GEOMETRICAL STRUCTURES FOR COSMOLOGICAL APPLICATIONS

M. I. WANAS

Department of Astronomy and Meteorology, Faculty of Science, Cairo University, Cairo, Egypt

(Received 9 May, 1986)

Abstract. A set of conditions for selecting geometrical structures appropriate for cosmological applications is suggested. These conditions are being applied to two geometrical structures constructed mainly for cosmological applications. The algebraic manipulation language REDUCE 2 has been used to carry out the relevant calculations. Without the help of a computer, the calculations involved are very tedious. The results obtained show that one of the two structures should be ruled out as a model for cosmological applications. The combination of the results of the present paper and those of a previous one support the procedure known as 'The Type Analysis'.

1. Introduction

One of the main purposes of theoretical cosmology is to build a mathematical model representing the actual universe. This model is expected to agree with accessible observations and predict the results of future observations. Several world models have been constructed since the formulation of the general theory of relativity. Some depend on that theory, while others depend on other field theories.

In 1965, Penzias and Wilson discovered the cosmic microwave background radiation, which confirmed that the Universe went through a hot phase. The model that has received the most support since then is 'The Hot Big-Bang' model. Although this model is a successful one, its main defect is the existence of a global time singularity. The model says nothing about the particle era (10^{-43}–10^{-3} s). Most of what happens in this era of the model is still conjectural. Some authors believe that the existence of space-time singularities indicates that the classical theory of gravitation is being broken-down, and that quantum gravitational effects will be important (Hawking, 1982). McCrea (1985) pointed out that modification of gravitation theory is almost certainly needed near the big-bang.

Several modifications of the gravitation theory have been carried out. Some follow a purely geometrical procedure. Others depend on a quantum approach, while a third class is based on the supersymmetry theory. The generalized field theory constructed by Mikhail and Wanas (1977), is one attempt to solve the problem geometrically. We believe that the results obtained by applying any geometrical field theory to problems in astronomy or cosmology depend not only on the structure of the theory, but also on the geometrical structure used in its applications. It is worth noticing that the geometry used in constructing the generalized field theory was that of the absolute parallelism (AP).

The aim of the present work is to suggest the conditions that will enable us to select

Astrophysics and Space Science **127** (1986) 21–25.

AP-spaces appropriate for cosmological applications. The calculations for selection of a geometrical structure in the present work is not a straight forward one and not at all easy. For this reason, the author has used the algebraic manipulation language 'REDUCE 2' to get the main results that conclude the work. In Section 2, widely accepted cosmological features are presented and translated into geometrical language, giving rise to the conditions necessary to select the appropriate geometrical structure. The application of these conditions to two different geometrical structures (AP- spaces) is carried out in Section 3. Carrying out the necessary calculations, we were able to show that one of the two structures is suitable for cosmological applications, while the other is to be rejected. The paper is concluded in Section 4.

2. Cosmological Features and the Selecting Conditions

The following cosmological features are to be taken into account, as basis, in constructing any cosmological model. Of course these are not the only features known from observations. We expect that a proper model of the Universe is liable to give rise to some more. Furthermore, we expect the model to predict new phenomena that may have not been observed before. The most commonly accepted features of the Universe are:

(i) Isotropy: on the largest scales, the actual Universe exhibits a very high degree of isotropy, which is favoured by the cosmic microwave background radiation (cf. Longair, 1982).

(ii) Homogeneity: the counts of galaxies and the linearity of Hubble's law support the spatial homogeneity of the Universe (cf. MacCallum, 1979).

(iii) Neutrality: there is a general agreement between cosmologists that matter in the Universe is electrically neutral. So, one does not expect any global electric (or magnetic) fields in the Universe.

(iv) Strength of the field: the gravitational field in the Universe is expected to be a giant field within a material distribution, especially in the very early stages of the Universe.

It is necessary to translate the above properties into geometrical language. This is done in order to get the conditions that will help in selecting geometrical structures approriate for cosmological applications. Firstly, to ensure homogeneity and isotropy, all mixed tensors of the second order P^{μ}_{ν} defined in the AP-space, should have the following two properties (cf. McCrea and Mikhail, 1956):

(a) Independent of spatial coordinates.

(b) $P^1_1 = P^2_2 = P^3_3$ and $P^{\mu}_{\nu} = 0$ for $\mu \neq \nu$.

The generalized field theory used is a theory for both gravity and electromagnetism. So, the above conditions (a) and (b) are not sufficient, and one has to impose some further conditions concerning electromagnetism. The geometrical elements representing the electromagnetic field and its strength are, respectively, $F_{\mu\nu}$, $Z_{\mu\nu}$. These two elements are second-order skew tensors defined in the AP-space. Thus, to ensure the neutrality of the Universe, the space selected should satisfy the condition

(c) $F_{\mu\nu} = 0$ and $Z_{\mu\nu} = 0$.

Finally, the geometrical elements which indicate the existence of matter in the model and the strength of the gravitational field are $T_{\mu\nu}$, Λ, respectively. The first is a second-order tensor, while the second is a scalar. Both are defined in the AP-space. To get a non-empty model showing a strong gravitational field, the AP-space should satisfy the condition:

(d) $T_{\mu\nu} \neq 0$ and $\Lambda \neq 0$.

Consequently, any AP-space appropriate for cosmological applications should satisfy the set of conditions (a), (b), (c), and (d).

3. AP-Spaces Used for Cosmological Applications

The two AP-spaces, constructed by Roberton (1932), are the most general spaces that have been found to satisfy the conditions (a) and (b). The structure of these two spaces is completely defined by the two tetrads listed in Table I (written in Cartesian like coordinates $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$). The two tetrads are used consecutively as input of the REDUCE program to carry out the calculations concerned.

TABLE I

Input of the program

Tetrad	λ_0^0	λ_0^α	λ_a^0	λ_a^α
I	1	0	0	$\frac{l^-}{4A}\,\delta_a^\alpha + \frac{k}{2A}\,x^\alpha x^a + \frac{k^{1/2}}{A}\,\varepsilon_{\alpha a\beta}x^\beta$
II	$\frac{l^-}{l^+}$	$\frac{k^{1/2}}{A}\,x^\alpha$	$4\,\frac{k^{1/2}}{l^+}\,x^a$	$\frac{l^+}{4A}\,\delta_a^\alpha - \frac{k}{2A}\,x^\alpha x^a$

where α, β, a each takes the values 1, 2, 3; A is an unknown function f time; $l^\pm = 4 \pm kr^2$; $r^2 = x^\alpha x^\alpha$; $k(= +1, 0, -1)$ is the curvature constant; and $\varepsilon_{\alpha a\beta}$ is skew-symmetric with respect to all indices with $\varepsilon_{123} = +1$.

Now to use any of these two tetrads in building up a cosmological model, the tetrad should satisfy the set of conditions (a), (b), (c), and (d). These tetrads already satisfy (a) and (b). Thus, one has to examine whether or not the other two conditions (c) and (d) are satisfied. In order to do so, it is necessary to calculate the tensors $F_{\mu\nu}$, $Z_{\mu\nu}$, $T_{\mu\nu}$, and Λ for both geometrical structures. This implies the calculations of the type of each of these spaces following the procedure proposed by Mikhail and Wanas (1981). This includes the calculations of the curvature tensor $R^\alpha_{\beta\gamma\delta}$ in addition to the previous tensors. This tensor indicates whether or not the space is flat. This type of calculation is very tedious without the help of a computer. The author has used the algebraic manipulation language 'REDUCE 2' to establish a program for carrying out these calculations (Wanas, 1985a). The computer used in solving the present problem was IBM 370/168 NUMAC at Newcastle, with 8 megabits main store, and the operating system MTS. The top level language version is REDUCE 2 (15 April, 1979 (MTS 8 August, 1980)). The implementation language is LISP.

The results obtained are summarized in Table II. To save space, the values of the non-vanishing components of the computed tensors are not listed, since these are not relevant in the present work. This table shows clearly that the two conditions (c) and (d) are satisfied for tetrad I only. Therefore, the geometrical structure II cannot be used for building a cosmological model, since it is only capable (as clear from its type) of representing a weak gravitational field in an empty space-time. The actual Universe is not empty, and the gravitational field is very strong, especially in its early stages of evolution. The application of the generalized field theory using tetrad I is to get the unknown function $A(t)$, and to select a value for the constant k. This work is being prepared for publication and will appear shortly.

TABLE II

Summary of the output of the program

Tetrad	$R^{\alpha}_{\beta\gamma\delta}$	$T_{\mu\nu}$	Λ	$F_{\mu\nu}$	$Z_{\mu\nu}$	Type
I	Non-zero	Non-zero	Non-zero	0	0	F0GIII
II	Non-zero	0	0	0	0	F0GI

4. Concluding Remarks

(i) A set of conditions for selecting geometrical structures suitable for constructing cosmological models has been suggested. Without using these conditions, one may use a wrong structure for such applications. This may lead to a wrong judgment of the field theory concerned.

(ii) The conditions (a) and (b) alone are not sufficient, since empty and flat spaces are homogeneous and isotropic. The actual Universe is neither empty nor flat.

(iii) The judgment on any geometrical field theory depends on the results of its applications. It is generally believed that these results depend on the structure of the theory only. As is clear from the present work, the geometrical structure used for application is an essential factor affecting the results obtained, and should be taken into consideration.

(iv) The type analysis has been proved to be a useful procedure (cf. Wanas, 1985b). It has been shown that this procedure can be used, before solving the field equations, to get some physical information about the geometrical structure used. Furthermore, in the present work, it is shown that the type analysis can be used for selecting geometrical structures appropriate for cosmological applications. Thus the type analysis is being confirmed for the second time.

(v) It is well known that in cases such as collapsed astronomical objects, and at the early stages of the Universe, strong gravitational fields exist. When dealing with such fields using general relativity, space-time singularities are bound to appear. There is speculation that the appearance of such singularities may be taken as indicative of the inadequacy of general relativity when describing strong gravitational fields. On the other hand, in the case of the generalized field theory, the situation is essentially different. The existence of a strong gravitational field is indicated by the cosmological function Λ being not equal to zero. This is satisfied for the selected model. We may speculate that the appearance of singularities indicates the inadequacy of geometrical structure, rather than general relativity, to describe strong gravitational fields.

Acknowledgement

The author wishes to thank the authorities of King's College, London University, for use of the computer facilities there.

References

Hawking, S. W.: 1982, in H. A. Brück, G. V. Coyne, and M. S. Longair (eds.), *Astrophysical Cosmology*, Pontifical Academy of Science, p. 563.

Longair, M. S.: 1982, in H. A. Brück, G. V. Coyne, and M. S. Longair (eds.), *Astrophysical Cosmology*, Pontifical Academy of Science, p. 583.

MacCallum, M. A. H.: 1979, in S. W. Hawking and W. Israel (eds.), *General Relativity; An Einstein Centenary Survey*, Cambridge University Press, Cambridge, p. 533.

McCrea, W. H.: 1985, *Rev. Mex. Astron. Astrophys.* **10**, 33.

McCrea, W. H. and Mikhail, F. I.: 1956, *Proc. Roy. Soc. London* **A235**, 11.

Mikhail, F. I. and Wanas, M. I.: 1977, *Proc. Roy. Soc. London* **A356**, 471.

Mikhail, F. I. and Wanas, M. I.: 1981, *Int. J. Theor. Phys.* **20**, 671.

Robertson, H. P.: 1932, *Ann. Math.* **33**, 496.

Wanas, M. I.: 1985a, *Bull. 10th Int. Cong. Statistics*, Computer Science, Social and Denographic Research, No. 2, p. 415.

Wanas, M. I.: 1985b, *Int. J. Theor. Phys.* **24**, 639.