OPEN AND CLOSED MAGNETOSPHERIC TAIL CONFIGURATIONS AND THEIR STABILITY

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Abstract. The asymptotic theory valid for magnetospheric tail configurations that vary only weakly in the antisolar direction is used to derive a number of explicit properties. The conditions under which the magnetopause converges to form a closed magnetosphere or diverges (open magnetosphere) are identified and discussed. It is shown that the presence of the high latitude low pressure tail lobes guarantees the open solution. The large value of the Mach-number of the unperturbed solar wind is the reason for the slow variation of the plasma and field quantities along the tail. Criteria for (two-dimensional) stability are discussed and it is shown that they can be expressed in terms of simple topological properties of the equilibria. Closed magnetospheres turn out to be stable, open magnetospheres with sufficiently stretched field lines are subject to an instability which – as shown earlier – may be the cause of magnetospheric substorms.

1. Introduction

Although there is no doubt that the ultimate cause of the tail of the magnetosphere is the solar wind, the details of the interaction as well as the reasons even for the gross geometric properties of the tail are still not clearly understood.

As far as analytical theory is concerned the major obstacles are the great difficulties involved in a satisfactory solution of Vlasov's equations. Until not long ago, only one-dimensional equilibria were considered.

In recent years, techniques were developed that make it possible to discuss several properties – including stability – of two-dimensional equilibria.

In applying the results to the tail of the magnetosphere one still has to ignore the variation along one of the space coordinates. Since the z_{SM}-dependence clearly dominates a choice has to be made between x_{SM} and y_{SM}. (For the definition of the solar magnetospheric coordinate system (SM) see Ness, 1965.) A general statement about which coordinate is preferable does not seem to be possible. An evaluation of the steady-cross-tail electric field requires taking the y_{SM}-dependence into account (Cowley, 1973; Bornatici and Schindler, 1974). On the other hand, for a discussion of the balance of forces in the quiet tail, limitation to the x_{SM}-z_{SM}-plane seems to be a reasonable approximation (Cole and Schindler, 1972).

In several papers (Schindler, 1972; Soop and Schindler, 1973; and Schindler, 1974) the latter line was followed and a systematic theory of equilibrium and stability was developed in selfconsistent form. It is the aim of the present analysis to use that theory to derive properties that are more explicit than the earlier results.

The pressure tensor that one uses in this kind of approach is static and isotropic in

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the x_{SM}-z_{SM}-plane. Although there is reasonable experimental justification for this choice (Hones *et al.*, 1973) one is faced with the difficulty that the solutions obtained are not unique, in that the plasma distribution with respect to the field lines can be prescribed arbitrarily. Since the actual plasma profiles are not known in detail we have to make sure that the properties discussed are sufficiently general so that one can hope that they apply to the actual cases too. To a certain extent this demand is met by the use of an expansion technique assuming that the x_{SM}-dependence of the magnetic field and of the plasma distribution functions is small (Schindler, 1974).

The theory is strictly selfconsistent in the sense that the momentum balance between particles and fields is satisfied for any volume element in the framework of Vlasov's theory. Therefore our method differs from the guiding-center approach (e.g. Bird and Beard, 1972) in that highly inflated field configurations with non-gyroscopic ions as expected to prevail just before the onset of a magnetospheric substorm (Schindler, 1974) are also taken into account. The theory provides solutions that are open-ended and other solutions in which all field lines are closed. (The terms open and closed magnetosphere are exclusively used in this manner.) For simplicity, the normal magnetic field component at the magnetopause is neglected. It is possible in principle to include a finite normal component, however as long as it is small compared with the tangential component it is not likely to change the present results significantly.

In the present picture the tail is kept in its stretched out form simply by the action of pressure gradients and magnetic field stresses. An appreciable amount of viscosity is not required. At least for quiet times this is consistent with the findings of Siscoe (1972) who concluded that the quiet tail may not be dominated by viscous forces. The concept of viscosity in the sense introduced by Axford and Hines (1961) may be applicable during substorms, if a considerable exchange of plasma between the plasma sheet and the magnetosheath takes place.

It seems worthwhile noting that – although we ignore steady flow – our model is not in conflict with the theory of convection. Actually, a fully quantitative convection theory would require information about the magnetic field configuration (Vasyliunas, 1970, 1972). As long as the convection speed is small compared with the characteristic magnetohydrodynamic velocity the first order convection theory involves the magnetic field only to zeroth order in the convection speed.

The analysis of the following sections is aimed at the understanding of the major geometric properties of the tail and at the influence they have on its stability.

2. The Unperturbed State

It has been shown (Schindler, 1972) that the problem of finding two-dimensional (say x- and z-dependent) isotropic equilibrium configurations reduces to the solution of Maxwell's equations. Assuming quasi-neutrality one obtains the equation

$$\Delta A + \mu_0 j(A) = 0, \tag{1}$$

where A is the y-component of the vector potential which is constant on the magnetic field lines, and j is the electric current density depending on A only; j is related to the isotropic pressure $p(A)$ by

$$j(A) = \frac{\mathrm{d}p}{\mathrm{d}A}. \tag{2}$$

Assuming that the solution of (1) varies with x only slowly, i.e. the ratio of characteristic lengths $\varepsilon = L_z/L_x \ll 1$, one may use an asymptotic expansion which gives after some integration

$$a(x) - z = \int_{A(x,z)}^{A_b} \frac{\mathrm{d}A}{\sqrt{2\mu_0(p_0 - p(A))}}, \tag{3}$$

where

$$a(x) = \int_{A_0(x)}^{A_b} \frac{\mathrm{d}A}{\sqrt{2\mu_0(p_0 - p(A))}} \tag{4}$$

is the position of the magnetopause assumed to be a field line $(A = A_b)$; $A_0 = A(x, 0)$ is the vector potential on the x-axis and $p_0 = p(A_0)$ the total pressure.

Equation (3) is valid to second order in ε.

If $p(A)$ is given, one can find the solution $A(x, z)$ by evaluating (3). We shall do that for an explicit case in Section 3. Here we shall discuss some general properties of the solution (3) with (4).

Since the function $a(x)$ depends on x only via A_0, one can write $a = a(p_0)$. Assuming that $p(A)$ is positive and monotonically decreasing with $\mathrm{d}p/\mathrm{d}A \neq 0$ everywhere we can rewrite (4) as

$$a(p_0) = \frac{1}{\sqrt{2\mu_0}} \int_{p_b}^{p_0} \left(-\frac{1}{\mathrm{d}p/\mathrm{d}A}\right) \frac{\mathrm{d}p}{\sqrt{p_0 - p}}, \tag{5}$$

where p_b is $p(A_b)$. As follows from the subsequent discussion the function $a(p_0)$ governs important topological features of the equilibria as well as their stability. Therefore we call $a(p_0)$ the 'characteristic function' associated with the pressure profile $p(A)$.

It is obvious that (5) has a real solution only for $p_0 \geqslant p_b$.

In the limit $p_0 \to p_b$ one finds that

$$a(p_0) \sim \frac{1}{\sqrt{2\mu_0}\left(-\dfrac{\mathrm{d}p}{\mathrm{d}A}\right)_{A=A_b}} \int_{p_b}^{p_0} \frac{\mathrm{d}p}{\sqrt{p_0 - p}} = \mathrm{const}\sqrt{p_0 - p_b}. \tag{6}$$

Because of our assumptions the constant is positive and finite. Therefore $a(p_0)$ starts out from 0 at $p_0=p_b$ and increases with growing p_0.

Now we shall study the other limit $p_0/p_b \to \infty$. First, let us assume that $p(A)$ varies proportional to $A^{-\gamma}$ with an exponent $\gamma>0$. Then we find

$$-\left(\frac{\mathrm{d}p}{\mathrm{d}A}\right)^{-1} \propto \frac{1}{\gamma} p^{-1/\gamma-1}.$$

We introduce a new variable s: $=p/p_0$ and split (5) into two integrals, one from p_b/p_0 to s_1 and one from s_1 to 1 with s_1 being a constant small compared to 1. Then we find

$$a(p_0) \propto \frac{1}{\sqrt{p_0}}\left\{\frac{1}{\gamma} p_0{}^{-1/\gamma} \int\limits_{p_b/p_0}^{s_1} s^{-1/\gamma-1} \frac{\mathrm{d}s}{\sqrt{1-s}} + \frac{1}{\gamma} p_0{}^{-1/\gamma} \int\limits_{s_1}^{1} s^{-1/\gamma-1} \frac{\mathrm{d}s}{\sqrt{1-s}}\right\}.$$

The second integral gives a constant value. In the first integrand we approximate $\sqrt{1-s}\approx 1$ because of $s_1 \ll 1$. Thus in the limit $p_b/p_0 \to 0$ we obtain

$$a(p_0) \propto \frac{1}{\sqrt{p_0}}\left\{p_0{}^{-1/\gamma}\left(\frac{p_b}{p_0}\right)^{-1/\gamma} + \text{const}\, p_0^{-1/\gamma}\right\}.$$

For large values of p_0/p_b the second term becomes small compared to the first one; hence,

$$a(p_0) \propto \frac{1}{\sqrt{p_0}} \quad \text{for} \quad p_0/p_b \to \infty. \tag{7}$$

From (6) and (7) we can derive the shape of the curve $a(p_0)$; an example is given in Figure 1. We can distinguish two regions:

(a) $p_0 \gtrsim p_b$, $a(p_0)$ increasing with growing p_0,

(b) $p_0 \gg p_b$, $a(p_0)$ decreasing with growing p_0.

Regions (a) and (b) are separated by an intermediate region with a maximum value of a at some point.

The variation of p_0, the pressure on the x-axis, which for small ε is practically equal to the total pressure, determines the shape of the magnetopause $a(x)$. If one assumes that p_0 decreases monotonically with increasing x two typical forms of the magnetopause can be distinguished:

if p_0 is not very different from p_b over the whole range of x, $a(x)$ is determined by region (a), which leads to a converging magnetopause ('closed magnetosphere', Figure 2a)

if p_0 is much larger than p_b over the range of x, a is determined by region (b), which leads to a diverging magnetopause ('open magnetosphere', Figure 2b).

The stability of the two cases will be discussed in Section 4.

In the derivation of (7) we have used that p varies with a negative power of A. If one assumes that for large values of p, p varies with some positive power α of $-A$, a similar consideration shows that $a(p_0)$ always decreases monotonically for $p_0/p_b \to \infty$ if $\alpha > 2$. As another example, an exponential dependence of p on A is considered in the following section, where an asymptotic variation similar to (7) is derived (see (11)).

3. An Explicit Example

Now we shall apply the results of the preceding section to a specific case. To get quantitative results we must specify the function $p(A)$ and the boundary condition at the magnetopause. Since $p(A)$ is related to the plasma population on each field line, $p(A)$ is determined by the entry and loss processes. To bypass these extremely difficult problems we simulate the actual plasma distribution by an *ad hoc* assumption about the shape of $p(A)$. Since observations suggest a monotonic decrease of plasma pressure away from the neutral sheet, a monotonic choice of $p(A)$ seems reasonable. We assume that

$$p(A) = p_b e^{-2A/A_c}, \tag{8}$$

where A_c is some characteristic value for A. The value of A at the boundary is set to zero. Then we get

$$-\frac{\mathrm{d}p}{\mathrm{d}A} = \frac{2}{A_c} p$$

and can integrate (5), which leads to

$$a(p_0) = \frac{A_c}{\sqrt{2\mu_0}} \frac{\operatorname{Arch} \sqrt{p_0/p_b}}{\sqrt{p_0}}, \tag{9}$$

which is the characteristic curve used in Figure 1.

For $p_0/p_b \to 1$ we obtain

$$a(p_0) \sim \frac{A_c}{\sqrt{2\mu_0 p_b}} \sqrt{\frac{p_0}{p_b} - 1},$$

which agrees with (6), and for $p_0/p_b \to \infty$

$$a(p_0) \sim \frac{A_c}{\sqrt{2\mu_0}} \frac{\log 2\sqrt{p_0/p_b}}{\sqrt{p_0}}. \tag{10}$$

Since for $p_0 \gg p_b$ the logarithmic variation of the numerator may be ignored in a first approximation, we note that (10) is similar to (7), i.e. the corresponding relation for the power law case discussed in Section 2. So we can approximate (10) by

$$a(p_0) = \frac{A_c K}{\sqrt{2\mu_0 p_0}}, \tag{11}$$

with K being a characteristic value of $\log 2\sqrt{p_0/p_b}$.

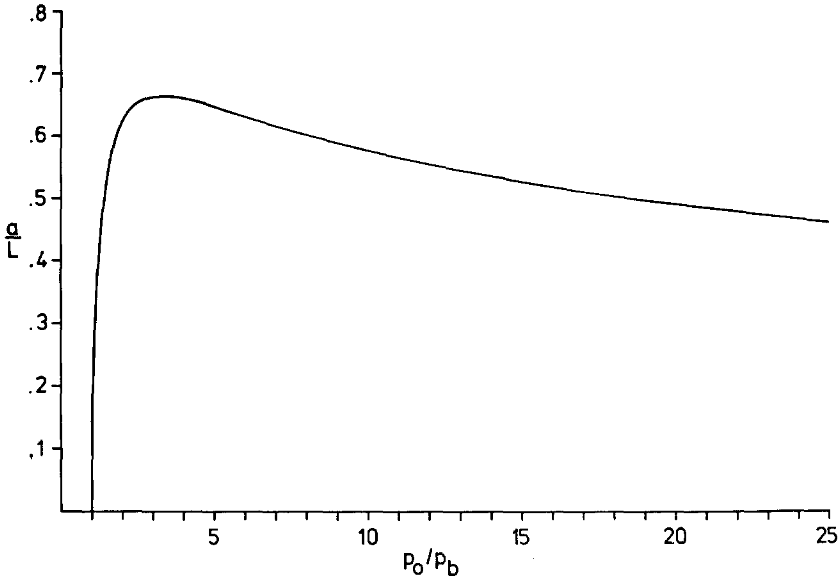


Fig. 1. Characteristic function $a(p_0)$. The position of the magnetopause a (divided by a characteristic length $L=A_c\sqrt{2\mu_0 p_0}$) is plotted against the pressure p_0 on the x-axis (normalized by the pressure at the magnetopause).

From (3) we find

$$z = \frac{A_c}{\sqrt{2\mu_0}} \frac{\mathrm{Arch}\sqrt{p_0/p(A)}}{\sqrt{p_0}}. \tag{12}$$

If we solve (12) for p we get

$$p(x, z) = p_0 \cosh^{-2} \frac{\sqrt{2\mu_0 p_0}z}{A_c}. \tag{13}$$

Using (8) we obtain A and $B_x = -\partial A/\partial z$ as functions of x and z in the form

$$A = A_c \ln\cosh \frac{\sqrt{2\mu_0 p_0}z}{A_c} + A_0, \tag{14}$$

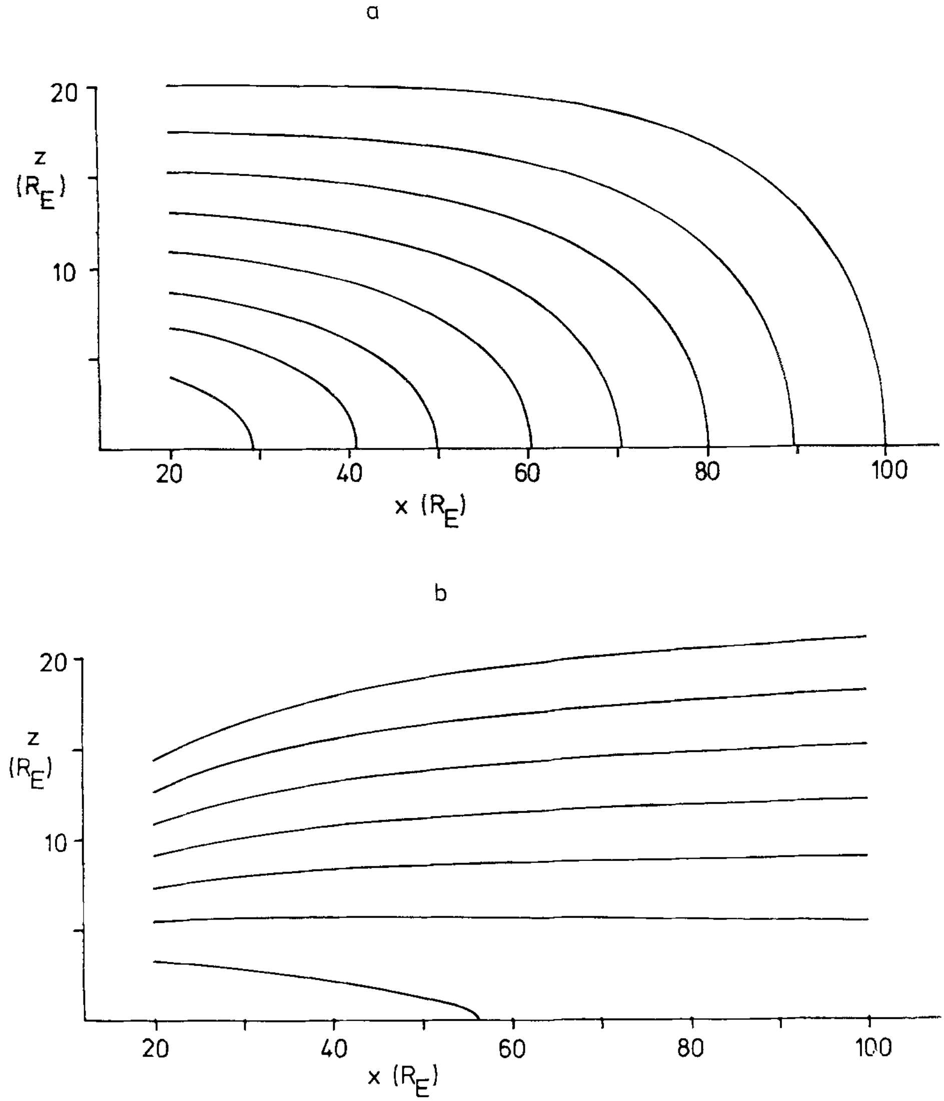
with

$$A_0 = A_c \ln\sqrt{p_b/p_0}$$

and

$$B_x = -\frac{\partial A}{\partial z} = -\sqrt{2\mu_0 p_0} \tanh \frac{\sqrt{2\mu_0 p_0}z}{A_c}. \tag{15}$$

If a and therefore p_0 are constant, the solutions (12)–(15) represent the well-known Harris sheet (Harris, 1962). In the general case, the magnetic field lines (A=const) follow from (12) explicitly if $p_0(x)$ is given. The examples plotted in Figure 2 correspond to a specific choice of $p_0(x)$ for each branch of the characteristic curve.



Figs. 2a–b. Examples of a closed (a) and an open (b) magnetosphere. The magnetic field lines are determined by means of arbitrary functions $p_0(x)$; case (a): $p_0/p_b = \exp\{1.225(1-(x/100)^2)\}$ and case (b): $p_0/p_b = 220(1+20/x)^2$ with x in units of the Earth radius R_E.

An alternative procedure is to prescribe $a(x)$ and to determine $p_0(x)$ from a given branch of the characteristic curve. This method is particularly useful if one formulates a boundary condition at the magnetopause. To give an example we take the simplified boundary condition used by Spreiter *et al.* (1966). It is based on the assumption that the pressure of the solar wind exerted on the boundary is given by the normal component of the momentum flux of the undisturbed incident stream $p_d \sin^2 \Psi$, where Ψ represents the angle between the direction of the incident stream and the tangent to the boundary, plus an additional constant p_s denoting the pressure of the solar wind gas. p_d is given by $\lambda\varrho_0 v_0^2$, where v_0 and ϱ_0 represent the velocity and density of the solar wind and λ is a constant lying between 1 or 2 depending on whether 'inelastic' or 'elastic' reflection of the solar particles at the boundary is assumed. Using the Mach number $M = v_0/v_{\text{sound}}$ of the unperturbed solar wind we can write the pressure balance in the form (γ is the ratio of the specific heats)

$$p_0 = p_s + \lambda\gamma M^2 \sin^2 \Psi, \quad \Psi > 0. \tag{16}$$

Clearly (16) is reasonable for $\Psi > 0$ only. Regions with $\Psi < 0$ would require a more realistic evaluation of the flow field. Our assumption that ε is small implies small values of Ψ such that we may replace $\sin \Psi$ by $\tan \Psi$ which is equal to $\mathrm{d}a/\mathrm{d}x$. We obtain

$$p_0 = p_s\left[1 + \lambda\gamma M^2\left(\frac{\mathrm{d}a}{\mathrm{d}x}\right)^2\right], \quad \frac{\mathrm{d}a}{\mathrm{d}x} > 0. \tag{17}$$

If we use the observational fact that the local pressure just inside the boundary is nearly equal to the magnetic pressure so that $p_0 \gg p_b$, we can express p_0 by means of (11) and write

$$\left(\frac{\mathrm{d}a}{\mathrm{d}x}\right)^2 = \frac{1}{\lambda\gamma M^2}\left(\frac{a_c^2}{a^2} - 1\right), \tag{18}$$

where a_c is a characteristic value of $a(x)$, assumed at, say, $x = x_0$.

The solution of (18) is an ellipse given by

$$\frac{a^2}{a_c^2} + \frac{(x - x_0)^2}{b_c^2} = 1, \tag{19}$$

with $b_c^2 = \lambda\gamma M^2 a_c^2$.

Because λ and γ are of order unity the ratio of the two semi-axes b_c/a_c is about M, which is large compared to unity. Thus, the reason for the observed weak x-dependence is the fact that the Mach number of the solar wind is large. The flare angle is by order of magnitude, equal to the Mach angle associated with the free solar wind.

Since $p_0 \gg p_b$, which leads to an open magnetosphere, we must take only that part of the ellipse which grows with increasing distance from the Earth (the closing part corresponds to $\mathrm{d}a/\mathrm{d}x < 0$, which had to be excluded). Of course we must also exclude the region where $\mathrm{d}a/\mathrm{d}x \gg \varepsilon$. That our solution is applicable only within a finite distance from the Earth is due to the fact that the approximation (16) becomes more and more invalid with increasing distance as it is shown by Spreiter *et al.* (1966).

From (19) we can gain $p_0(x)$ by means of (11) and with that the magnetic field lines which are plotted in Figure 3.

Figure 4 shows the variation of the magnetic field strength $B \simeq |B_x|$ at the magnetopause and of B_z along the x-axis, where $B_z = \partial A/\partial x$ is derived from (14) as

$$B_z = -\frac{A_c}{2p_0}\left(1 - \frac{\sqrt{2\mu_0 p_0}\,z}{A_c} \tanh \frac{\sqrt{2\mu_0 p_0}\,z}{A_c}\right)\frac{\mathrm{d}p_0}{\mathrm{d}x},$$

and $\mathrm{d}p_0/\mathrm{d}x$ is determined from (11) and (19).

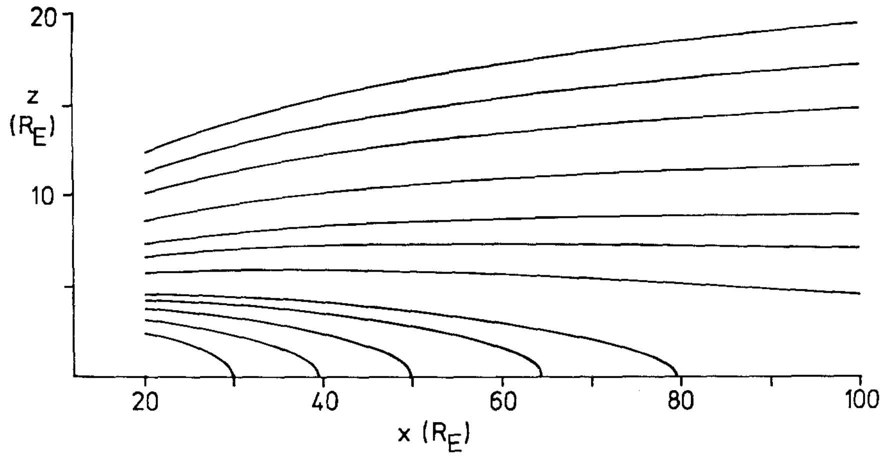


Fig. 3. Magnetic field lines for the example of a magnetosphere given in Section 3. $p_0(x)$ is determined from $a(x)$ by means of (11) with $a_c = 25\ R_E$ and $b_c = 200\ R_E$ (i.e. $M\sqrt{\lambda y} = 8$).

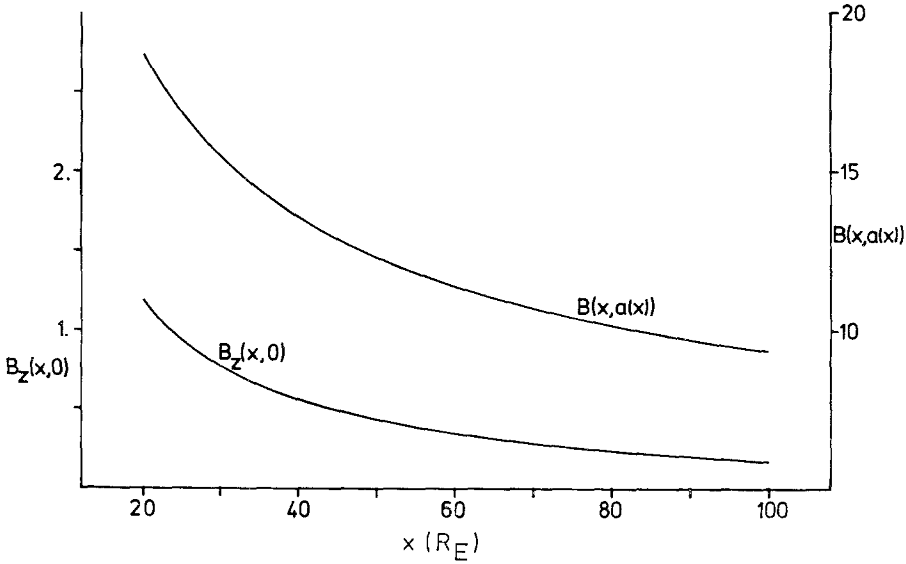


Fig. 4. Variation of the magnetic field $B(x, a)$ at the boundary and the z-component $B_z(x, 0)$ on the x-axis, normalized by $\sqrt{2\mu_0 p_b}$, for the example of Section 3.

4. Stability

Several properties concerning the stability of two-dimensional equilibria have already been discussed. Schindler (1972, 1974) showed that in the framework of two-dimensional stability theory ($\partial/\partial y = 0$) the following criterion holds. Consider the function $w(x)$ defined by

$$-B_z(x,0)B_x'(x,0)\int_{w(x)}^{a(x)}\frac{dz}{B_x(x,z)^2}=\frac{da(x)}{dx}, \tag{20}$$

where the prime denotes differentiation with respect to z.

An equilibrium for which $w(x)>a(x)$ holds for all x is stable; $w(x)<a(x)$ for all x with ε sufficiently small implies instability. The function $w(x)$ has a simple physical interpretation, as being the local magnetopause position that would marginally stabilize the plasma sheet, if we treated the plasma sheet cross section for any value of x as a one-dimensional sheet. The purpose of this section is to discuss the stability of the equilibria (2), (3) in the framework of that theory and to relate the stability properties to the characteristic function $a(p_0)$.

First we show that the function $w(x)$ that governs stability can be related to the characteristic function in a simple manner. We write

$$\frac{da(x)}{dx}=\frac{da}{dp_0}\frac{dp_0}{dA_0}\frac{dA_0}{dx}=\frac{da}{dp_0}\left[\frac{1}{\mu_0}B_x'(x,0)\right]B_z(x,0), \tag{21}$$

so that (20) takes the form

$$-\mu_0\int_w^a\frac{dz}{B_x^2}=\frac{da}{dp_0}. \tag{22}$$

It follows immediately that magnetosphere solutions that lie entirely on the closed branch ($da/dp_0>0$) are stable, because in that case (22) implies $w(x)>a(x)$ for all x. Purely open magnetospheres on the other hand have $da/dp_0<0$ and therefore can be unstable for sufficiently small ε in accordance with the earlier result of Schindler (1974).

This stability criterion may be understood also in terms of the variation of the B_z component across the plasma sheet. For simplicity we confine the discussion to the case where the current density is unidirectional ($j<0$) and to configurations that are symmetric with respect to the plane $z=0$ with B_x vanishing at the plane of symmetry only. We can then confine the discussion to the region $z\geqslant 0$ and assume $B_x<0$. Obviously, for the magnetosphere this assumption is not stringent. Starting from (3) one finds after partial integration

$$a-z=\sqrt{\frac{2}{\mu_0}}\left\{\frac{\sqrt{p_0-p}}{j}-\frac{\sqrt{p_0-p_b}}{j_b}-\int_{p_b}^{p}\frac{d}{d\xi}\left(\frac{1}{j(\xi)}\right)\sqrt{p_0-\xi}\,d\xi\right\}, \tag{23}$$

where j is understood to be a function of p, as obtained by eliminating A from $j(A)$ and $p(A)$. Differentiating (23) with respect to x at constant z and integrating by parts we find that

$$\frac{B_z}{B_x \dfrac{\mathrm{d}p_0}{\mathrm{d}x}} = \frac{\mathrm{d}a}{\mathrm{d}p_0} + \frac{1}{2\sqrt{2\mu_0}} \int\limits_A^{A_b} \frac{\mathrm{d}A}{(p - p(A))^{3/2}}, \tag{24}$$

where we have used $B_x = -\sqrt{2\mu_0[p_0 - p(A)]}$.

With the aid of (20) we finally obtain

$$\frac{B_z}{B_x \dfrac{\mathrm{d}p_0}{\mathrm{d}x}} = \mu_0 \int\limits_z^w \frac{\mathrm{d}z}{B_x^2}. \tag{25}$$

This equation immediately yields the following properties:

(a) If there is a point (x, z) where $B_z = 0$ its z-coordinate coincides with $w(x)$.

(b) A configuration with $B_z > 0$ and $\mathrm{d}p_0/\mathrm{d}x < 0$ is stable.

Property (b) tells us that the occurrence of a point where B_z vanishes is necessary for an instability to develop.

The latter criterion is a special case of a more general sufficient stability criterion for two-dimensional equilibria, which are not restricted to small values of ε. This criterion can be stated as follows*:

If there is a non-vanishing cartesian component of the equilibrium magnetic field $\mathbf{B}_0$, say B_{z0}, the configuration is stable. To prove that statement we note that stability is guaranteed if the functional (Schindler *et al.*, 1973)

$$Z = \int \left[(\nabla A_1)^2 - \mu_0 \frac{\mathrm{d}j_0}{\mathrm{d}A_0} A_1^2 \right] \mathrm{d}x\, \mathrm{d}z \tag{26}$$

is positive for all functions A_1 for which the operations involved are defined, and which vanish at the boundaries. We express $\mathrm{d}j_0/\mathrm{d}A_0$ in terms of B_{z0} by differentiating (1) with respect to x,

$$\mu_0 \frac{\mathrm{d}j_0}{\mathrm{d}A_0} = -\frac{\Delta B_{z0}}{B_{z0}}; \tag{27}$$

which allows writing (26) in the form

$$Z = \int \left[(\nabla A_1)^2 + \frac{\Delta B_{z0}}{B_0} A_1^2 \right] \mathrm{d}x\, \mathrm{d}z. \tag{28}$$

Substituting $A_1 = \Psi B_{z0}$ one finds

$$Z = \int [\Psi^2 (\nabla B_{z0})^2 + 2\Psi B_{z0} \nabla\Psi \cdot \nabla B_{z0} + \\ + B_{z0}^2 (\nabla\Psi)^2 + \Delta B_{z0} B_{z0} \Psi^2]\, \mathrm{d}x\, \mathrm{d}z.$$

* A preliminary account of this criterion has already been given by Schindler (1970).

Since $B_{z0} \neq 0$, Ψ is continuous and the last term can be integrated by parts. Several terms cancel out and one finds that

$$Z = \int \Psi^2 (\nabla B_{z0})^2 \, dx \, dz > 0, \tag{29}$$

which proves our claim.

Thus, the closed magnetosphere solutions (Figure 2a) corresponding to positive slope of the characteristic function are stable. B_z does not change its sign. The open magnetospheres violate that criterion and may therefore be subject to an instability. The conditions for which the stability actually occurs are such that the instability can explain the onset of magnetospheric substorms (Schindler, 1974).

5. Summary and Discussion

We have discussed topological properties and stability of equilibria that may represent instantaneous states of the tail of the magnetosphere.

The approach uses the asymptotic theory for slow variation along the tail and contains two functions of primary importance, the plasma pressure profile $p(A)$ and the characteristic function $a(p_0)$. The function $p(A)$ specifies the way in which the plasma pressure is distributed on the field lines. A theory of $p(A)$ would imply a detailed discussion of plasma gain and loss in the magnetosphere which is out of the reach of present magnetospheric theory. However, since $p(A)$ can be chosen largely* arbitrarily it is easy to choose pressure profiles that are consistent with the observations. The most important features to be built in are the monotonic decrease of plasma pressure away from the plane of symmetry ($z=0$) such that there are low pressure tail lobes and the fact that the total pressure ($p_0 = p + B^2/2\mu_0$) decreases away from the Earth. Once the function $p(A)$ is fixed (e.g. using an exponential as in (8)) the characteristic function $a(p_0)$ can be determined. Here a and p_0 denote the magnetopause position and the pressure at $z=0$ evaluated at the same value of x.

Without having to specify the boundary condition of the magnetopause one can relate gross topological features of the solution to the shape of the characteristic curve. If the solution is confined to the region of positive slope ($da/dp_0 > 0$) the magnetosphere boundary converges away from the Earth (Figure 2a) while a ngeative slope region implies an open solution (Figure 2b).

The presence of the low pressure tail lobes automatically guarantees the open case. A closed magnetosphere (Johnson, 1960) is selfconsistently possible, however it would require that the plasma pressure of the boundary p_b is not too different from the plasma pressure on the axis ($z=0$). A closed magnetosphere cannot be reconciled with the lack of plasma pressure on the high latitude field lines as actually observed. Thus, the model explains the gross shape of the magnetosphere in terms of the internal spatial plasma distribution.

* The constraint that distribution functions must be positive (Schindler, 1972) is not stringent for most practical purposes.

So far, the boundary shape $a(x)$ did not need specification. In fact, the selfconsistent theory for the domain inside the magnetopause contains $a(x)$ as an arbitrary function. In an actual case, the boundary shape is of course determined by a boundary condition. A complete treatment would require a full gas dynamic solution of the solar wind in the magnetosheath, which exceeds our present (analytical) treatment. We expect that the dominant features would follow already from a simple Chapman-Ferraro type of boundary condition. In this case the magnetopause is represented by the (diverging part) of an ellipse (Figure 3). An important conclusion follows from the consideration of the ratio of the semi-axes, which is of the order of $1/M$, where M is the Mach number of the free solar wind. Since $M \gg 1$, this is the physical explanation for the fact that the variation of the tail along the x-direction is small. Incidentally, this fact justifies – a posteriori – using the asymptotic theory for small values of the parameter ε which we can now identify with $1/M$.

A stability analysis shows that purely closed cases are stable whereas the actual case (open, $\mathrm{d}a/\mathrm{d}p_0 < 0$) may be subject to an instability if ε is sufficiently small. One can understand that stability result in terms of the way in which B_z varies across the tail in the z-direction. An instability requires that B_z changes its sign. Obviously, that is the case in open magnetospheres, where $B_z > 0$ on the axis ($z = 0$) and $B_z < 0$ at the boundary (Figure 3). The instability is of the spontaneous merging type; it may be the cause of magnetospheric substorms (Schindler, 1974).

Some final remarks are directed at the various approximations made. The assumption that the particle distribution functions are assumed to be static and isotropic in the v_x, v_z-plane may not be too stringent for quiet times, as was already discussed in the introduction. The two-dimensionality may be a more severe limitation. Future work will show to what extent a variation along the y-direction changes the present conclusions. In any case, a quantitative modelling of the tail will require a 3-dimensional model. For instance, it seems that a quantitative model has to take into account the tail flaring in the y-direction and its influence on the radial dependence of the magnetic field. Therefore, the variation of $|B_x|$ and B_z as plotted in Figure 4 can have only qualitative significance.

The stability analysis treats the magnetopause as unperturbed. This is probably justified in view of the small growth time of the instability (Schindler, 1974); a final answer is, however, not yet available.

These deficiencies seem to be the price present magnetospheric theory has to pay for a fully selfconsistent treatment.

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