2 - 1/2)\omega_{B\perp}^{\nu+1}}{2^{2\nu+5/2}\Gamma(\nu/2)\omega_{st}^2 \omega_0^{\nu-1}} \right]. \tag{85}$$

Integrating Equation (49) in accordance with (84), at $\gamma_2 \gg \gamma_1$ we obtain

$$P_1(\omega) = \frac{2^{\nu+1}\Gamma(\nu/2) K_e \gamma_2^{2\nu+1-\xi}}{3\pi^{1/2}(\nu+2)(2\nu+1-\xi)\Gamma(\nu/2 - \frac{1}{2})} \frac{e^2}{c} \omega_{\mathrm{st}}^2 \omega_0^{\nu-1} \omega^{-\nu} . \tag{86}$$

Thus, for a power-law energy spectrum of the electrons with a cut-off at certain energy $\mathscr{E}_2 = mc^2 \gamma_2$, the high-frequency 'tail' of the radiation spectrum is determined by random fields and the radiation spectrum repeats the turbulence spectrum; this takes place up to frequencies $\omega_{\max} = c\gamma_2^2 / l_{\min}$, $l_{\min}$ being the minimal turbulence scale (Toptygin and Fleishman, 1983, 1984). This result is valid even if the spectrum cut-off is not sharp (it is sufficient to have a power-law spectral fall with an index $\xi > 2\nu + 1$).

2. *In the intermediate frequency range* $\omega_{**} \ll \omega \ll \omega_*$, where $\omega_{**} \approx \omega_p(\omega_p \gamma_1 / \omega_{B\perp})^{1/2}$, at $\omega_{B\perp} \gg \omega_{st}$ the main contribution into the radiation comes from the large-scale magnetic field; integration over energies of the radiating electrons yields

$$P_2(\omega) = \frac{3^{\xi/2}\Gamma(\xi/4 - \frac{1}{12})\Gamma(\xi/4 - \frac{19}{12}) K_e}{\xi + 1} \frac{e^2}{c} \omega_{B\perp}^{\alpha+1} \omega^{-\alpha} , \tag{87}$$

where $\alpha = (\xi - 1)/2$. In this case there appears a power-law spectral part with an index α, typical for the synchrotron radiation (Ginzburg and Syrovatskii, 1964). If the small-scale field is dominant and $\omega_{st} \gg \omega_{B\perp}$, the frequency-dependence of the spectral power is again a power-law, with the same index $\alpha = (\xi - 1)/2$. However, the numerical factor cannot be found analytically, and requires numerical calculations.

In a narrow vicinity of ω_* Equations (86) and (87) are invalid. In this vicinity the intensity falls down exponentially with the increase of ω. The fall leads to a smooth transition from (86) to (87). Quantitatively, this fall may be characterized by the ratio P_1/P_2 taken near ω_*, for instance, at $\omega = \omega_{B\perp}\gamma_2^2$. This gives

$$\frac{P_1}{P_2} = \frac{2^{\nu+1}(\xi+1)\Gamma(\nu/2)}{3^{1+\xi/2}\pi^{1/2}(\nu+2)(2\nu+1-\xi)\Gamma(\nu/2-1/2)\Gamma(\xi/4-1/12)\Gamma(\xi/4+19/12)} \times$$

$$\times\, \frac{\omega_{st}^2\,\omega_0^{\nu-1}}{\omega_{B\perp}^{\nu+1}}\,. \tag{88}$$

Note that the product $\omega_{st}^2\omega_0^{\nu-1}$ (ω_{st}^2 being determined by the energy of random fields with scales $l \leq L_0$) is independent of the choice of the critical scale L_0.

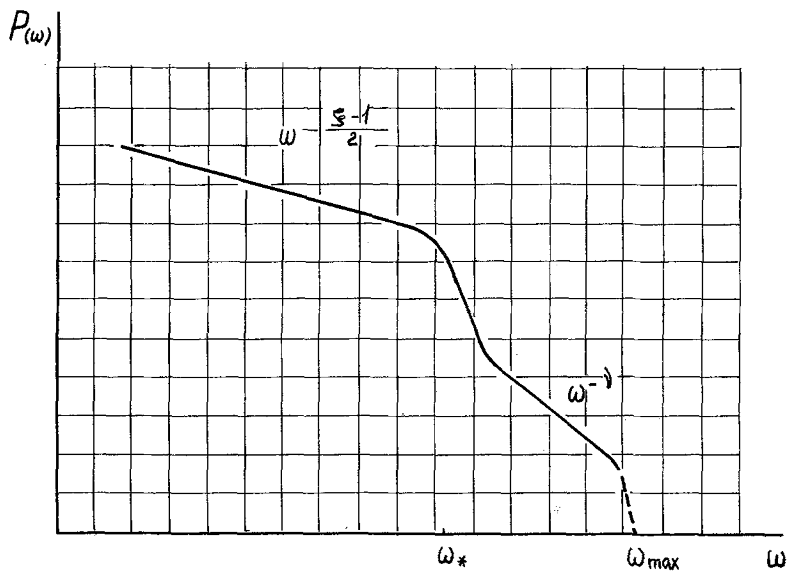

Fig. 2. The radiation spectrum produced by electron ensemble in a magnetic field with random inhomogeneities. Electrons have power-law energy distribution with a cut-off at $\mathscr{E} = mc^2\gamma_2$. The high-energy part of the radiation spectrum and the exponential intensity decrease (jump) near $\omega_* \approx \omega_{B\perp}\Lambda\gamma_2^2$ are shown.

Figure 2 schematically displays the spectrum of the synchrotron radiation with account for small-scale turbulence.

The spectrum becomes even more complicated, if Langmuir turbulence is excited in a wide interval of scales in addition to the large-scale field. Langmuir waves with $k > \omega_p/c$ behave as a small-scale turbulence. At $\omega \gg \omega_p\gamma_2^2$ they lead to the radiation spectrum $P_1^{st}(\omega)$ of the form (86), where ω_{st}^2 is determined by the value of $\langle E_{st}^2 \rangle$ at $k > \omega_p/c$. For $\omega_{B\perp}\gamma_2^2 < \omega < \omega_p\gamma_2^2$, the radiation spectrum is determined by large-scale Langmuir harmonics with $k < \omega_p/c$. The phase velocity of these harmonics exceeds c.

Therefore, one needs to take account of the dependence of the correlator on the time-argument. Without writing down the spectrum of one particle (see, e.g., Kaplan and Tsytovich, 1972), let us present the spectral power of the radiation for an ensembe of electrons

$$P_l(\omega) = \frac{2^{\xi/2-3/2}(\xi^2+4\xi+11)}{(\xi+1)(\xi+3)(\xi+5)} K_e \frac{e^2}{c} \frac{\omega_E^2}{\omega_p}\left(\frac{\omega_p}{\omega}\right)^{\alpha}, \quad \alpha = \frac{\xi-1}{2}; \tag{89}$$

where $\omega_E^2 = e^2\langle E^2\rangle/m^2c^2$, $\langle E^2\rangle$ is the square electric field of large-scale plasmons.

At $\omega \lesssim \omega_{B\perp}\gamma_2^2$ the dominant contribution into the synchrotron radiation comes from the large-scale magnetic field and the spectrum is given by Equation (87). Hence, in this case the radiation spectrum contains three power-law parts with indices α, α, and ν, respectively (Figure 3). The first two parts have equal indices but different levels. The corresponding jump near the frequency $\omega = \omega_{B\perp}\gamma_2^2$ can be easily shown to be equal to

$$\frac{P_1}{P_2} = \frac{2^{\xi/2-3/2}(\xi^2+4\xi+11)}{3^{\xi/2}(\xi+3)(\xi+5)\Gamma(\xi/4-\frac{1}{12})\Gamma(\xi/4+\frac{19}{12})} \frac{\omega_E^2}{\omega_p\omega_{B\perp}}\left(\frac{\omega_p}{\omega_{B\perp}}\right)^{\alpha}. \tag{90}$$

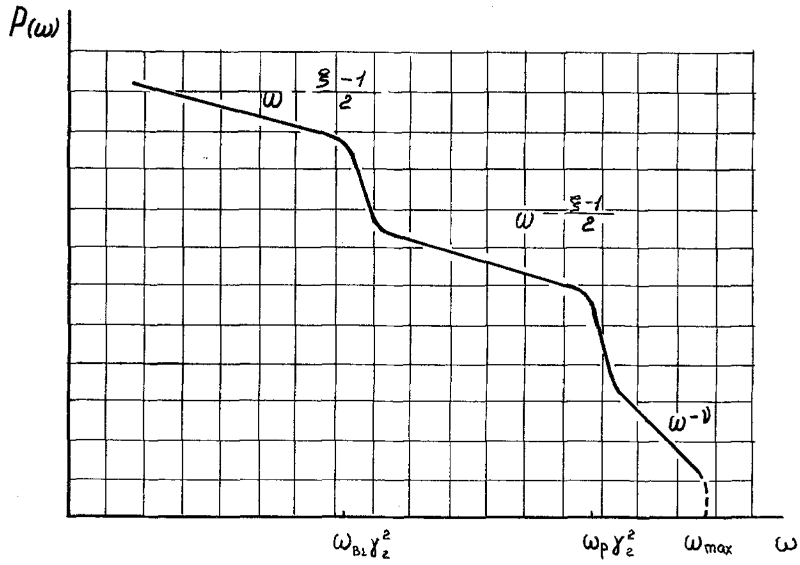

Fig. 3. The radiation spectrum from electron ensemble in a magnetic field superimposed by Langmuir turbulence with wide interval of wave numbers. The spectrum has two jumps in high-frequency range (if $\omega_{B\perp} \ll \omega_p \ll c/l_{\min}$).

The jump between the second and the third parts is also easily evaluated as

$$\frac{P_1^{st}(2\omega_p\gamma_2^2)}{P_1(2\omega_p\gamma_2^2)} = \frac{4\Gamma(\nu/2)(\xi+1)(\xi+3)(\xi+5)}{3\pi^{1/2}(\nu+2)(2\nu+1-\xi)(\xi^2+4\xi+11)\Gamma(\nu/2-1/2)} \frac{\omega_{st}^2}{\omega_E^2}. \tag{91}$$

This jump is determined by the ratio between the energies contained in Langmuir pulsations and large-scale plasmons.

3. *Now let us analyse the low-frequency range $\omega \lesssim \omega_{**}$.* The main contribution into this range may come either from the intrinsic radiation of the relativistic particle or from the transition radiation. After integration over the electron spectrum, the radiation spectrum of the relativistic particle can be written as

$$P_3(\omega) = \frac{2^{\nu+5}\Gamma(\nu/2)K_e}{9\pi^{1/2}(\xi+1)\Gamma(\nu/2-\frac{1}{2})} \frac{e^2}{c} \frac{\omega_{st}^2\omega_0^{\nu-1}}{\omega_p^{\nu}\gamma_1^{\xi-1}} \left(\frac{\omega}{\omega_p}\right)^{\nu+2}. \tag{92}$$

As shown in Section 5, the transition radiation for a developed MHD turbulence is determined by fluctuations of the electron number density in magneto-acoustic and shock waves. However, in contrast to the synchrotron radiation, the transition radiation may be produced not only by the relativistic electrons but also by the nucleon component of cosmic rays. The latter component is commonly dominant; for instance, at $\mathscr{E} \gtrsim 1$ GeV the number of nuclei exceeds the number of electrons by about two orders of magnitude. That is why let us integrate the intensity of the transition radiation (81) over the spectrum of all cosmic rays, electrons and nuclei,

$$\mathrm{d}N_{cr} = K_{cr}\gamma^{-\xi}\,\mathrm{d}\gamma, \quad \gamma > \gamma_1 \tag{93}$$

(for low frequencies, the particle spectrum at $\gamma \gg \gamma_1$ plays little role). Performing integration, we obtain

$$\begin{aligned} J &= \int_{\gamma_1}^{\infty} \left(\gamma^{-2} + \frac{\omega_p^2}{\omega^2}\right)\gamma^{-\xi}\,\mathrm{d}\gamma \\ &= \left(\frac{\omega}{\omega_p}\right)^{2\nu+1-\xi} \left(\frac{\omega_p^2\gamma_1^2}{\omega^2}\right)^{-\alpha} \frac{1}{\xi-1} F\left(\nu, \frac{\xi-1}{2}, \frac{\xi+1}{2}; -\frac{\omega^2}{\omega_p^2\gamma_1^2}\right). \end{aligned} \tag{94}$$

At high frequencies, $\omega \gg \omega_p\gamma_1$ (actually, this means that $\omega \gg \omega_p$) one can use the expansion of the hypergeometric function at large values of the argument. This yields

$$J \approx \frac{\Gamma(\xi/2+\frac{1}{2})\Gamma(\nu-\xi/2+\frac{1}{2})}{(\xi-1)\Gamma(\nu)} \left(\frac{\omega}{\omega_{\mathrm{p}}}\right)^{2\nu-\xi+1}. \tag{95}$$

Making use of the latter equation and inserting A_ν, we get

$$\begin{aligned} P^m(\omega) &= \frac{2^{\nu-2}(\nu-1)\Gamma(\xi/2+\frac{1}{2})\Gamma(\nu-\xi/2+\frac{1}{2})K_{cr}}{\nu(\xi-1)\Gamma(\nu+2)} \times \\ &\times \frac{q^2\omega_{st}^2}{c\,\omega_p}\left(\frac{\omega_0}{\omega_p}\right)^{\nu-1}\left(\frac{\omega_p}{\omega_B}\right)^2\left(\frac{\omega_p}{\omega}\right)^{\xi+1-\nu}. \end{aligned} \tag{96}$$

If fluctuations of the electron number density in the radiation region are absent although static magnetic inhomogeneities and Alfvén waves are available, the transition radiation from a fast particle is determined by Equation (70). Integrating this equation with the particle spectrum (93), we obtain

$$P^a(\omega) = \frac{(\nu - 1)\Gamma(\xi/2 + \frac{1}{2})\Gamma(\nu - \xi/2 + \frac{1}{2})K_{cr}}{\nu(\xi - 1)\Gamma(\nu + 2)} \frac{q^2\omega_{st}^2}{c\omega_p}\left(\frac{2\omega_0}{\omega_p}\right)^{\nu - 1}\left(\frac{\omega_p}{\omega}\right)^{\xi + 3 - \nu}. \tag{97}$$

The spectrum (97) decreases with frequency quicker than in the case of magneto-acoustic waves; at $\omega \approx \omega_p$ the intensity (96) appears to be a factor of $(\omega_p/\omega_B)^2 \gg 1$ larger than (97).

The total radiation power in the low-frequency region is given by $P(\omega) = P_3(\omega) + P^m(\omega)$. Comparing (96) and (92), we see that the transition radiation dominates the synchrotron one at low enough frequencies, Figures 4 and 5 display different situations which may appear at various parameter values. The first situation occurs when the synchrotron radiation is suppressed by the density effect at those frequencies at which it becomes comparable with the transition radiation. This leads to the appearance of the minimum in the spectral curve. If the density effect is not still significant at the above frequencies, the radiation spectrum will show a kink at some frequency and a rise in the low-frequency range (Figure 5). For typical parameter values, the frequency which separates the spectral ranges of dominant synchrotron and transition radiations is about $(20–200)\omega_p$.

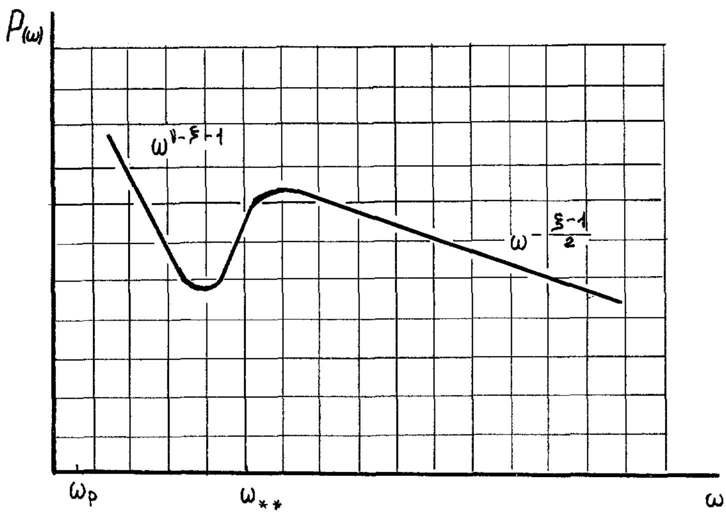

Fig. 4. The low-frequency range of the radiation spectrum. The increase of the spectrum due to the transition radiation appears at those frequencies at which the intrinsic radiation of particles is suppressed by the density effect.

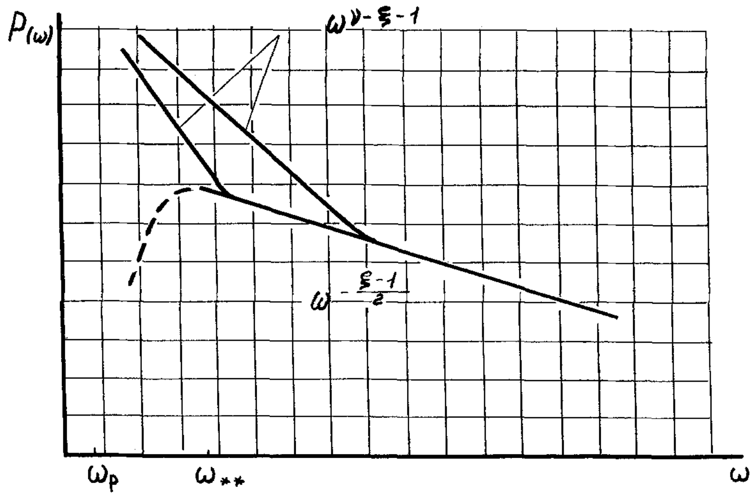

Fig. 5. The low-frequency range of radiation spectrum. The increase of the spectrum due to the transition radiation takes place when the density effect is absent.

The transition radiation may also be easily evaluated for an ensemble of weak shocks, by making use of the well-known expression for the radiation produced by a small density jump (Ginzburg and Tsytovich, 1984). Let the mean distance between fronts be $L \gg L_f$, L_f being a typical length along which the transition radiation is formed. The radiation intensity of one particle will be written as

$$I_\omega^{tr} = \frac{q^2}{6\pi L}\left(\frac{\Delta N}{N_0}\right)^2 [1 + \omega^2/(\omega_p^2\gamma^2)]^2 ; \tag{98}$$

and integration over the electron spectrum yields

$$P^{tr}(\omega) = \frac{q^2 K_{cr}}{12\pi L}\,\Gamma(\xi/2 - \tfrac{1}{2})\Gamma(\tfrac{5}{2} - \xi/2)\left(\frac{\Delta N}{N_0}\right)^2\left(\frac{\omega_p}{\omega}\right)^{\xi - 1}, \quad \omega \gg \omega_p . \tag{99}$$

The spectrum of the transition radiation is steeper than the synchrotron spectrum and gives larger contribution into the low-frequency range.

7. Numerical Estimates and Interpretation of Observed Spectra of Cosmic Radio Sources

Nowadays, radiospectra from many cosmic objects are measured in wide frequency bands. These spectra often possess kinks which are interpreted as the kinks in the energy spectra of radiating electrons (Kardashev, 1962) or are associated with the anisotropy of radiating particles (Galeev, 1984).

In this work we have obtained jumps of the spectral radiation power at high frequencies, in addition to kinks at low frequencies. Such high-frequency jumps are, indeed, observed in the radiation spectra of cosmic objects (Capps *et al.*, 1982). In this section we shall perform numerical estimates and try to interpret the spectra from some radio sources on the ground of the proposed theory.

To understand the nature of various spectral parts it is important to know the magnitude and frequency-dependence of the radiation polarization. For the synchrotron radiation in a quasi-uniform field, the polarization degree at high frequencies is larger than at low ones. However, when frequency increases, the polarization becomes lower, again, in the range, where the radiation is determined by a small-scale turbulence (and falls down to zero for an isotropic turbulence). Such an inverse behaviour of the frequency-dependence of the polarization may be regarded as a typical indication that the radiation is affected by small-scale turbulent fields. If the turbulence is quasi-one-dimensional (wave-vectors of MHD or Langmuir waves are aligned along the large-scale magnetic field), then, according to the calculation, the polarization becomes lower and the predominant direction of E-vector (the position angle) turns at an angle of 90°.

7.1. Radiation due to small-scale turbulence

Let us estimate a jump at a frequency $\omega_{B\perp}\gamma_2^2$ in the radiation spectrum. At $\nu = 1$–2 we approximately have

$$\frac{P_1}{P_2} \approx 0.1 \left(\frac{\omega_{st}}{\omega_{B\perp}}\right)^2 \left(\frac{\omega_0}{\omega_{B\perp}}\right)^{\nu-1} . \tag{100}$$

For an order-of-magnitude estimate, let us make use of the parameters of the magnetic fields and turbulence known for the interplanetary space (see Toptygin, 1985, for review):

$$\begin{aligned} &B_0 = 4.1 \times 10^{-5}\,\mathrm{G}\,; \qquad \langle B_{st}^2\rangle = 3.6 \times 10^{-10}\,\mathrm{G}^2\,; \\ &L_0 = 2 \times 10^{11}\,\mathrm{cm}\,. \end{aligned} \tag{101}$$

There is clearly no ground to assume that the turbulence properties in radio sources such as supernova remnants, radiogalaxies and quasars are the same as in the interplanetary medium. However, at present one has no detailed data on the turbulence in the above objects. On the other hand, the circumsolar plasma provides an example of cosmic plasma with well-known parameters, and may be used to verify some hypotheses concerning distant objects.

Substituting the above values, we obtain $P_1/P_2 \approx 0.9 \times 10^{-3}$. Under realistic conditions, such a jump may be really observed. Moreover, the local radiation intensity at high frequencies may be much larger, if the magnetic field component transverse to the line-of-sight is small.

In cosmic objects the regular magnetic field is, actually, non-uniform and has different directions in different spatial regions. Hence, on the average, we have $\omega_{B\perp} \approx \omega_B$. However, in high-resolution observations one often gets images of objects with dark lines or regions (Jura, 1982) created probably owing to local decreases of the component of the large-scale magnetic field transverse to the line-of-sight. High-resolution observations of the radiation from these regions might give useful information on the properties of the small-scale turbulence (on its intensity and spectral index ν). The main turbulence scale in these objects may be estimated from distance between neighbouring dark lines (regions).

Note that if the radiation from dark regions is determined by small-scale turbulence, its polarization degree ought to be lower than for radiation from bright regions.

To get a power-law tail (with index ν) in the radiation spectra of electrons, one needs the minimum turbulence scale $l_{\min}$ to be much smaller than the scale mc^2/eB along which the synchrotron radiation is formed. According to the data on interplanetary medium (Beinroth and Neubauer, 1981; Kennel *et al.*, 1982), $mc^2/eB \approx 4 \times 10^7$ cm and $l_{\min} \approx 10^5$ cm. In this case the tails may occupy frequency intervals whose boundary frequencies differ by two or three orders of magnitude.

As an example of objects where the radiation due to the small-scale turbulence is important let us consider the object OJ 287. The optical and radio spectra of this object were presented by Ennis *et al.* (1982) (Figure 6). Analyzing these spectra, one can assume that the radioemission is of the synchrotron nature (as was pointed out by Ennis

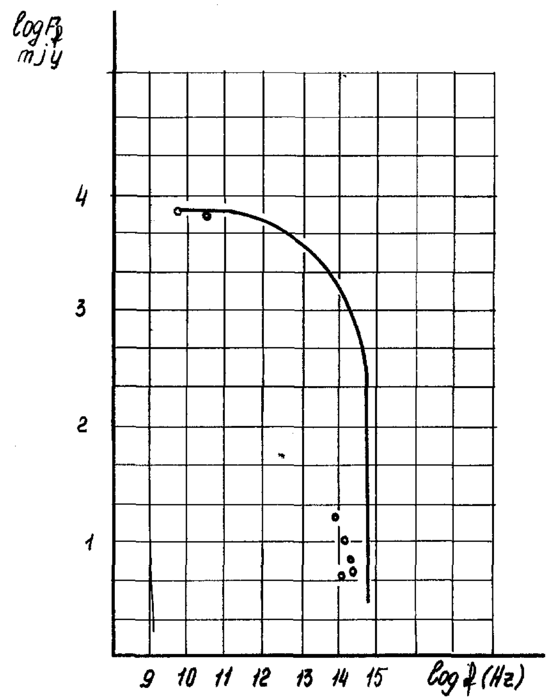

Fig. 6. The radiation spectrum for the object OJ 287 (Ennis *et al.*, 1982). Circles – observational data; solid curve – theory of synchrotron radiation with energy cut-off, but without turbulence. One can see, that such theory explains the observations unsatisfactorily.

et al., 1982), and the optical emission is formed due to the small-scale turbulence. According to Ennis *et al.* (1982) the polarization of the radio emission is $P(3.7\text{ cm}) = 16.6\%$. The polarization in optics shows temporal variations from 2.8 to 13.5% (Visvanathan, 1973), and from 1 to 32% (Ennis *et al.*, 1982), for larger observation period. The polarization grows simultaneously with the increase of the radiation intensity and with the rotation of position angle at about 90° (Visvanathan, 1973).

The above behaviour of the optical radiation may be explained by assuming that the quantity $\omega_c = \omega_{B\perp}\gamma_2^2$ suffers deviations from its mean value $\overline{\omega}_c$. According to König and Arnab (1985) this may be associated with a shock propagating in the object and illuminating successively regions with different strengths of the regular magnetic field. Then, with decreasing ω_c the optical synchrotron radiation fals down exponentially, so that all the optical radiation becomes determined by small-scale turbulence. This assumption explains the behaviour of the polarization described above.

In the frame of this hypothesis, one may estimate some parameters of the turbulence. The spectral turbulence index coincides with the index $\nu = 1.25$ (Visvanathan, 1973) of the optical radiation, and the relative turbulence level is determined by Equation (88) which gives $\langle B_{st}^2\rangle/B_0^2 \approx 4\times 10^{-3}$. Note that a close spectral index ($\nu = 1.2$) was reported to be realized in 1962 in the interplanetary space (Toptygin, 1985).

7.2. Radiation due to long-wavelength plasmons

Tsytovich and Chikhachev (1969) were first who considered this radiation. As follows from the results of Section 6, the radiation due to long-wavelength plasmons may be observed if $\omega_p \gg \omega_{B\perp}$ that is commonly the case in astrophysical objects. The spectral jump at $\omega \approx \omega_{B\perp}\gamma_2^2$ is determined by the ratio of the plasmon energy to the energy of the quasi-uniform magnetic field,

$$\frac{P_1(\omega)}{P_2(\omega)} \approx 0.1\,\frac{\omega_E^2}{\omega_{B\perp}^2}\,, \qquad \xi = 3\,, \qquad \omega_E = \frac{e\,\langle E^2\rangle^{1/2}}{mc}\,. \tag{102}$$

At $\omega \approx \omega_p\gamma_2^2$ the spectrum suffers the second jump accompanied by variation of the spectral index; the radiation at $\omega \gg \omega_p\gamma_2^2$ is determined by small-scale pulsations of electric fields. The corresponding jump is equal to

$$\frac{P_{st}^1(2\omega_p\gamma_2^2)}{P_1(2\omega_p\gamma_2^2)} \approx \frac{\omega_{st}^2}{\omega_E^2}\,. \tag{103}$$

From the positions of these two jumps one can find the ratio $\omega_p/\omega_{B\perp}$.

Let us consider the radiation spectrum obtained by Henry *et al.* (1984), and presented in Figure 7 from a jet of the quasar 3C 273. The radiation from this object is of the synchrotron nature which is confirmed by high polarization, $P(2.7\text{ GHz}) = 8\%$. The polarization of the optical radiation is lower, $P < 4\%$; the spectral indices in radio and optics are equal, within the measurement errors. It is also important that the position angle in radio and optics differs by 90°. These data better agree with the assumption

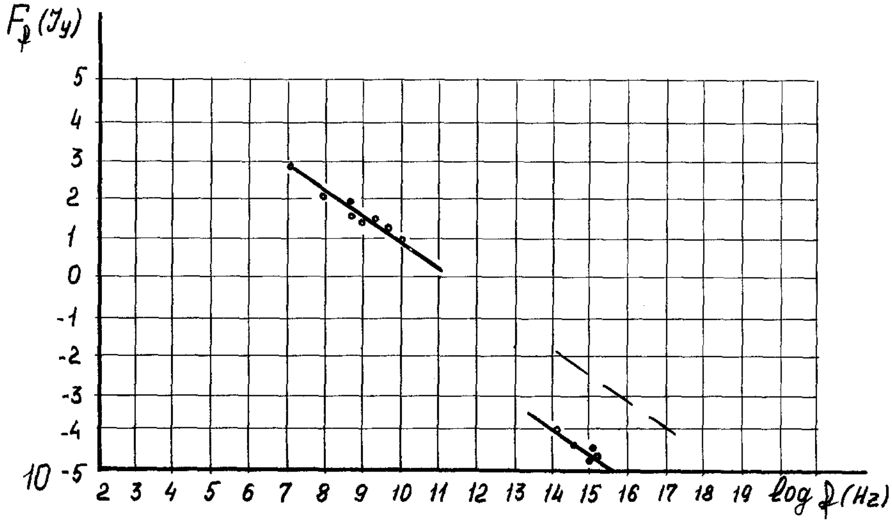

Fig. 7. The emission spectrum from a jet of quasar 3C 273 (Henry *et al.*, 1984).

on the plasmon (due to long-wavelength plasmons) nature of the optical radiation than with the assumption on the synchrotron nature (Henry *et al.*, 1984). However, to get the observed intensity of the optical radiation one needs a high level of the Langmuir turbulence; from the observed jump $P_1/P_2 \approx \frac{1}{30}$ we find $\langle E_{st}^2 \rangle / B_\perp^2 \approx 0.3$. But according to Bodo and Ferrari (1982) and Pelletier and Zaninetti (1984) the turbulence in jets may be, indeed, rather intensive.

7.3. Transition Radiation due to Inhomogeneities of Magnetic Field and Electron Number Density

As has been shown in Section 5, in the presence of the developed MHD turbulence the main contribution into the emission of relativistic particles comes from fluctuations of electron number density in magneto-acoustic waves. This conclusion remains valid when Langmuir waves are present too (and their level does not exceed the level of MHD turbulence) because at comparable levels of turbulent pulsations the intensity of the transition radiation due to Langmuir waves is a factor of $(\omega_p/\omega_B)^2 \gg 1$ lower than the intensity of the radiation due to magneto-acoustic waves. Let us discuss the possibility to observe the transition radiation in a turbulent plasma. For this purpose consider the ratio of the transition to synchrotron radiation intensities of electrons. With account for the values of numerical factors we obtain

$$\frac{P^m(\omega)}{P_2(\omega)} \approx 3 \times 10^{-2} \frac{\overline{Z^2 K_{cr}}}{K_e} \left(\frac{\omega_{st}}{\omega_B}\right)^2 \left(\frac{\omega_0}{\omega_p}\right)^{\nu - 1} \times$$

$$\times \left(\frac{\omega_p}{\omega_{B\perp}}\right)^{\xi/2 + 1/2} \left(\frac{\omega_p}{\omega}\right)^{\xi/2 + 3/2 - \nu} . \qquad (104)$$

Adopting the parameter values (101), typical for interplanetary space and taking $\omega_p = 1.3 \times 10^{-5}\,\mathrm{s}^{-1}$, $\nu = 1.5$, $\xi = 3$, we can rewrite (104) in the form

$$\frac{P^m(\omega)}{P_2(\omega)} \approx 0.6\, \frac{\overline{Z}^2 K_{cr}}{K_e} \left(\frac{\omega_p}{\omega}\right)^{3/2} . \tag{105}$$

In this case $\overline{Z}^2 K_{cr}/K_e > 1$ because the numerator takes into account all relativistic particles, while the denominator takes into account only the electrons. Equations (104) and (105) neglect the suppression of the synchrotron radiation at low frequencies, $\omega < \omega_{**} \approx \omega_p(\omega_p \gamma_1^e/\omega_{B\perp})^{1/2}$ due to the density effect. That is why, actually, the estimation (105) should be larger and the transition radiation becomes dominant when the density effect takes place.

If the magnetic fields are weak enough, the transition radiation may exceed the synchrotron one even at higher frequencies when the density effect does not still occur. This is because with the decrease of the quasi-uniform magnetic field the synchrotron radiation becomes less intensive whereas the transition radiation becomes more intensive, so that the ratio (104) increases rapidly (as $B^{-\xi/2-5/2}$) with decreasing magnetic field.

The transition radiation may also be effectively generated by shocks. The intensity ratio of this and the synchrotron radiation is

$$\frac{P^{tr}(\omega)}{P_2(\omega)} \approx 2 \times 10^{-2}\, \frac{\overline{Z}^2 K_{cr}}{K_e}\, \frac{c}{\omega_p L} \left(\frac{\omega_p}{\omega_{B\perp}}\right)^2 \frac{\omega_p}{\omega} , \tag{106}$$

where L is a mean distance between shock fronts; for estimation, it has been assumed that $\Delta N \approx N$ and $\xi = 3$. Putting $(\omega_p/\omega_{B\perp}) \approx 3 \times 10^4$ (as for the interplanetary space), we obtain

$$\frac{P^{tr}(\omega)}{P_2(\omega)} \approx 6 \times 10^2\, \frac{\overline{Z}^2 K_{cr}}{K_e}\, \frac{c}{\omega_p L}\, \frac{\omega_p}{\omega} .$$

At $\overline{Z}^2 K_{cr}/K_e \approx 1$ one has $P^{tr}(\omega)/P_2(\omega) > 1$, if distance between fronts does not exceed $L_{\max} = 10^8$ cm. However, in other cases an estimate may be more optimistic. For instance, according to Fedorenko and Samsonov (1979), the typical parameters for radiogalaxies are $N_e \approx 10^{-3}$–$10^{-4}\,\mathrm{cm}^{-3}$, $\omega_p \approx 10^3\,\mathrm{s}^{-1}$, and $L_{\max} \approx 10^{10}$ cm. The above estimates reveal that the transition radiation in a plasma with MHD turbulence may noticeably change the spectrum of the synchrotron radiation and should be taken into account while analysing observational data.

Let us notice that the conclusions of Section 7.1 on favourable conditions for observation of the radiation due to the small-scale turbulence (from dark regions) are fully valid for observation of the transition radiation. Since we discuss radio spectral range, one can use VLBA data to study the turbulence from radiation spectra of cosmic objects.

Simultaneous analysis of the low-frequency spectral range (form mainly by magneto-acoustic and shock waves) and the high-frequency range (associated with small-scale fields of all turbulence modes) would allow one to estimate the relative contribution of different modes into the turbulence.

In conclusion let us consider available observational data. According to Braude *et al.* (1971) the spectra of some radiosources (mainly, of radiogalaxies) at low frequencies, $f \approx 10$–5000 MHz, show a spectral rise as compared to the law appropriate to the synchrotron radiation. The authors explain this phenomenon by assuming that the energy spectra of electron differ from (83) and are given by

$$\mathrm{d}N(\mathscr{E}) = K_0 \mathscr{E}^{-\xi} \exp(K \mathscr{E}^{-2}), \quad \mathscr{E} > \mathscr{E}_{min}. \tag{107}$$

Braude and Kaner (1972) made an attempt to derive the spectrum (107) from theoretical consideration. In this case the turbulence energy w_T should be much larger than the magnetic field energy w_B as well as the thermal energy NkT.

Even without discussing extremely exotic character of these assumptions, let us notice that at $w_T \gg w_B$ the emission spectrum will be determined not by synchrotron but by plasma mechanisms considered above. We think that the spectral rises at low frequencies are more naturally explained by assuming that they are produced by the transition radiation of energetic particles in a turbulent plasma with magneto-acoustic and shock waves.

This mechanism quite well explains an empirically discovered property of the observed spectra with rises: the higher the frequency at which a rise becomes pronounced, the steeper is the spectrum at $f < f_0$. The frequency f_0 falls commonly in the interval $30 \lesssim f_0 \lesssim 300$ MHz.

8. Conclusions

The plasma turbulence leads to a wealth of features in the radiation spectra of relativistic particles. A discovery of these features in observed radio and optical spectra allows one to obtain valuable and unique information on physical properties of radiation sources. Below we outline briefly the possibilities investigated in this paper.

(1) An intrinsic radiation of a single electron at low ($\omega < \omega_p \gamma$) and high ($\omega > \omega_{B\perp} \gamma^2$) frequencies is determined by small-scale fields in a plasma.

(2) Inhomogeneities of a turbulent plasma lead to appearance of a new radiation mechanism, the transition radiation, which dominates at low frequencies, $\omega < \omega_p \gamma$.

(3) The related effects are shown to occur also in the radiation produced by an ensemble of particles with a power-law spectrum; to get a high-frequency power-law tail in the radiation spectrum with the spectral index equal to the index ν of the turbulence, one needs to have a sharp enough cut-off of the energy spectrum of electrons at $\mathscr{E} = mc^2 \gamma_2$. The transition radiation may lead to a spectral rise at low frequencies.

(4) It is concluded that at $\omega \gtrsim \omega_{B\perp} \gamma_2^2$ the polarization becomes lower and the position angle turns at 90°, if an anisotropic (quasi-one-dimensional) turbulence is present.

(5) It is shown that the above phenomena may be studied by analysing the emission

from dark regions (observed in some objects) because the synchrotron radiation from these regions is weak due to the smallness of the transverse (with respect to line-of-sight) magnetic field $B_\perp \ll B$.

(6) It is pointed out that a non-thermal radiation from some objects may be explained on the ground of the developed scheme. In particular, a spectral rise at low frequencies observed in some objects may be explained by the transition radiation of relativistic particles due to the presence of shocks and magneto-acoustic waves.

(7) The observations of the above features of radiation permit to find parameters of small-scale turbulence. The large-scale turbulence may be determined by means of observation of intensity fluctuations in radio emission from two near directions (Chibisov and Ptuskin, 1981).

Appendix. Evaluation of Radiation Spectrum at High Frequencies by the Method of Equivalent Photons

To simplify the kinetic equation we have made the approximation (see Equation (16)):

$$\theta^2 + \gamma^{-2} = a\gamma^{-2}, \tag{A.1}$$

where a is a constant. Below we will determine this constant from the requirement that the obtained result be coincident with the result of the perturbation theory (the method of equivalent photons) at high frequencies, $\omega \gg \omega_{B\perp}\gamma^2$.

In the rest-frame of the relativistic particle random magnetic fields look as a set of almost transverse pseudo-photons which are scattered by the particle, leading to the generation of electromagnetic wave. First of all let us find the electromagnetic field in this frame, assuming that in a laboratory reference frame only random magnetic fields are available. At $\gamma \gg 1$ we have

$$\mathbf{B}'_\perp = \gamma \mathbf{B}_\perp\,, \qquad \mathbf{E}'_\perp = [\mathbf{v}, \mathbf{B}_\perp]\gamma/c\,, \qquad E'_\parallel = 0\,, \qquad B'_\parallel = B_\parallel\,. \tag{A.2}$$

In this case $\parallel$ and $\perp$ label the field components with respect to the particle velocity $\mathbf{v}$.

The radiation intensity may be written as

$$\mathrm{d}I_{\mathbf{n},\,\omega} = \hbar\omega\,\mathrm{d}N(\mathbf{n}, \omega)\,, \tag{A.3}$$

where $\mathrm{d}N$ is a number of pseudo-photons scattered per unit time. It is expressed through the number density $n(\omega)$ of incident pseudo-photons. To find $n(\omega)$ let us evaluate the Poynting vector in the primed reference frame,

$$\mathbf{S}' = -\frac{\gamma^2\mathbf{v}}{4\pi}\,\langle B_\perp^2(r', t')\rangle\,. \tag{A.4}$$

Introducing Fourier-components of random fields and making use of Equation (10), we obtain

$$\mathbf{S}' = -\frac{\gamma\mathbf{v}}{8\pi}\int \langle B_\perp^2(\mathbf{k}, \omega)\rangle\,(1 - k_\parallel^2/k^2)\,\mathrm{d}^3k\,\mathrm{d}\omega\,. \tag{A.5}$$

After integration over $d\omega$ and insertion of the spectral function of random fields in the form (69) we get

$$\mathbf{S}' = -\frac{\gamma^2 A_\nu \mathbf{v}}{8\pi} \int\limits_{k \geq k_0} d^3k(1 - k_\parallel^2/k^2)k^{-\nu-2}. \tag{A.6}$$

In this integral we introduce the new variables $k'_\perp$, $\omega' = -\gamma c k_\parallel$. Writing down the energy current density in the form

$$\mathbf{S}' = \int\limits_0^\infty \mathbf{v}\hbar\omega' n(\omega')\, d\omega', \tag{A.7}$$

we find the spectral number density of the quanta,

$$n(\omega') = \begin{cases} \dfrac{(\nu+1)A_\nu}{\nu(\nu+2)\hbar c} \dfrac{\gamma^{\nu+1}c^\nu}{\omega'^{\,\nu+1}}, & \omega' \geq \gamma c k_0; \\[2ex] \dfrac{A_\nu \gamma}{2\nu k_0^\nu \hbar c}\left\{1 + \dfrac{\nu\omega'^2}{(\nu+2)\gamma^2 c^2 k_0^2}\right\}\dfrac{1}{\omega'}, & \omega' < \gamma c k_0. \end{cases} \tag{A.8}$$

The number of pseudo-photons scattered per unit time in the particle rest-frame into a solid angle $d\Omega'_2$ at a frequency ω'_2 is given by

$$dN'(\mathbf{n}'_2, \omega'_2) = cn(\omega'_1)\, d\sigma(\omega'_1, \omega'_2, \theta'), \tag{A.9}$$

where

$$d\sigma(\omega'_1, \omega'_2, \theta') = \frac{d\Omega'_2}{2}\left(\frac{e^2}{mc^2}\right)^2 \left(\frac{\omega'_2}{\omega'_1}\right)^2 \left(\frac{\omega'_1}{\omega'_2} + \frac{\omega'_2}{\omega'_1} - \sin^2\theta'\right) \tag{A.10}$$

is the cross-section of the Compton-scattering with frequency variation fro ω'_1 to ω'_2; θ' is an angle between the particle velocity and propagation direction of the scattered quanta. In the laboratory reference frame the number of radiating quanta is expressed through (A.9) as

$$dN(\mathbf{n}_2, \omega_2) = \frac{dN'}{d\Omega'_2}\, \frac{\omega_2}{\gamma\omega'_2}\, d\Omega_2. \tag{A.11}$$

The quantities ω'_2 and θ' which enter this equation should be expressed through their values ω_2 and θ in the laboratory frame,

$$\omega' = \gamma(1 - \frac{v}{c}\cos\theta)\omega, \qquad \cos\theta' = \left(\cos\theta - \frac{v}{c}\right)\Big/\left(1 - \frac{v}{c}\cos\theta\right). \tag{A.12}$$

If the quantum recoil may be neglected, i.e., $\hbar\omega \ll mc^2\gamma$, then $\omega'_2/\omega'_1 \approx 1$. With allowance for (A.12) we obtain

$$\omega_2'/\omega_2' + \omega_2'/\omega_1' - \sin^2\theta' \approx 2\,. \tag{A.13}$$

Now, using (A.3), we find the angular and spectral distribution of the radiation intensity

$$I_{\mathbf{n},\,\omega} = \frac{(\nu+1)A_\nu r_0^2 c^\nu}{\nu(\nu+2)\gamma^2}\left(1 - \frac{v}{c}\cos\theta\right)^{-\nu-2}\omega^{-\nu}\,,$$
$$\frac{k_0 c}{\omega} < 1 - \frac{v}{c}\cos\theta\,, \tag{A.14}$$

where $r_0 = e^2/mc^2$ is the classical electron radius. Integration over angles leads to the spectral distribution of radiation

$$I_\omega = \frac{2^{\nu+2}\pi r_0^2}{\nu(\nu+2)}\,A_\nu\gamma^{2\nu}c^\nu\omega^{-\nu}\,, \quad \omega > 2k_0 c\gamma^2\,. \tag{A.15}$$

The radiation intensity calculated for these frequencies from the general equation (35) is given by

$$I_\omega = \frac{2^{\nu+4}\pi r_0^2}{3\nu}\,A_\nu\gamma^{2\nu}c^\nu\omega^{-\nu}a^{-\nu}\,. \tag{A.16}$$

Let us find the constant a in question by requiring (A.15) and (A.16) to be equal. From this we obtain

$$a = [\tfrac{4}{3}(\nu+2)]^{1/\nu}\,. \tag{A.17}$$

This quantity is seen to be close to 2 that confirms the assumption $\theta^2 \approx \gamma^{-2}$ made at the derivation.

References

Axford, W. I.: 1981, *Proc. 17th ICRC, Paris* **12**, 155.

Beinroth, H. J. and Neubauer, F. M.: 1981, *J. Geophys. Res.* **86**, 7755.

Bekefi, G.: 1966, *Radiation Processes in Plasma*, John Wiley and Sons, New York.

Bel'kov, S. A., Nikolaev, Yu.A., and Tsytovich, V. N.: 1980, *Izw. Vuzov, Radiofiz.* **23**, 261.

Berezinsky, V. S., Bulanov, S. V., Ginzburg, V. L., Dogiel, V. A., and Ptuskin, V. S.: 1984, *Astrophysics of Cosmic Rays*, Nauka, Moscow.

Bodo, G. and Ferrari, A.: 1982, *Astron. Astrophys.* **114**, 394.

Braude, S. Ya., Zhuck, I. N., and Rjabov, B. P.: 1971, Preprint IRE AN Ukr. SSR, No. 6, Khar'kov.

Braude, S. Ya. and Kaner, E. A.: 1972, Preprint IRE AN Ukr. SSR, No. 17, Khar'kov.

Capps, R. W., Sitco, M. L., and Stein, W. A.: 1982, *Astrophys. J.* **255**, 413.

Chibisov, G. V. and Ptuskin, V. S.: 1981, *Proc. 17th ICRC, Paris* **2**, 233.

Ennis, D. J., Neugebauer, G., and Werner, M.: 1982, *Astrophys. J.* **262**, 460.

Fedorenko, V. N. and Samsonov, A. M.: 1979, Preprint No. 602, Phys. Techn. Inst., Leningrad.

Galeev, A. A.: 1984, *Zh. Eksperim. Teor. Fiz.* **86**, 1655.

Ginzburg, V. L. and Frank, I. M.: 1946, *Zh. Eksperim. Teor. Fiz.* **16**, 15.
Ginzburg, V. L. and Syrovatskii, S. I.: 1964, *The Origin of Cosmic Rays*, Pergamon Press, Oxford.
Ginzburg, V. L. and Tsytovich, V. N.: 1984, *The Transition Radiation and Transition Scattering*, Nauka, Moscow.
Henry, J. P., Becklin, E. E., and Telesco, C. M.: 1984, *Astrophys. J.* **280**, 98.
Jura, M.: 1982, *Astrophys. J.* **258**, 59.
Kaplan, S. A. and Tsytovich, V. N.: 1969, *Usp. Fiz. Nauk* **97**, 77.
Kaplan, S. A. and Tsytovich, V. N.: 1972, *Plasma Astrophysics*, Nauka, Moscow.
Kardashev, N. S.: 1962, *Astron. Zh.* **39**, 393.
Kennel, C. F., Scarf, F. L., Coroniti, F. V., Smith, E. J., and Gurnett, D. A.: 1982, *J. Geophys. Res.* **87**, 17.
König, A. and Arnab, R.: 1985, *Astrophys. J.* **289**, 188.
Landau, L. D. and Lifshitz, E. M.: 1960, *Electrodynamics of Continuous Media*, Pergamon Press, New York.
Migdal, A. B.: 1954, *Dokl. Akad. Nauk SSSR* **96**, 749.
Pelletier, G. and Zaninetti, L.: 1984, *Astron. Astrophys.* **136**, 313.
Tamoykin, V. V.: 1978, *Astrophys. Space Sci.* **53**, 3.
Ter Mikaeljan, M. L.: 1954, *Dokl. Akad. Nauk SSSR* **94**, 1033.
Toptygin, I. N.: 1980, *Space Sci. Rev.* **26**, 157.
Toptygin, I. N.: 1985, *Cosmic Rays in Interplanetary Magnetic Fields*, D. Reidel Publ. Co., Dordrecht, Holland.
Toptygin, I. N. and Fleishman, G. D.: 1983, in 'XV Soviet Conference on Radioastronomy', *Abstracts of Reports*, p. 68, IRE AN Ukr. SSR, Khar'kov.
Toptygin, I. N. and Fleishman, G. D.: 1984, *Izv. Akad. Nauk SSSR, Fiz.* **48**, 2240.
Tsytovich, V. N.: *Vestnik MGU, Fiz.* **4**, 21.
Tsytovich, V. N. and Chikhachev, A. S.: 1969, *Astron. Zh.* **46**, 486.
Visvanathan, O.: 173, *Astrophys. J.* **185**, 145.