AXIALLY SYMMETRIC EXPLOSION IN MAGNETOGASDYNAMICS

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Abstract. A point explosion in a spheroid with axially symmetric exponential density distribution is investigated by generalizing the method of Laumbach and Probstein to include the effects of a magnetic field. It is shown that the shock velocity decreases and tends to zero. Also, the elongation of the shock envelope along the axis of symmetry is much reduced and the blowout of the shock wave is removed on account of the magnetic field.

1. Introduction

Axisymmetric disturbances are of considerable physical interest – for example, the propagation of shocks in stellar atmospheres, supernova explosions, high-altitude nuclear detonation, and phenomena associated with laser production of plasmas.

The problem of a strong shock wave propagating from a point energy source in an inhomogeneous atmosphere whose density varies exponentially, has been studied by several authors. Kompaneets (1960) and Andriankin *et al.* (1962) analysed the problem approximately by assuming uniform pressure behind the shock front. Laumbach and Probstein (1969) obtained an explicit analytical solution by taking the flow field as 'locally radial' and using an integral method with an energy constraint. Although different approximations have been made to simplify this problem, all these methods lead to excellent agreement, even with exact numerical results (Sachdev, 1972). Sakashita (1971), Bhowmick (1976) and Möllenhoff (1976) applied the method of Laumbach and Probstein to a point explosion in an axially symmetric spheroid with exponential or Gaussian density distribution. In magnetogasdynamics the propagation of a spherical shock wave in an inhomogeneous medium has been studied by Summers (1975), Verma and Vishwakarma (1978) and Verma and Singh (1979). In these problems, the magnetic field is assumed to be an idealized field such that the lines of force lie on hemispheres whose centre is the point of explosion and directed tangentially to the advancing shock front. Rosenau and Frankenthal (1976) and Rosenau (1977) considered the propagation of axisymmetric magnetohydrodynamic shocks using similarity methods. As far as we are aware, the Laumbach and Probstein method to obtain analytical solutions for strong shocks propagating in an inhomogeneous medium has not been developed in magnetogasdynamics.

The aim of the present paper is to generalize the method of Laumbach and Probstein by taking into account the interaction with the azimuthal magnetic field and investigate

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the effects of a magnetic field on the axially symmetric explosion in a spheroid with exponential density distribution. In Section 2, the method of Laumbach and Probstein is generalized to include the magnetogasdynamic effects. In Section 3, as an example, the point explosion in a spheroid (Sakashita, 1971) is analysed and the effects of a magnetic field are discussed in Section 4.

2. Generalization of Laumbach and Probstein's Method in Magnetogasdynamics

We consider a point explosion in a medium with general axially symmetric distribution of density. Thus the density ϱ_0 may be taken as $\varrho_0\,(r_0, \theta)$, where θ is the angle between the axis of symmetry and the radius vector, and r_0 is the position of a fluid particle at $t = 0$, the time of explosion. Following Laumbach and Probstein, we assume the flow field to be locally radial – i.e., gradients in the θ direction are neglected. If r is the Eulerian coordinate of a fluid particle of thickness dr, the equation of continuity for any polar angle is of the form

$$\varrho_0 r_0^2\,\mathrm{d}r_0 = \varrho r^2\,\mathrm{d}r, \tag{1}$$

where the Lagrangian formulation is adopted with r_0 and t as independent variables. The electrical conductivity of the gas is assumed to be infinite and permeated by an azimuthal magnetic field. Thus, the momentum equation and the equation for a magnetic field in Lagrangian coordinates are given (cf. Summers, 1975; Rosenau and Frankenthal, 1976; Rosenau, 1977), respectively, by

$$\frac{\partial^2 r}{\partial t^2} + \frac{r^2}{\varrho_0 r_0^2}\left(\frac{\partial p}{\partial r_0} + h\,\frac{\partial h}{\partial r_0}\right) + \frac{h^2}{\varrho r} = 0 \tag{2}$$

and

$$\frac{\partial h}{\partial t} + \left(\frac{\varrho r^2}{\varrho_0 r_0^2}\right) h\,\frac{\partial u}{\partial r_0} + \frac{hu}{r} = 0, \tag{3}$$

where the symbols have their usual meanings. The flow is assumed to be adiabatic, and the energy equation accordingly reduces to

$$\frac{p(r_0, t)}{p_s(r_0)} = \left[\frac{\varrho(r_0, t)}{\varrho_s(r_0)}\right]^{\gamma}, \tag{4}$$

where the subscript s refers to conditions when the particle is at the shock front.

With the strong shock assumption we have

$$\varrho_s = \frac{\gamma + 1}{\gamma - 1}\,\varrho_0, \tag{5}$$

$$p_s = \frac{2}{\gamma + 1}\,\varrho_0 \dot{R}^2, \tag{6}$$

and

$$h_s = \frac{\gamma + 1}{\gamma - 1} h_0, \tag{7}$$

where R is the location of shock front and the dot represents differentiation with respect to time.

In the initial phase, the shock wave generated by the explosion propagates outwards isotropically with its spherical shape. Also, in the example analysed by us (Section 3), the shock envelope is nearly spherical throughout. Thus the equation of continuity in Lagrangian coordinates with spherical symmetry holds good approximately in the form

$$\frac{\partial \varrho}{\partial t} + \left(\frac{\varrho r^2}{\varrho_0 r_0^2}\right) \varrho \frac{\partial u}{\partial r_0} + \frac{2\varrho u}{r} = 0. \tag{8}$$

The magnetic field equation (Equation (3)) with the continuity equation (8) give the 'frozen-in field' condition

$$\frac{h}{\varrho r} = \text{const} = \frac{h_s}{\varrho_s R} = \frac{h_0}{\varrho_0 r_0}, \tag{9}$$

as the shock location $R(t)$ takes the same value in both Lagrangian or Eulerian coordinates (see Equation (19)). Using Equation (9) in Equation (1), we get the following equation for a magnetic field analogous to the continuity equation (1):

$$h_0 r_0 \, \mathrm{d}r_0 = hr \, \mathrm{d}r. \tag{10}$$

Integrating the momentum equation (2) and using Equation (10), we obtain

$$p^*(r_0, \theta, t) - p_s^*(R, \theta) = \int_{r_0}^{R} \frac{1}{r^2} \frac{\partial^2 r}{\partial t^2} \varrho_0 r_0^2 \, \mathrm{d}r_0 + \int_{r_0}^{R} \frac{1}{r^3} \left(\frac{\partial r}{\partial r_0}\right)^{-1} h_0^2 r_0^2 \, \mathrm{d}r_0, \tag{11}$$

with

$$p^* = p + \frac{h^2}{2}.$$

Now, the magnetic field equation (3), with the aid of Equations (1) and (10), may be written in the form

$$\frac{\partial h}{\partial r_0} + \left(\frac{1}{ru^2} \frac{\partial u}{\partial t} + \frac{1}{r^2}\right) h_0 r_0 = 0,$$

which on integration gives

$$h(r_0, \theta, t) - h_s(R, \theta) = \int_{r_0}^{R} \left[\frac{1}{r^2} + \frac{1}{ru^2} \left(\frac{\partial^2 r}{\partial t^2}\right)\right] h_0 r_0 \, \mathrm{d}r_0. \tag{12}$$

For strong shocks, assuming that most of the mass is concentrated near the shock front, which is the idea used in Laumbach and Probstein's method, Equations (11) and (12) may be approximated as

$$p^* - p_s^* = \frac{1}{R^2}\left(\frac{\partial^2 r}{\partial t^2}\right)_s \int_{r_0}^{R} \varrho_0 r_0^2 \, dr_0 + \frac{1}{R^3}\left(\frac{\partial r}{\partial r_0}\right)_s^{-1} \int_{r_0}^{R} h_0^2 r_0^2 \, dr_0 \tag{13}$$

and

$$h - h_s = \left[\frac{1}{R^2} + \frac{1}{R u_s^2}\left(\frac{\partial^2 r}{\partial t^2}\right)_s\right] \int_{r_0}^{R} h_0 r_0 \, dr_0 . \tag{14}$$

Combining Equations (13) and (14), we obtain

$$p - p_s = \frac{1}{R^2}\left(\frac{\partial^2 r}{\partial t^2}\right)_s \left[J - \frac{RLh_s}{u_s^2}\right] - \frac{Lh_s}{R^2} - \\ - \frac{1}{2}\left[\frac{1}{R^2} + \frac{1}{R u_s^2}\left(\frac{\partial^2 r}{\partial t^2}\right)_s\right]^2 L^2 + \frac{1}{R^3}\left(\frac{\partial r}{\partial r_0}\right)_s^{-1} K, \tag{15}$$

with

$$J = \int_{r_0}^{R} \varrho_0 r_0^2 \, dr_0, \qquad K = \int_{r_0}^{R} h_0^2 r_0^2 \, dr_0$$

and

$$L = \int_{r_0}^{R} h_0 r_0 \, dr_0 .$$

As the flow field is locally radial, the mass contained within a differential solid angle is constant; therefore, the integral energy conservation equation assumes the form

$$\frac{E}{4\pi} = \int_0^R \frac{p}{\gamma - 1} r^2 \, dr + \int_0^R \frac{h^2}{2} r^2 \, dr + \int_0^R \frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2 \varrho_0 r_0^2 \, dr_0 , \tag{16}$$

where E is the total energy of the flow field and is equal to the explosion energy. The first integral on the right-hand side is the internal energy per unit solid angle; the second term is the magnetic energy per unit splid angle; and the third term is the kinetic energy per unit solid angle.

On the basis of the assumption that most of the mass is concentrated near the shock front, the dependent Eulerian variable r is expanded in a Taylor series about the location of the shock front as

$$r(r_0, t) = R + \left(\frac{\partial r}{\partial r_0}\right)_s (r_0 - R) + \frac{1}{2}\left(\frac{\partial^2 r}{\partial r_0^2}\right)_s (r_0 - R)^2 + \cdots, \tag{17}$$

Where we retain terms only up to the order of $(r_0 - R)^2$. From the continuity equation (1) and strong shock condition (5) we have

$$\left(\frac{\partial r}{\partial r_0}\right)_s = \left(\frac{\varrho_0 r_0^2}{\varrho r^2}\right)_s = \frac{\gamma - 1}{\gamma + 1} . \tag{18}$$

Using the above relation and differentiating (17) with respect to t we obtain

$$r_s = R \tag{19}$$

$$\left(\frac{\partial r}{\partial t}\right)_s = u_s = \frac{2}{\gamma + 1}\dot{R} \tag{20}$$

and

$$\left(\frac{\partial^2 r}{\partial t^2}\right)_s = \frac{2(\gamma - 1)}{(\gamma + 1)(\gamma + 1 + 2q^2)} \times \times \left[\frac{2(q^2 + 2\gamma - 1)}{(\gamma - 1)}\ddot{R} + \left(\frac{\partial \ln \varrho_0}{\partial r_0}\right)_s \dot{R}^2 + q^2\left(\frac{\partial \ln \varrho_0}{\partial r_0}\right)_s \dot{R}^2 + + \frac{4\gamma + (\gamma + 1)q^2}{(\gamma + 1)}\frac{\dot{R}^2}{R}\right], \tag{21}$$

with

$$q^2 = \frac{h_s^2}{p_s} = \frac{(\gamma + 1)^3}{2(\gamma - 1)^2} M_h^{-2},$$

where $M_h = \sqrt{\varrho_0}\dot{R}^2/h_0^2$ is the Alfvén Mach number of the shock. The solution of Equation (21), which is performed by use of the conservation equations and boundary conditions, is somewhat lengthy and is given in the Appendix.

Since the mass is highly concentrated near the shock front, for any r different from R the corresponding value of r_0 may be taken as zero. Using this approximation, replacing the term $(\partial r/\partial t)^2$ by $(\partial r/\partial t)_s^2$ and using Equations (18)–(20), we finally obtain, from Equations (14)–(16), a second-order differential equation for the shock radius R in the form

$$\frac{E}{4\pi} = \frac{\varrho_0(R, \theta)\dot{R}^2}{3(\gamma + 1)} \times \times \left[\left(\frac{2}{\gamma - 1} + q^2\right)R^3 - \frac{2(2 - \gamma)q^2 RL(R, \theta)}{(\gamma + 1)h_0(R, \theta)} + \frac{2q^2 K(R, \theta)}{(\gamma + 1)h_0^2(R, \theta)} - - \frac{(2 - \gamma)(\gamma - 1)q^2 L^2(R, \theta)}{(\gamma + 1)^2 R h_0^2(R, \theta)}\right] + \frac{2\dot{R}^2}{(\gamma + 1)^2} J(R, \theta) + \frac{R}{3} \times \times \left[\frac{J(R, \theta)}{\gamma - 1} - \frac{(2 - \gamma)q^2 R\varrho_0(R, \theta)L(R, \theta)}{2h_0(R, \theta)} - - \frac{(2 - \gamma)(\gamma - 1)q^2 \varrho_0(R, \theta)L^2(R, \theta)}{2(\gamma + 1)R h_0^2(R, \theta)}\right]\left(\frac{\partial^2 r}{\partial t^2}\right)_s - - \frac{(\gamma + 1)(2 - \gamma)(\gamma - 1)q^2 R\varrho_0(R, \theta)L^2(R, \theta)}{48\dot{R}^2 h_0^2(R, \theta)}\left(\frac{\partial^2 r}{\partial t^2}\right)_s^2, \tag{22}$$

with

$$J(R, \theta) = \int_0^R \varrho_0 r_0^2 \, \mathrm{d}r_0, \qquad K(R, \theta) = \int_0^R h_0^2 r_0^2 \, \mathrm{d}r_0,$$

$$L(R, \theta) = \int_0^R h_0 r_0 \, \mathrm{d}r_0 \tag{23}$$

and Equation (21). In Equation (22) the angle θ is contained as a parameter. If we integrate Equation (22) for any given initial density distribution and magnetic field distribution, we obtain the propagation of shock front and the resultant flow field.

3. Point Explosion in a Spheroid

For astrophysical interest, such as in rotating stars, we take the initial density distribution as a spheroid in which the density decreases exponentially outwards from the centre. The stretching of the dipolar magnetic field by the radial flow of the plasma then produces an azimuthal magnetic field component and a relatively negligible radial component. For simplicity, we take the initial magnetic field distribution parallel to the initial density distribution. Thus, the initial density and magnetic field laws may be written as

$$\varrho(r_0, \theta) = \varrho_c \exp(-\xi) \tag{24}$$

and

$$h(r_0 \theta) = h_c \exp(-\xi), \tag{25}$$

with

$$\xi = r_0 \Theta \tag{26}$$

and

$$\Theta(\theta) = \left[1 + \frac{e^2 \cos^2 \theta}{1 - e^2}\right]^{1/2}, \tag{27}$$

where ϱ_c, h_c and e denote the central density, magnetic field intensity and the eccentricity of the spheroid, respectively. The eccentricity e is taken to be constant. To obtain the solution of the present model we use the generalized Laumbach and Probstein method developed in Section 2.

For the density and magnetic field distributions given by Equations (24) and (25), the function $J(R, \theta)$, $K(R, \theta)$ and $L(R, \theta)$ in Equation (22) can be evaluated; and therefore Equation (22), after a lengthy calculation, is reduced to

$$-g_1(\eta) \frac{\ddot{\eta}^2}{\dot{\eta}^2} + g_2(\eta)\ddot{\eta} + g_3(\eta)\dot{\eta}^2 = \frac{E}{4\pi \varrho_c} \Theta^5, \tag{28}$$

where

$$g_1(\eta) = \frac{(2-\gamma)(\gamma-1)q^2(q^2+2\gamma-1)^2\eta}{3(\gamma+1)(\gamma+1+2q^2)^2\,e^{-\eta}}\cdot[1-e^{-\eta}(1+\eta)]^2,$$

$$g_2(\eta) = \frac{4(q^2+2\gamma-1)\eta}{3(\gamma+1)(2q^2+\gamma+1)}(F_1-F_2),$$

$$g_3(\eta) = \frac{1}{3(\gamma+1)}\times$$

$$\times\left[\left(\frac{2}{\gamma-1}+q^2\right)e^{-\eta}\eta^3+\frac{q^2}{2(\gamma+1)\,e^{-\eta}}\times\right.$$

$$\times\{1-(2\eta^2+2\eta+1)\,e^{-2\eta}\}+\frac{12}{(\gamma+1)}\times$$

$$\times\left\{1-\left(\frac{\eta^2}{2}+\eta+1\right)e^{-\eta}\right\}-\frac{2(2-\gamma)q^2\eta}{(\gamma+1)}\times$$

$$\left.\times\{1-(1+\eta)\,e^{-\eta}\}-\frac{(2-\gamma)(\gamma-1)q^2}{(\gamma+1)^2\eta\,e^{-\eta}}\{1-(1+\eta)\,e^{-\eta}\}^2\right]+$$

$$+\frac{2(\gamma-1)\eta}{3(\gamma+1)(2q^2+\gamma+1)}\left[\frac{4\gamma+(\gamma+1)q^2}{(\gamma+1)\eta}-(1+q^2)\right]\times$$

$$\times(F_1-\tfrac{1}{2}F_2),$$

$$F_1 = \frac{2}{\gamma-1}\left[1-\left(\frac{\eta^2}{2}+\eta+1\right)e^{-\eta}\right]-\frac{(2-\gamma)}{2}q^2\eta\times$$

$$\times[1-(1+\eta)\,e^{-\eta}]-\frac{(2-\gamma)(\gamma-1)q^2}{2(\gamma+1)\eta\,e^{-\eta}}[1-(1+\eta)\,e^{-\eta}]^2,$$

$$F_2 = \frac{(2-\gamma)(\gamma-1)^2q^2}{4(\gamma+1+2q^2)\,e^{-\eta}}[1-(1+\eta)\,e^{-\eta}]^2\times$$

$$\times\left[\frac{4\gamma+(\gamma+1)q^2}{(\gamma+1)\eta}-(1+q^2)\right]$$

and

$$\eta = \xi(R,\theta) = R\Theta. \tag{29}$$

In the same way as in Sakashita (1971), we transform the time t to the reduced time t^* by

$$t^* = t\left[\frac{E}{4\pi\varrho_c}\Theta^5\right]^{1/2}. \tag{30}$$

With the aid of the above transformation, Equation (28) becomes

$$-g_1(\eta)\left(\frac{\eta''}{\eta'}\right)^2+g_2(\eta)\eta''+g_3(\eta)\eta'^2=1, \tag{31}$$

where the prime represents differentiation with respect to the reduced time t^*. If we take η as the independent variable instead of t^*, Equation (31) is reduced to

$$-g_1(\eta)\left(\frac{d\eta'}{d\eta}\right)^2 + g_2(\eta)\eta'\frac{d\eta'}{d\eta} + g_3(\eta)\eta'^2 = 1. \tag{32}$$

Numerical integration of this equation gives the reduced shock velocity η' as a function of the reduced shock location η. The initial condition can be taken from the gas-dynamic solution (Sakashita, 1971), as the magnetic field does not affect the shock velocity in the vicinity of the explosion point (Verma and Vishwakarma, 1979). The time development of the shock front can be obtained from the integral

$$t^* = \int_0^\eta \frac{d\eta}{\eta'}. \tag{33}$$

4. Results and Discussion

The solutions of Equations (32) and (33) are plotted with solid lines in Figures 1 and 2, where we have taken $\gamma = \frac{5}{3}$ and $M_h = 10$. Dashed lines correspond to gasdynamic shock velocity. It is shown that in the beginning the velocity of magnetogasdynamic shock decreases rapidly, similar to gasdynamic shock. Afterwards, it decreases rather slowly and tends to zero, which is in contrast to ordinary gasdynamics where it starts to increase and tends to infinity.

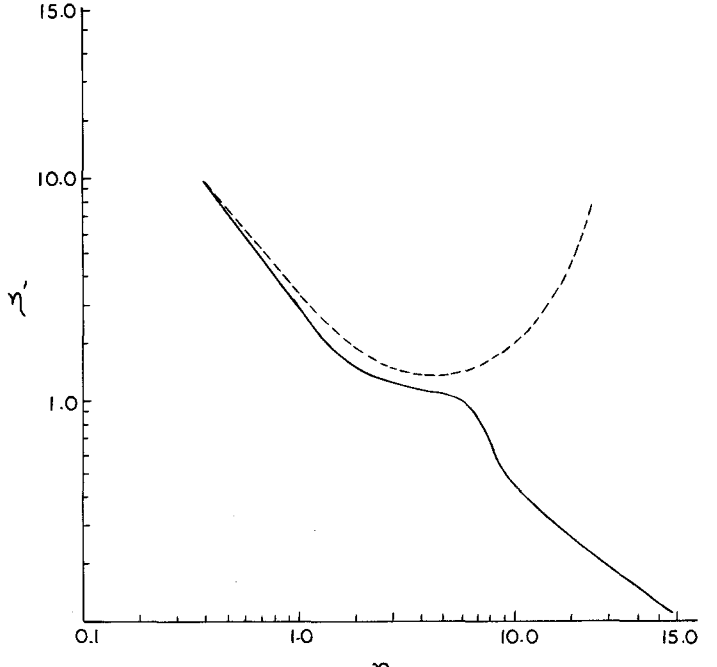

Fig. 1. Variation of reduced shock velocity with reduced shock radius. —— Magnetogasdynamic shock velocity; – – – gasdynamic shock velocity.

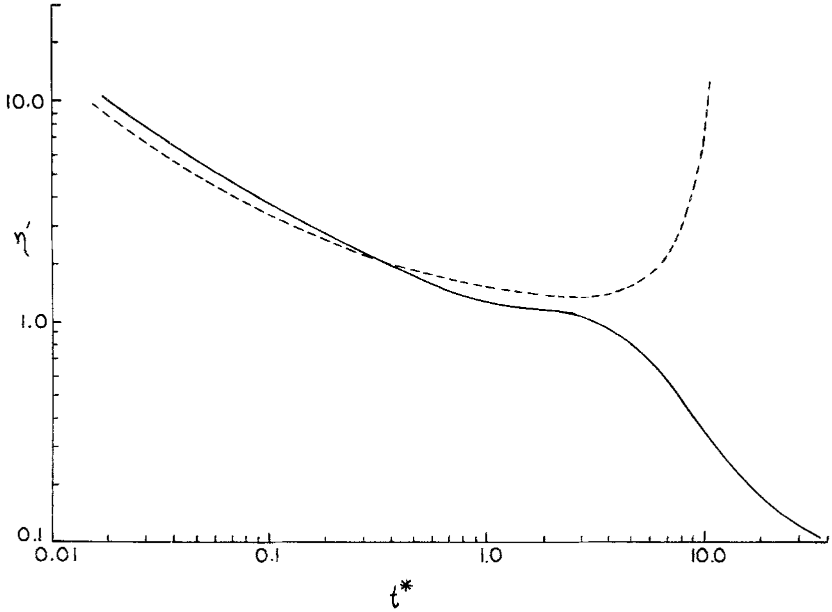

Fig. 2. Variation of reduced shock velocity with reduced time. —— Magnetogasdynamic shock velocity; - - - gasdynamic shock velocity.

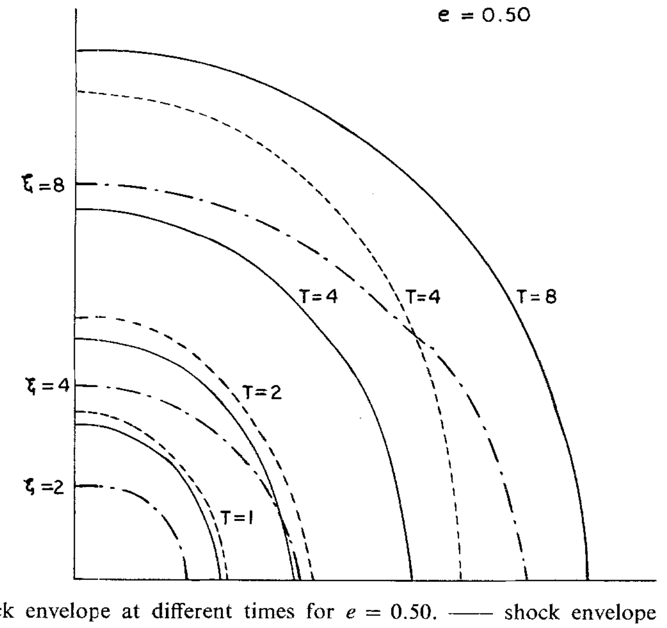

Fig. 3. Shock envelope at different times for $e = 0.50$. —— shock envelope in magnetogasdynamics; - - - shock envelope in ordinary gasdynamics; —·— equidensity surfaces of initial spheroid. $T = t(E/4\pi\varrho_c)^{1/2}$.

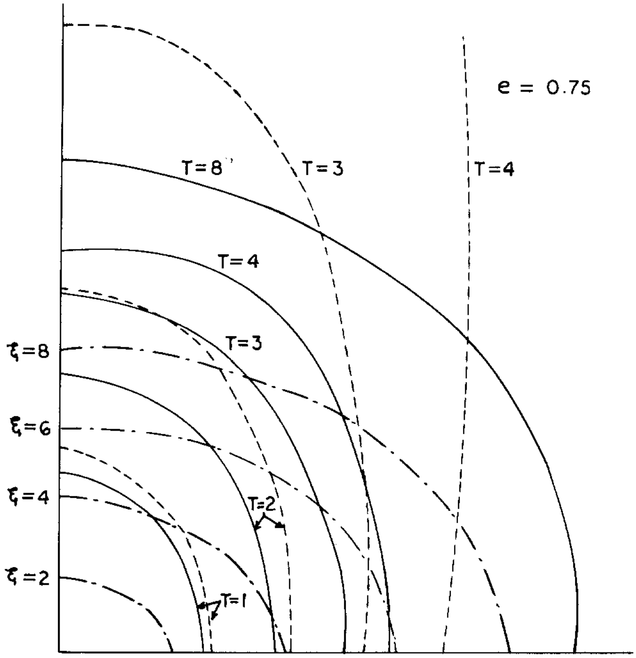

Fig. 4. Shock envelope at different times for $e = 0.75$.

In order to obtain the time development of the shock envelope, the reduce time t^* is deduced from Equation (30) for any given real time t and angle θ. The corresponding η can be obtained from the integration results shown in Figures 1 and 2, and finally η can be transformed to the real location R by Equation (29). Shock envelopes at various times in units of $(4\pi\varrho_c/E)^{1/2}$ are shown in Figures 3 and 4 for the two cases of eccentricity, $e = 0.50$ and $e = 0.75$. Solid and dashed lines represent the shock envelopes in magnetogasdynamics and ordinary gasdynamics, respectively.

It can be seen from Figures 3 and 4 that the shock propagation is greatly influenced by a magnetic field along the symmetry axis. The elongation of the shock envelope along the axis of symmetry is much less than in the gasdynamic case. On the other hand, along the equatorial plane (perpendicular to the axis of symmetry) the effects of the magnetic field appear to be less and the differences from the gasdynamic case are small. In the gasdynamic model of Sakashita (1971), a blow-out of the shock wave occurs along the axis of symmetry if the eccentricity of spheroid exceeds a certain critical value. An interesting feature of the present magnetogasdynamic model is that blow-out of the shock wave does not occur.

5. Summary and Conclusions

The above anomalous features of the present model are perhaps due to the term $h^2/\varrho r$ in the momentum equation and the magnetic field distribution adopted. For the density and magnetic field distributions (24) and (25), the total pressure (gasdynamic pressure + magnetic pressure) inside the shock envelope decreases faster than the total external pressure, and the shock velocity decreases and tends to zero. Due to the decrease of shock velocity, elongation of the shock envelope along the axis of symmetry is small and blow-out of the shock wave does not occur.

A model may be constructed with a steeper density gradient and more suitable magnetic field distribution in which the total pressure inside the shock envelope may decrease more slowly than the total external pressure. Thus, the shock velocity, after reaching a minimum due to spherical damping, may increase and tend to infinity as in the gasdynamic case (Sakashita, 1971; Möllenhoff, 1976). Also, a blow-out of the shock wave may occur along the axis of symmetry.

In conclusion, in the present model the magnetic field has a damping effect on the propagation of a shock wave and its velocity tends to zero. Also, breaking of the shock envelope is removed.

Appendix

Using Equation (18) in Equation (17), and differentiating twice with respect to t, we obtain

$$\frac{\partial^2 r}{\partial t^2} = \frac{2}{\gamma + 1}\ddot{R} + \left(\frac{\partial^2 r}{\partial r_0^2}\right)_s \dot{R}^2 - \left(\frac{\partial^2 r}{\partial r_0^2}\right)_s (r_0 - R)\ddot{R}. \tag{A1}$$

From the energy Equation (4) and the strong shock condition (5).

$$\left(\frac{1}{p}\frac{\partial p}{\partial r_0}\right)_s = \left(\frac{1}{p_s}\frac{\partial p_s}{\partial r_0} + \frac{\gamma}{\varrho}\frac{\partial \varrho}{\partial r_0} - \frac{\gamma}{\varrho_0}\frac{\partial \varrho_0}{\partial r_0}\right)_s, \tag{A2}$$

while

$$\left(\frac{1}{p_s}\frac{\partial p_s}{\partial r_0}\right)_s = \left(\frac{1}{\varrho_0}\frac{\partial \varrho_0}{\partial r_0}\right)_s + 2\frac{\ddot{R}}{\dot{R}^2} \tag{A3}$$

is obtained from (6). From the continuity equation (1) it follows that

$$\left(\frac{1}{\varrho}\frac{\partial \varrho}{\partial r_0}\right)_s = \left(\frac{1}{\varrho_0}\frac{\partial \varrho_0}{\partial r_0}\right)_s + \left(\frac{4}{\gamma + 1}\right)\frac{1}{R} - \left(\frac{\gamma + 1}{\gamma - 1}\right)\left(\frac{\partial^2 r}{\partial r_0^2}\right)_s, \tag{A4}$$

where $(\partial r/\partial r_0)_s$ has been replaced by (18). Finally, from Equation (9) we have

$$\left(\frac{1}{h}\frac{\partial h}{\partial r_0}\right)_s = \left(\frac{1}{h_0}\frac{\partial h_0}{\partial r_0}\right)_s + \left(\frac{1}{\varrho}\frac{\partial \varrho}{\partial r_0}\right)_s - \left(\frac{1}{\varrho_0}\frac{\partial \varrho_0}{\partial r_0}\right)_s - \left(\frac{2}{\gamma + 1}\right)\frac{1}{R}, \tag{A5}$$

in which Equations (5), (7) and (18) have been used. Eliminating $\partial^2 r/\partial t^2$ between Equation (2) – the momentum equation evaluated at the shock – and Equation (A1), we have

$$-\left(\frac{\partial^2 r}{\partial r_0^2}\right)_s \dot{R}^2 = \left(\frac{1}{\varrho_0}\frac{\partial p}{\partial r_0}\right)_s + \left(\frac{h}{\varrho_0}\frac{\partial h}{\partial r_0}\right)_s + \left(\frac{h^2}{\varrho r}\right)_s + \frac{2}{\gamma + 1}\ddot{R}. \tag{A6}$$

Using (A2)–(A5) and conditions (5)–(7) in (A6), we determine the expression for $(\partial^2 r/\partial r_0^2)\,\dot{R}^2$. From (A1) the result given by (21) follows.

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