VORTICES IN STRONGLY MAGNETIZED NONUNIFORM ELECTRON-POSITRON-ION PLASMAS

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Abstract. The nonlinear mode coupling equations for electrostatic and electromagnetic waves in strongly magnetized nonuniform electron-positron-ion plasmas are derived. It is found that a small fraction of stationary ions (or high-Z charged impurities) can be responsible for the formation of coherent vortices which are forbidden when the ions are absent. Such vortices might significantly affect the transport properties of electron-positron plasmas in external magnetic fields.

It is well known (Rees, 1971, 1983; Michel, 1982; Begelman *et al.*, 1984) that electron-positron plasmas appear in the polar cap regions of pulsar magnetospheres, in the early universe, and in the inner region of the accretion disks surrounding the central black holes in active galactic nuclei. Recently, several authors (Gedalin *et al.*, 1985; Shukla *et al.*, 1986; Iwamoto, 1993; Zhao *et al.*, 1994, 1996; Greaves and Surko, 1994, 1995; Verheest, 1996) have investigated collective effects in strongly magnetized electron-positron plasmas. Such studies generally focus on the linear properties of electrostatic and electromagnetic waves and their generation mechanisms, and numerous nonlinear phenomena including the formation of coherent structures (Shukla *et al.*, 1986; Yu *et al.*, 1986; Zhao *et al.*, 1996). The latter involve solitary wave patterns and vortical motions which are associated with microstructures in plasmas. Specifically, Yu *et al.* (1986) have presented an analytical description of double vortices in strongly magnetized, nonrelativistic, uniform electron-positron plasmas.

However, Hoshino and Arons (1991) and Hoshino *et al.* (1992) have suggested that electron-positron plasmas also contain a small fraction of heavy ions. Recent investigations (Berezhiani *et al.*, 1992a, 1992b; Berezhiani and Mahajan, 1995) have shown that a component of immobile ions can lead to new nonlinear effects in uniform unmagnetized electron-positron plasmas. In this paper, we shall incorporate the effects of stationary ions in the study of the nonlinear coupling of low-frequency (in comparison with the electron gyrofrequency) electrostatic and electromagnetic waves in a strongly magnetized electron-positron plasma with an equilibrium density gradient. It will be found that in such a plasma there is a non-zero $\mathbf{E} \times \mathbf{B}_0$ particle current connected with a new type of wave spectrum and a

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new class of localized vortex structures, which would not have appeared if the ions were not present.

Let us consider an electron-positron-ion plasma with an equilibrium density gradient $\partial n_{j0}/\partial x$ in the presence of a uniform magnetic field $B_0\hat{\mathbf{z}}$, where n_{j0} is the unperturbed number density of the particle species j (j equals e for electrons, p for positrons, and i for ions), B_0 is the strength of the magnetic field, and $\hat{\mathbf{z}}$ is the unit vector along the z axis. At equilibrium, we thus have $n_{e0}(x) = Z_i n_{i0}(x) + n_{p0}(x)$, where $Z_i e$ is the charge of the positive ions, and e is the magnitude of the electron charge. We assume that the ions are so heavy that they do not participate in the dynamics of the system as the frequencies involved are much larger than the ion plasma and ion gyrofrequencies. Furthermore, when $|d_t|$ is much smaller than the electron (or positron) gyrofrequency ($\omega_c = eB_0/mc$, where m is the electron mass, and c is the speed of light), then in the wave fields the perpendicular (to $\hat{\mathbf{z}}$) component of the particle velocities in our cold plasma are

$$\mathbf{v}_{e\perp} \approx \mathbf{v}_E + \frac{c}{B_0\omega_c} d_t \nabla_\perp \phi + v_{ez}\mathbf{B}_\perp/B_0, \tag{1}$$

and

$$\mathbf{v}_{p\perp} \approx \mathbf{v}_E - \frac{c}{B_0\omega_c} d_t \nabla_\perp \phi + v_{pz}\mathbf{B}_\perp/B_0, \tag{2}$$

where $\mathbf{v}_E = (c/B_0)\hat{\mathbf{z}} \times \nabla\phi$ is the $\mathbf{E} \times \mathbf{B}_0$ drift velocity, $\mathbf{E} = -\nabla\phi - \hat{\mathbf{z}}(1/c)\partial_t A_z$ is the wave electric field, $\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{z}}$ is the wave magnetic field, ϕ is the scalar potential, and A_z is the z component of the vector potential. Furthermore, $d_t = \partial_t + \mathbf{v}_E \cdot \nabla$, with $\mathbf{v}_E \cdot \nabla \gg v_{ez,pz}\partial_z$, where $v_{ez}(v_{pz})$ is the electron (positron) velocity along the $\hat{\mathbf{z}}$ direction. We have neglected the compressional magnetic field perturbations.

Let us first derive the governing equations for electrostatic disturbances. Thus, we set $\mathbf{B}_\perp = 0$ in (1) and (2) and substitute them into the equation for the conservation of the charge density. The resulting equation can be written as

$$\left(1 + \frac{\omega_p^2}{\omega_c^2}\right) d_t \nabla_\perp^2 \phi - \frac{Z_i \omega_p^2}{\omega_c} \boldsymbol{\kappa} \cdot \nabla\phi - 4\pi \partial_z j_z = 0, \tag{3}$$

where $\nabla_\perp^2 = \partial_x^2 + \partial_y^2 \gg \partial_z^2, \omega_p^2 = 4\pi n_0 e^2/m, n_0 = n_{e0} + n_{p0}, \boldsymbol{\kappa} = (\hat{\mathbf{z}} \times \nabla n_{i0})/n_0$, and $j_z = -e(n_{e0}v_{ez} - n_{p0}v_{pz})$. The latter is given by (Shukla *et al.*, 1986)

$$d_t j_z = -(n_0 e^2/m)\partial_z \phi. \tag{4}$$

Second, we focus on electromagnetic perturbations for which we have from Ampére's law

$$j_z = -(c/4\pi)\nabla_\perp^2 A_z, \tag{5}$$

where the displacement current has been neglected as we are dealing with disturbances with phase velocities much smaller than the speed of light.

Substituting for $\mathbf{v}_{e\perp,p\perp}$ from (1) and (2) into the equation for the charge conservation equation, and making use of (5), we obtain the vorticity equation for electromagnetic disturbances. We have

$$\left(1+\frac{\omega_p^2}{\omega_c^2}\right) d_t\nabla_\perp^2\phi - \frac{Z_i\omega_p^2}{\omega_c}\boldsymbol{\kappa}\cdot\nabla\phi + c d_z\nabla_\perp^2 A_z = 0, \tag{6}$$

where $d_z = \partial_z + B_0^{-1}(\nabla A_z \times \hat{\mathbf{z}})\cdot\nabla$.

Furthermore, instead of (4), we here have (Yu *et al.*, 1986)

$$(1-\lambda^2\nabla^2)d_t A_z + c\partial_z\phi = 0, \tag{7}$$

where $\lambda = c/\omega_p$ is the collisionless skin depth of the electron-positron plasma. The λ^2-term in (7) is the contributions of the parallel inertia of the electrons and the positrons.

Equations (3) and (4) as well as (6) and (7) are, respectively, the coupled nonlinear equations which describe the dynamics of three-dimensional convective cells and shear Alfvén waves in strongly magnetized, cold electron-positron-ion plasmas containing an equilibrium density gradient. In the absence of this density gradient, our system of equations are identical to those of Shukla *et al.* (1986) and Yu *et al.* (1986).

In the linear limit, the local dispersion relations can be derived from (3), (4), (6), and (7) by assuming that ϕ and A_z are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$, where ω is the wave frequency and $\mathbf{k}$ is the wave vector. Accordingly, for convective cells and shear Alfvén waves, we have, respectively,

$$\omega^2 - \omega\omega_* - \omega_{cc}^2 = 0, \tag{8}$$

and

$$\omega^2 - \omega\omega_* - \omega_{sa}^2 = 0, \tag{9}$$

where $\omega_* = Z_i(\omega_p^2/ak_\perp^2\omega_c)\boldsymbol{\kappa}\cdot\mathbf{k}$, $a = 1+\omega_p^2/\omega_c^2$, $\omega_{cc}^2 = k_z^2\omega_p^2/ak_\perp^2$, and $\omega_{sa}^2 = k_z^2c^2/a(1+k_\perp^2\lambda^2)$. Here $k_z(\mathbf{k}_\perp)$ is the component of the wave vector along (across) the z axis. We note that for $k_\perp^2\lambda^2 \gg 1$, we have $\omega_{sa} = \omega_{cc}$, whereas the flute-mode is characterized by $\omega = \omega_*$ which is a new eigenmode of a nonuniform electron-positron-ion plasma in an external magnetic field.

In the following, we shall present quasi-stationary nonlinear solutions of the pair of Equations (3) and (4) as well as (6) and (7). In the quasi-stationary frame, we let $\xi = y + \alpha z - Vt$, where α and V are constants giving the angle and speed of the nonlinear structures. Thus, (3) and (4) can be cast in the form

$$D_{\xi\phi}(\nabla_\perp^2\phi - \delta\phi) = 0, \tag{10}$$

where $D_{\xi\phi} = \partial_\xi - (c/VB_0)(\partial_x\phi\partial_\xi - \partial_\xi\phi\partial_x)$, $\delta = P/aV$, $P = -Z_i(\omega_p^2/n_0\omega_c)(\partial n_{i0}/\partial x) - \omega_p^2\alpha^2/V$, and $\nabla_\perp^2 = \partial_x^2 + \partial_\xi^2$.

On the other hand, for the shear Alfvén waves we have from (6) and (7)

$$D_{\xi\phi}(\nabla_\perp^2\phi + \mu\phi) = \frac{\alpha c}{aV} D_{\xi A}\nabla_\perp^2 A_z, \tag{11}$$

and

$$(1 - \lambda^2\nabla_\perp^2)A_z = \frac{c\alpha}{V}\phi, \tag{12}$$

where $\mu = Z_i(\omega_p^2/aV\omega_c)(\partial n_{i0}/\partial x)/n_0$ and $D_{\xi A} = \partial_\xi - (1/\alpha B_0)(\partial_x A_z \partial_\xi - \partial_\xi A_z \partial_x)$.

From (11) and (12) we readily obtain

$$D_{\xi\phi}\left[\nabla_\perp^2\phi + \left(\mu + \frac{\alpha^2 c^2}{aV^2\lambda^2}\right)\phi - \frac{\alpha c}{aV\lambda^2}A_z\right] = 0. \tag{13}$$

We now discuss the solutions of (10) and (13). We see from (10) that for $\delta = 0$, the resulting equation is satisfied by the ansatz

$$\nabla_\perp^2\phi = \frac{4\phi_s K^2}{a_s^2}\exp[-(2/\phi_s)(\phi - VB_0 x/c)], \tag{14}$$

where ϕ_s, K and a_s are arbitrary constants. The solution of (14) is given by (Shukla *et al.*, 1995)

$$\phi = \frac{VB_0}{c}x + \phi_s \ln[2\cosh(Kx) + 2(1 - a_s^{-2})\cos(K\xi)], \tag{15}$$

which exhibits chains of vortices for $a_s^2 > 1$. On the other hand, for $a_s = 1$ we have zonal flow.

Furthermore, for $\delta > 0$ or $\partial n_{i0}/\partial x < 0$ with $|\partial n_{i0}/\partial x|V/n_0 > \alpha^2\omega_c$, (10) admits both monopolar and dipolar vortices. The structures of the latter are presented in Shukla *et al.* (1995). On the other hand, we note that for $\delta < 0$ Equation (10) does not admit localized vortex solutions. Thus, the ion density gradient plays a key role for the formation of vortices.

Finally, we discuss the properties of shear Alfvén dipolar vortices which are governed by

$$\nabla_\perp^4\phi + C_1\nabla_\perp^2\phi + C_2\phi - \frac{F_3}{\lambda^2}\frac{VB_0}{c}x = 0, \tag{16}$$

where $C_1 = \beta_1 - F_3 - 1/\lambda^2$, $C_2 = (F_3 - \beta_1)/\lambda^2 + c\alpha\beta_2/V\lambda^2$, $\beta_1 = \mu + \alpha^2c^2/aV^2\lambda^2$, $\beta_2 = \alpha c/aV\lambda^2$, and F_3 is an arbitrary constant of integration. We note that (16) has been derived from the solution of (13) by employing (12).

The dipolar vortex solution of (16) is

$$\phi = [Q_1K_1(s_1r) + Q_2K_1(s_2r)]\cos\theta, \tag{17}$$

in the outer region ($r = (x^2 + \xi^2)^{1/2} > R$), where $F_3 = 0$, and

$$\phi = \left[Q_3J_1(s_3r) + Q_4I_1(s_4r) + \frac{F_3}{\lambda^2}\frac{VB_0}{cC_2}r\right]\cos\theta, \tag{18}$$

in the inner region ($r < R$). Here, R is the vortex radius, $\cos\theta = x/r$, Q_j are arbitrary constants, and K_1 and I_1 are modified Bessel functions. Furthermore, we have defined $s_{1,2}^2 = [-\alpha_1 \pm (\alpha_1^2 - 4\alpha_2)^{1/2}]/2$ for $\alpha_1^2 > 4\alpha_2 > 0$ and $s_{3,4} = [C_1^2 - 4C_2)^{1/2} \pm C_1]/2$ for $C_2 < 0$. Here, $\alpha_1 = \beta_1 - 1/\lambda^2$ and $\alpha_2 = -(\beta_1/\lambda^2) + c\alpha\beta_2/V\lambda^2$. The constant Q_j can be determined by matching the inner and outer solutions of ϕ, A_z, $\partial_r\phi$, $-\partial_r A_z$, $\nabla_\perp^2\phi$, and j_z at the vortex interface $r = R$. We note that the dipolar vortex profiles, as given by (17) and (18), are significantly different from those presented by Yu *et al.* (1986), as the ion density gradient is the reason why we can have a bounded outer solution.

In summary, we have derived the nonlinear equations for low-frequency electrostatic and electromagnetic disturbances in a nonuniform electron-positron-ion plasma in an external magnetic field. It is found that the presence of a stationary ion component in strongly magnetized electron-positron plasmas gives rise to a novel flute-like mode the frequency of which is proportional to the gradient of the ion number density. Physically, the mode arises because the divergence of the particle $\mathbf{E} \times \mathbf{B}_0$ currents in an equilibrium ion density gradient is exactly balanced by the currents arising from the particle polarization drifts and the deviation from the quasi-neutrality condition. Furthermore, we have found that the inhomogeneous (the κ-) term in (3) and (6) is responsible for the formation of vortex streets and well behaved double vortices, which would have been otherwise absent if the ions were not taken into consideration. The coherent vortex structures might be responsible for the convective transport of particles across the external magnetic field lines (Shukla *et al.*, 1986).

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