VACUUM FRIEDMANN COSMOLOGY BASED ON LYRA'S MANIFOLD

(Letter to the Editor)

A. BEESHAM

Department of Applied Mathematics, University of Cape Town, Rondebosch, South Africa

and

Department of Mathematics and Applied Mathematics, University of Durban-Westville, Durban, South Africa

(Received 5 June, 1986)

Abstract. Vacuum Friedmann–Robertson–Walker cosmological models are derived in Lyra's manifold and some properties of the solutions are discussed. In addition to the usual de Sitter universe, several new solutions are obtained.

1. Introduction

In 1917 Einstein introduced the cosmological constant into his field equations in order to obtain a static cosmological model (Lorenz-Petzold, 1984 and references given therein) since his equations without the cosmological constant admitted only nonstatic solutions. After the discovery of the redshift of the galaxies and its explanation as being due to the expansion of the Universe, Einstein regretted his introduction of the cosmological constant. Recently, there has been much interest in the cosmological term within the context of quantum field theories, quantum gravity, supergravity theories, Kaluza–Klein theories, and the inflationary-universe scenario (Banerjee and Banerjee, 1985 and references given therein). However, as Sen (1957) has pointed out, there is no theoretical justification for the cosmological term in general relativity based on a Riemannian manifold.

Lyra (1951) proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. Sen (1957) studied a static cosmological model in Lyra's manifold, and others who have considered models based on this modified geometry are Halford (1970), Bhamra (1974), Kalyanshetti and Waghmode (1982), and most recently Reddy and Innaiah (1985). In this paper we study the vacuum Friedmann models based on Lyra's manifold and show that a wider class of solutions is obtained compared to models based on Riemannian geometry.

2. Field Equations and Solutions

The field equations in normal gauge in Lyra's manifold (cf. Sen, 1957) are

$$4R_{ab} - 2Rg_{ab} + 6\varphi_a\varphi_b - 3\varphi_c\varphi^c g_{ab} = T_{ab}\,, \tag{1}$$

where as in Riemannian geometry R_{ab} is the Ricci tensor, R the Ricci scalar, g_{ab} the metric tensor, and T_{ab} the energy-momentum tensor. φ_a is a displacement vector. We use units in which $c = 8\pi G = 1$ and Latin indices are assumed to take values 0 to 3. As usually considered by all the authors who have studied Lyra's manifold and its cosmologies, we take φ_a to be the constant vector

$$\varphi_a = (\beta = \text{constant}, 0, 0, 0) . \tag{2}$$

We shall not restrict β to be real since some authors consider ti to be real (for example, Halford, 1970) whilst others consider it to be purely imaginary (for example, Sen, 1957). The vacuum Friedmann-type equation is, from the Robertson–Walker metric and (1),

$$4H^2 + 4k/R^2 - 3\beta^2 = 0 , \tag{3}$$

where $H = \dot{R}/R$ is the Hubble parameter, the overhead dot denoting a derivative with respect to time.

The solutions to (3) are:

(i) $k = 0$, $\beta^2 > 0$, β real:

$$R = m \exp(\beta t/2) , \tag{4}$$

where m = constant of integration.

(ii) $k = +1$, $\beta^2 > 0$, β real:

$$R = (2/\beta) \cosh(\beta t/2) , \tag{5}$$

where we have chosen the constant of integration to be zero so that $R(0) = 2/\beta$.

(iii) $k = -1$, $\beta^2 > 0$, β real:

$$R = (2/\beta) \sinh(\beta t/2) , \tag{6}$$

where we have chosen the constant of integratio to be zero so that $R(0) = 0$.

(iv) $k = -1$, $\beta^2 < 0$, β pure imaginary:

$$R = (2/\alpha) \sin(\alpha t/2) , \tag{7}$$

where we have set $\beta^2 = -\alpha^2$ and where, once again, we choose the constant of integration to be zero so that $R(0) = 0$.

(v) $k = -1$, $\beta^2 = 0$:

We obtain the usual Milne Universe.

3. Discussion

The solution (4) is identical to the usual de Sitter universe as we would expect. However, in addition, we also have the solutions (5), (6), and (7). Both (5) and (6) tend asymptotically to (4) for large t but the solution (7) represents an oscillating-type universe. Thus we have a wider class of solutions than in the corresponding general relativistic case with the cosmological constant. It is also interesting to note that the solutions (4) and (6) are formally identical to the corresponding Brans–Dicke vacuum Friedmann models with constant scalar field (Lorenz-Petzold, 1983). Our model (5) expands from a finite value of R at $t = 0$. In conclusion, the advantage of cosmological models based on Lyra's manifold rather than a Riemannian manifold is that the cosmological constant arises naturally from the geometry rather than being introduced in an arbitrary *ad hoc* fashion.

Acknowledgement

I am grateful to the Council for Scientific and Industrial Research of South Africa for financial assistance in the award of a grant.

References

Banerjee, A. and Banerjee, N.: 1985, *Astrophys. Space Sci.* **114**, 165.
Bhamra, K. S.: 1974, *Australian J. Phys.* **27**, 541.
Halford, W. D.: 1970, *Australian J. Phys.* **23**, 863.
Kalyanshetti, S. B. and Waghmode, B. B.: 1982, *Gen. Rel. Grav.* **14**, 823.
Lorenz-Petzold, D.: 1983, *Astrophys. Space Sci.* **96**, 451.
Lorenz-Petzold, D.: 1984, *Astrophys. Space Sci.* **100**, 461.
Lyra, G.: 1951, *Math. Z.* **54**, 52.
Reddy, D. R. K. and Innaiah, P.: 1985, *Astrophys. Space Sci.* **114**, 285.
Sen, D. K.: 1957, *Z. Phys.* **149**, 311.