SPONTANEOUS MAGNETIC RECONNECTION MECHANISMS IN PLASMA

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Abstract. Using the kinetic theory and model collision integral of Bhatnagar–Gross–Krook we obtain the general dispersion relation for different regimes of the tearing-mode instability development in configuration with sheared magnetic field. Thus, we can construct the general picture of the applicability of different mechanisms of the tearing-mode dependent on collision frequency and value of the shear.

1. Introduction

The release of magnetic energy stored in a plasma configuration is usually associated with the magnetic field-line reconnection, and the rapid explosional start of this phenomenon with some spontaneous process. Then it is necessary to distinguish between the forced quasi-stationary reconnection, in the course of which the main magnetic energy dissipation and plasma heating occur, and the spontaneous reconnection which characterizes the rapid magnetic topology reconstruction and provides conditions for following effective magnetic energy dissipation.

In this paper we are treating the tearing instability as a mechanism of spontaneous reconnection process. Such an instability is used by investigators to explain a wide range of physical phenomena (magnetospheric substorms, solar flares, disruptive instabilities in tokamaks, etc.). A great number of theoretical works has been published where different physical mechanisms of magnetic field reconnection were investigated. That means that a region must appear in plasma in which magnetic field frozen-in condition breaks and longitudinal (i.e., directed along the magnetic field **B**) electric field $\tilde{E}_{\parallel}$ differs from zero. It is known that this can be connected with the following mechanisms: (1) Coulomb collisions (2) finite electron inertia, (3) collisionless Landau damping, (4) collective collisions due to the development of anomalous resistivity.

The last dissipation mechanism connected with plasma turbulence, concerns itself with several problems (particularly, for exciting of the instabilities in plasma, very narrow current layers are necessary) and that is why we confine ourselves by examining the first three ones. Accordingly, in theoretical works made in MHD as well as in kinetic approximations, different regimes of tearing instability development have been investigated: a resistive MHD regime of Furth–Killen–Rosenbluth (Furth *et al.*, 1963), MHD inertial regime (Coppi, 1965), kinetic regimes (Coppi *et al.*, 1966; Drake and Lee, 1977;

Galeev and Zeleny, 1977; Zeleny and Taktakishvili, 1981). But there still has not been the unified view on the development conditions of those different modes. In this paper, using the kinetic theory, we try to construct a general picture of the applicability of different mechanisms of the tearing instability. We believe this simple step to be necessary for the correct use of theoretical results in astrophysical and laboratory researches.

2. Plasma Configuration and General Equations

As the initial plasma and magnetic field configuration we take the so-called generalized self-consistent Harris distribution (Galeev, 1984):

$$\mathbf{B} = \mathbf{B}_z + \mathbf{B}_y = B_{0z}\,\mathrm{th}(x/L)\hat{\mathbf{e}}_z + b_y B_{0z}\hat{\mathbf{e}}_y\,,$$
$$f_{0j} = \frac{n(x)}{(m_j/2\pi T_j)^{-3/2}}\exp\left[-\frac{m_j}{2T_j}\{v_x^2 + (v_y - u_j)^2 + v_z^2\}\right], \tag{1}$$
$$n(x) = n_0\,\mathrm{ch}^{-2}(x/L)\,;$$

where L is a current density width, $b_y \equiv |\mathbf{B}_y|/B_{0z}$ and the main current is directed along the y-axis (magnetic field shear), m_j, T_j, and u_j are the mass, the temperature, and the current velocity of the particles of species j, respectively. The distribution function satisfies Vlasov equations for any $\mathbf{B}_y$ = const. and this proves possibility of applying an arbitrary magnetic field $\mathbf{B}_y$ on a self-consistent Harris distribution (Harris, 1962). It is more convenient to represent perturbations of electromagnetic values as Fourier components of vector and scalar potentials $\tilde{A}'_{\parallel}(x) = \tilde{A}_{\parallel}(x)\exp(-i\omega t + i\mathbf{kr})$ (sign $\parallel$ means the direction along magnetic field), $\varphi'(x) = \varphi(x)\exp(-i\omega t + i\mathbf{kr})$.

The plane model (Equation (1)), of course, has restricted applicability for coronal loops and tokamak plasma configurations where one must take into account cylindric and toroidal geometry, respectively. But, because in this case we are interested mainly in the physical mechanisms of magnetic energy dissipation and not in the particular reconnection geometry, the applicability of plane model (1) obviously has a sense. Taking into account the cylindricity gives appropriate numerical coefficients to the expression for instability growth rate (Furth *et al.*, 1963) and gives the dependence of growth rate on the azimuthal wave number. In the case of solar coronal loops, field $\mathbf{B}_y$ in Equation (1) corresponds to the longitudinal magnetic field $B_{\parallel}$ of the loop and to the toroidal field B_T in tokamak. And the sign-reversing field $\mathbf{B}_z$ corresponds to the azimuthal field B_θ connected with the longitudinal current $j_{\parallel}$ along the loop, in the solar case, and to the poloidal field B_p, in the case of tokamak.

If the condition $b_y < \varepsilon_j^{1/2} \equiv (\rho_j/L)^{1/2} \equiv (v_{T_j} m_i c/e_j B_{0z} L)^{1/2}$ is satisfied, then the particle rotation (Larmor) radius in the $\mathbf{B}_y$ field, is greater than the 'singular layer' (the region which gives the greatest contribution to the tearing instability (Dobrovolny, 1968)) width $\rho_{y_j} > d_j \equiv \varepsilon_j^{1/2} L$, for the neutral Harris sheet. Therefore, particles are not magnetized in the region $|x| < d_j$ (as in the case of neutral sheet), and we can neglect the influence of $\mathbf{B}_y$ field; thus, the expressions for growth rates obtained by Coppi *et al.* (1966) and Zeleny and Taktakishvili (1981) are correct. So in the further discussion we shall assume $b_y \gg e_j^{1/2}$. In addition, for validity of our assumption of magnetized particles, the condition $\nu_j < \Omega_{y_j}$ has to be satisfied too (here $\nu_j \equiv \Sigma_l \nu_{jl}$, ν_{jl} is the collision frequency between particles of j and l species, Ω_{y_j} is the frequency in the B_y field). In other words we must have: $b_y > \varepsilon_j \nu_j / k v_{T_j} (kK \lesssim 1)$. This unequality gives the upper limit for collision frequency that is valid in the frame of our theory.

The particles magnetized by $\mathbf{B}_y$ field move along the total magnetic field $\mathbf{B} = \mathbf{B}_y + \mathbf{B}_z$, drifting simultaneously normally to $\mathbf{B}$ because of inhomogeneous $\mathbf{B}_z = B_{0z}\,\mathrm{th}(x/L)\hat{\mathbf{e}}_z$. As the detailed analysis of the equations of particles motion shows this magnetic drift is negligible in the singular layer and, therefore, does not play any role in the development of the instability. We can use the drift approximation in the layers magnetized by $\mathbf{B}_y$ field (Galeev, 1984).

In the sheared current sheets, unlike 'pure neutral' sheets (see, e.g., Dobrovolny, 1968), the electrostatic component of the perturbed field can be rather important, i.e., the scalar potential φ comes to the play (Coppi, 1965). Also, nonlocal effects connected with the finite ion-Larmor radius ρ_{y_i} are essential under certain conditions (Drake and Lee, 1977; Galeev and Zeleny, 1977).

Using the model collision operator of Bhatnagar–Gross–Krook (Bhatnagar *et al.*, 1954) and integration over unperturbed trajectories we have tried to construct some general picture of the tearing instability regimes dependent on collision frequency ν_j and the value of magnetic shear b_y. We must mention that by Mahajan *et al.* (1979) authors using the so-called variational method got general dispersion relations for tearing modes in different regimes. Our method seems to us more simple and as one can see below gives the possibility to map all regimes on a 'collision frequency-shear value' diagram. The Maxwell–Vlasov system of equations for perturbed vector and scalar potentials takes the form

$$[\mathrm{d}^2/\mathrm{d}x^2 - (k^2 + V_0)]\tilde{A}_\parallel(x) = -(4\pi/c)\sum_j e_j \int f_j v_\parallel\, \mathrm{d}^3 v\,, \tag{2}$$

$$[\mathrm{d}^2/\mathrm{d}x^2 - (k^2 + M_0)]\varphi(x) = -4\pi \sum_j e_j \int f_j\, \mathrm{d}^3 v\,, \tag{3}$$

$$V_0 = -2L^{-2}\,\mathrm{ch}^{-2}(x/L)\,, \qquad M_0 = \mathrm{ch}^{-2}(x/L)\sum_j \tilde{d}_j^{-2}\,,$$

$$\tilde{d}_j^{-2} \equiv 4\pi e_j^2 n_0/T_j\,, \tag{4}$$

$$f_j = \frac{e_j f_{0j}}{cT_j}\left[\int_{-\infty}^{0} i(\omega \mathbf{v}\tilde{\mathbf{A}} - \omega_j c\varphi)\, e^{-i(\omega_j\tau - \mathbf{kr}(\tau))}\,\mathrm{d}\tau\right] +$$

$$+ \sum_l \nu_{jl} f_{0j} \int_{-\infty}^{0} \left(\frac{\tilde{n}_j}{n_0} + \frac{m_j}{T_j}\mathbf{v}\tilde{\mathbf{v}}\right) e^{-i(\omega_j\tau - \mathbf{kr}(\tau))}\,\mathrm{d}\tau\,, \tag{5}$$

$$f_{0j}^D = n(x)\,(m_j/2\pi T_j)^{3/2} \exp\left\{-\frac{m_j}{2T_j}\,[v_\parallel - u_j B_y/B)^2 + V_\perp^2\,]\right\}, \tag{6}$$

$$\tilde{n}_j \equiv \int f_j\,\mathrm{d}^3 v\,, \qquad \tilde{\mathbf{v}} \equiv \int \mathbf{v} f_j\,\mathrm{d}^3 v\,, \qquad \omega_j \equiv \omega + i\nu_j\,, \qquad \nu_j \equiv \sum_l \nu_{jl}\,;$$

where ω is the frequency of perturbations (for one-dimensional perturbations $k = k_z\hat{\mathbf{e}}_z \equiv k\hat{\mathbf{e}}_z$ which we investigate here, $\operatorname{Re}\omega = 0$; and, therefore, the growth rate $\gamma = \operatorname{Im}\omega = -i\omega$.

The total plasma volume can be divided formally into two parts: (1) The singular region, where the longitudinal component of the wave vector is small $k_\parallel(x) \equiv \mathbf{kB}/|\mathbf{B}| \simeq kx/Lb_y \simeq 0$. In this case the electrostatic field $-\nabla_\parallel\varphi(x) = -ik_\parallel\varphi(x)$ does not cancel the inductive one, $-c^{-1}\,\partial\tilde{A}_\parallel(x)/\partial t$, and the longitudinal electric field $\tilde{E}_\parallel = -c^{-1}\,\partial\tilde{A}_\parallel(x)/\partial t - \nabla_\parallel\varphi(x)$ differs from zero. Thus, the frozen-in condition breaks near the region where $k_\parallel \to 0$, i.e., near the $x = 0$ plane. (2) The outer region, where $\tilde{E}_\parallel \simeq 0$ and the ideal MHD approximation is applicable. The dispersion relation for tearing instability can be obtained by matching the solutions of the above equations for these two regions. At the vicinity of the singular surface $x = 0$, giving the greatest contribution in the perturbed current $\tilde{j}_\parallel$, the Maxwell equations (2)–(3) take the form

$$\frac{\mathrm{d}^2\varphi(x)}{\mathrm{d}x^2} = G(x)\,, \tag{7}$$

$$\frac{\mathrm{d}^2\tilde{A}_\parallel(x)}{\mathrm{d}x^2} = \frac{\rho_{y_i}^2}{2d_i^2}\,\frac{\omega}{k_\parallel c}\,G(x)\,, \tag{7'}$$

$$G(x) = \left(\varphi(x) - \frac{\omega\tilde{A}_\parallel(x)}{k_\parallel c}\right)\left(\frac{\rho_{y_i}^2}{2\tilde{d}_i^2}\right)^{-1}\times$$

$$\times\left[\sum_j Z_{1j}(1 + X_{j'})\tilde{d}_j^{-2}D_j^{-1}\right],\quad j \neq j'\,; \tag{8}$$

$$X_j \equiv [2i\nu_{jj'}\,\omega/(k_\parallel v_{T_j})^2]Z_{1j}D_j^{-1}\,,\quad j \neq j'\,;$$

$$Z_{n_j} \equiv \pi^{-1/2}\int_{-\infty}^{\infty} x^n\,e^{-x^2}(x - \xi_j)^{-1}\,\mathrm{d}x\,;\quad \xi_j \equiv \omega/k_\parallel v_{T_j}\,;$$

$$D_j = 1 + i\nu_j Z_{0j}/k_\parallel v_{T_j} + 2i\nu_{jj'}\,\omega Z_{1j}/(k_\parallel v_{T_j})^2\,.$$

We use the following assumptions in deriving (7)–(8): (1) It is possible do not into account the difference of the distribution function (Equation (6)) from Maxwellian in the singular region, i.e., we can neglect u_j there. (2) In the singular region $\mathrm{d}^2/\mathrm{d}x^2 \gg k_0^2$, V_0. (3) $\tilde{A}_\parallel(x)$ is a slowly-varying function of x; and thus we assume $\tilde{A}_\parallel(x) \simeq$ const. bringing it out of the integral over unperturbed trajectories in Equation (5), but the changing of $\mathrm{d}\tilde{A}_\parallel(x)/\mathrm{d}x$ will be taken into account. (4) Unlike $\tilde{A}_\parallel(x)$ the scalar potential $\varphi(x)$ is rapidly varying function of x and we expand it in a Taylor's series leaving the derivatives of the second order:

$$\varphi(x') = \varphi(x) + (x' - x)\left(\frac{\mathrm{d}\varphi(x')}{\mathrm{d}x'}\right)_{x'=x} + \frac{1}{2}\,(x' - x)^2\left(\frac{\mathrm{d}^2\varphi(x')}{\mathrm{d}x'}\right)_{x'=x}$$

(it is easy to show that the $(\mathrm{d}\varphi(x')/\mathrm{d}x')$ term automatically comes to zero). (5) We assume that $\rho_{y_i} \gg d_e, d_i$ limiting thus the $\mathbf{B}_y$ field: $b_y < c(2T_i m_i)^{1/2}/(T_e + T_i)$.

When moving in the singular region a particle 'feels' the Doppler-shifted perturbed field: $\omega' = \omega - \omega_D$. For collisionless limit $(\gamma > \nu_e)$ we have particles free motion along magnetic field and, therefore, $\omega_D = k_{\parallel}(x)v_{\parallel} \simeq k_{\parallel}(x)v_{T_e}$. And for rather strong collisions $(\gamma < \nu_e)$, when instead of free motion we have the diffusion of electrons along $\mathbf{B}$, the Doppler frequency ω_D takes the form: $\omega_D = k_{\parallel}^2(x)D_e = k_{\parallel}^2(x)v_{T_e}^2\nu_e^{-1}$, where D_e is the longitudinal diffusion coefficient (Drake and Lee, 1977). The condition for the effective acceleration of particles by the perturbed $\tilde{E}_{\parallel}$ field is that the particle must 'feel' a quasi-stationary rather than variable (due to ω_D-shift) electric field. That means that the particle will be accelerated effectively if the value of Doppler-shift is much smaller than the characteristic frequency of the field: $|\omega_D(x)| \ll |\omega|$. This, condition defines the singular region width and finally we obtain:

$$\Delta_s = \begin{cases} \Delta_s^0 = \bar{\gamma} b_y L & \text{collisionless limit ;} \quad (9) \\ \Delta_s^c = (\bar{\gamma}\bar{\nu}_e)^{1/2}\, b_y L & \text{collisional limit ;} \quad (10) \end{cases}$$

where $\bar{\gamma} \equiv \gamma/kv_{T_e}$, $\bar{\nu}_e \equiv \nu_e/kv_{T_e}$. Naturally, in what follows we shall come to the same expressions just from the direct analysis of Equations (7)–(8). The dispersion relation for the tearing instability is obtained from the matching condition for the jump of the logarithmic derivative of $\tilde{A}_{\parallel}$ in adiabatic (where the frozen-in condition is satisfied and the ideal MHD approximation is valid) and nonadiabatic (singular) regions. The solution of Equation (2) in adiabatic region is expressed in associated Legendre functions: $\tilde{A}_{\parallel}^{\mathrm{ad}}(x) = P_1^{-m}(\pm\,\mathrm{th}\,(x/L))$, $m \equiv kL$. Thus, finally we come to the dispersion relation

$$(1 - m^2)/m = \tilde{A}_{\parallel}^{-1} L \int_{-\infty}^{\infty} (\mathrm{d}^2\tilde{A}_{\parallel}(x)/\mathrm{d}x^2)\,\mathrm{d}x\,, \tag{11}$$

for $\Delta_s \ll L$, where $\tilde{A}_{\parallel}$ must be taken from Equation (7′).

The left-hand side of Equation (11) corresponds to the free-energy store of the instability, which is released during the attracting and merging of the parallel current filaments in the current layer. This merging is energetically favourable if the left-hand side of Equation (11) is positive – i.e., $kL < 1$. That means that only rather large bundles of current filaments with $\lambda > 2\pi L$ could be formed during the merger. The right-hand side of Equation (11) represents the particular mechanism of the dissipation, responsible for absorbtion of the releasing energy; therefore, it defines the growth rate of perturbations.

3. Different Regimes of Tearing-Mode

Qualitatively, one can say that the role played by the scalar potential is merely the following: at the distance $x > \delta_\varphi$ (where δ_φ is the characteristic spatial scale of φ, which is defined from Equation (7)), the perturbed electrostatic field $-\nabla_{\parallel}\varphi(x)$ cancels the perturbed inductive field $-c^{-1}\,\partial\tilde{A}_{\|}(x)/\partial t$. Now it is clear that when $\delta_\varphi < \Delta_s$ the taking into account the scalar potential leads to the 'cutting off' of the region of the perturbed current localization: $d^2\tilde{A}_{\|}/dx^2 \sim \tilde{j}_{\|} \sim \tilde{E}_{\|} \to 0$ in the region $x > \delta_\varphi$. When $\delta_\varphi > \Delta_s$, the influence of φ is negligible because the Doppler effect lead to the rapid decrease of conductivity $\sigma_{\|} = \tilde{j}_{\|}/\tilde{E}_{\|}$ far earlier than the decrease of $\tilde{E}_{\|}$ sets in. A detailed analysis of Equation (7) gives the natural result that the minimum spatial scale of φ is of the order of ion-Larmor radius ρ_{y_i}. This result is easy to understand; for the distribution of φ is defined by ions and they 'feel' electric field that is averaged over the Larmor orbit of the $2\rho_{y_i}$ diameter and all smaller scale variations of φ are averaged in the course of this Larmor motion. In our case $|\mathbf{k}| = k_z = k$, $\delta_\varphi^{\min}$ is equal to $\rho_{y_i}\sqrt{T_i/2T_e}$. Therefore, two different cases of the tearing-mode development are possible: (a) $\delta_\varphi \sim \rho_{y_i} < \Delta_s$, when all characteristic scales in plasma configuration exceed ρ_{y_i} and, consequently, the MHD approach (inertial or resistive) is acceptable; (b) $\Delta_s < \delta_\varphi \sim \rho_{y_i}$, when the region of the current localization is narrower than ion-Larmor radius and MHD approach becomes principally unacceptable and it is necessary a kinetic description.

Let us now consider the different limits.

3.1. Collisionless limit $\gamma \gg \nu_e$

Assuming in (7) and (7′), ν_{jj}/kv_{T_j}, $\gamma/kv_{T_j} \ll 1$, we obtain

$$\frac{\rho_{y_i}^2}{2}\,\varphi'' = \left(\varphi - \frac{\omega\tilde{A}_{\|}}{k_{\|}c}\right)\left[Z_{1e}\,\frac{T_i}{T_e} + Z_{1i}\right](1 - Z_{1i})^{-1}\,, \tag{12}$$

$$\tilde{A}_{\|}'' = \frac{\omega}{k_{\|}c}\left(\varphi - \frac{\omega\tilde{A}_{\|}}{k_{\|}c}\right)d_i^{-2}\left[Z_{1e}\,\frac{T_i}{T_e} + Z_{1i}\right](1 - Z_{1i})^{-1}\,; \tag{13}$$

$(\,)'' \equiv d^2(\,)/dx^2$.

It is easy to see from Equation (12) that in our case of aperiodic perturbations $\operatorname{Re}\omega = 0$ the behaviour of φ is controlled by electron term and they play the determining role in the development of the modes which we investigate below. Thus we can rewrite Equation (12) in the form

$$\varphi'' = \frac{2T_i}{\rho_{y_i}^2 T_e}\left(\varphi - \frac{\omega\tilde{A}_{\|}}{k_{\|}c}\right)Z_{1e}\,. \tag{12′}$$

Equation (12′) describes two regimes of the tearing-mode instability:

(1a) The kinetic regime, when $\delta_\varphi > \Delta_s^0$ and, consequently, the influence of φ can be neglected. We can get from (12′) the solution of φ in the region $x \gg \Delta_s^0$ (in this case $Z_{1e} \to 1$):

$$\varphi = -\frac{\omega\tilde{A}_{\|}}{k_{\|}c}\,\frac{x^2}{(\Delta_\varphi^K)^2}\int_0^\infty \frac{\sin\theta}{\theta^2 + x^2/(\Delta_\varphi^K)^2}\,d\theta\,, \tag{14}$$

where

$$(\Delta_\varphi^K)^2 = \tilde{d}_e^2\left(1 + \frac{\rho_{y_i}^2}{2\tilde{d}_i^2}\right) \simeq \rho_{y_i}^2 \frac{T_e}{2T_i} .$$

It is clear that for spatial scale of the scalar potential we have

$$\delta_\varphi^K = \Delta_\varphi^K \simeq \rho_{y_i} \sqrt{\frac{T_e}{2T_i}}$$

The condition $\delta_\varphi^k > \Delta_s^0$ defines the margin of kinetic regime

$$b_y \leq \sqrt{\frac{m_i}{2m_e}}\ \varepsilon_e^{-1} . \tag{15}$$

Matching condition Equations (11) and (13) (if we neglect φ) give the growth rate of collisionless kinetic tearing-mode (Drake and Lee, 1977) by

$$\bar{\gamma}_0' = \pi^{-1/2}\left(1 + \frac{T_i}{T_e}\right)\frac{1 - m^2}{m}\ \frac{\varepsilon_e^2}{b_y} . \tag{16}$$

(1b) The MHD inertial regime, when $\delta_\varphi < \Delta_s^0$. In this case the solution of φ in the region $x < \Delta_s^0(Z_{1e} \approx x^2/2(\Delta_s^0)^2)$, coming from (12′) takes the form

$$\varphi(x) = \frac{\omega\tilde{A}_\parallel}{k_\parallel c}\ \frac{x^2}{2(\Delta_\varphi^M)^2} \int_0^{\pi/2} \mathrm{d}\theta \sqrt{\sin\theta} \exp\left[-\frac{x^2\cos\theta}{2(\Delta_\varphi^M)^2}\right]; \tag{17}$$

where the parameter $\Delta_\varphi^M = \sqrt{2\delta_\varphi^K \Delta_s^0}$ defines the region of current localization. It is easy to obtain from Equation (11) the expression for the growth rate of the collisionless MHD mode: i.e.,

$$\bar{\gamma}_0'' = \varepsilon_e^3 \sqrt{\frac{m_e}{m_i}}\left[4\left(1 + \frac{T_i}{T_e}\right)\frac{T_i}{T_e I}\right]^2 \left(\frac{1 - m^2}{m}\right)^2 . \tag{18}$$

The integral I appears from the integration in Equation (11) and is of the form

$$I = \int_{-\infty}^{\infty} \mathrm{d}t\left(1 - \frac{t^2}{2}\int_0^{\pi/2} \mathrm{d}\theta \sqrt{\sin\theta} \exp\left[-\frac{t^2\cos\theta}{2}\right]\right) . \tag{19}$$

Changing the order of integration we come to the expression: $I = 2\pi\Gamma(\frac{3}{4})/\Gamma(\frac{1}{4})$. The growth rates (16) and (18) match accurate to the factor of order of unity at the margin (15) $b_y^* = \sqrt{(m_i/2m_e)}\ \varepsilon_e^{-1}$ (Galeev and Zeleny, 1977).

3.2. COLLISIONAL LIMIT $\gamma < \gamma_e$

Using the asymptotics $\nu_j/k_{\|} v_{T_j} \gg 1$ from Equations (7) and (7′) we obtain

$$\frac{\rho_{y_i}^2}{2} \varphi'' = \left(\varphi - \frac{\omega A_{\|}}{k_{\|} c}\right) \frac{T_i}{T_e} \frac{k_{\|}^2 v_{T_e}^2}{2\gamma \nu_e + k_{\|}^2 v_{T_e}^2}, \tag{20}$$

$$\tilde{A}_{\|}'' = \frac{\omega}{k_{\|} c}\left(\varphi - \frac{\omega \tilde{A}_{\|}}{k_{\|} c}\right) \tilde{d}_e^{-2} \frac{k_{\|}^2 v_{T_e}^2}{2\gamma \nu_e + k_{\|}^2 v_{T_e}^2}. \tag{21}$$

Equation (20) describes now two collisional regimes:

(2a) When $\delta_\varphi^c > \Delta_s^c$, kinetic regime. It is seen that in the region $x > \Delta_s^c$ the solution of Equation (20) is the same as in the collisionless case (1a) (see Equation (14)) and, therefore $(\delta_\varphi^K)_{\text{coll}} \simeq \rho_{y_i} \sqrt{T_e/2T_i}$. Neglecting φ in Equation (21) with the aid of Equation (11), we get the semi-collisional mode of Drake and Lee (1977) as

$$\bar{\gamma}_{sc} = \bar{\nu}_e^{1/3} \varepsilon_e^{4/3} b_y^{-2/3} \left[\pi^{-1/2}\left(1 + \frac{T_i}{T_e}\right) \frac{1 - m^2}{m}\right]^{2/3}. \tag{22}$$

(2b) When $\delta_\varphi^c < \Delta_s^c$, the MHD regime. Here we also obtain the same solution of (20) as in the collisionless case (1b) (see Equation (17)), but instead of collisionless singular layer width Δ_s^0 we must insert the collisional one into the parameter Δ_φ: $(\Delta_\varphi^M)_{\text{coll}} = \sqrt{2\delta_\varphi^K \Delta_s^c}$. By analogy to (1b) we can get now from Equations (11), (17), and (21) the resistive MHD mode of Furth–Killen–Rosenbluth (Furth *et al.*, 1963) of the form

$$\bar{\gamma}_{\text{FKR}} = \bar{\nu}_e^{3/5} \varepsilon_e^{6/5} \mu^{1/5} \left[\frac{1 - m^2}{m} \frac{2(T_e + T_i)}{T_e I}\right]^{4/5}; \tag{23}$$

where we must keep in mind that, in Equations (22) and (23), we use the dimensionless parameters $\varepsilon_e = c/\omega_{p_e} L$, $\mu = m_e/m_i$, $\bar{\gamma} = \gamma/k v_{T_e}$, $\bar{\nu}_e = \nu_e/k v_{T_e}$.

The growth rates $\bar{\gamma}_{sc}$ and $\bar{\gamma}_{\text{FKR}}$ match for $\Delta_s^c = \delta_\varphi^c \sim \rho_{y_i}$. In addition, the semi-collisional growth rate γ_{sc} smoothly match the collisionless one $\bar{\gamma}_0' = \varepsilon_e^2/b_y$ when $\bar{\gamma}_0' = \nu_e$.

4. The Development of the Instability for Large Shear

As it is clear from analysis of the previous section, the collisionless growth rate $\bar{\gamma}_0' \simeq \varepsilon_e^2/b_y$ (see Equation (16)) match the collisionless growth rate for 'pure neutral' ($b_y = 0$) sheet $\bar{\gamma}_0 \simeq \varepsilon_e^{3/2}$ (Coppi *et al.*, 1966) at the point $b_y = \varepsilon_e^{1/2} \ll 1$. This value of shear corresponds to the marginal condition for magnetized electrons $\rho_{y_e} < d_e = \sqrt{\varepsilon_e}\, L$ (see Section 2). This matching occurs till the value of collision frequency $\bar{\nu}_e \lesssim \varepsilon_e^{3/2}$. However, for $b_y > \varepsilon_e^{1/2}$, when the growth rate of the instability $\bar{\gamma}_0' \simeq \varepsilon_e^2 b_y^{-1}$ becomes smaller than the collision frequency $\bar{\nu}_e > \bar{\gamma}_0' = \varepsilon_e^2/b_y$ (for $b_y > \varepsilon_e^{1/2}$ this inequality corresponds to $\bar{\nu}_e > \varepsilon_e^{3/2}$) the semi-collisional regime of Drake and Lee (1977) applies ($\bar{\gamma}_{sc} \sim \bar{\nu}_e^{1/3}$). While, for $b_y < \varepsilon_e^{1/2}$, the collisionless regime of Coppi *et al.* (1966) is prolonged till $\bar{\nu}_e = 1$

(see, e.g., Zeleny and Taktakishvili, 1981). Thus, at the first glance the matching for strong collisions does not take place. But here we should take into account that even for small longitudinal magnetic field $b_y < \varepsilon_e^{1/2} \ll 1$ there do exist in distribution (1) a number of particles (electrons) with small velocities normal to magnetic field $v_\perp < v_{T_e}$ for which the Larmor radius in B_y field is smaller than $d_e = \varepsilon_e^{1/2} L$ width: i.e.,

$$\rho_{y_e}^{<} = v_\perp / \Omega_{y_e} < \varepsilon_e^{1/2} L \,, \tag{24}$$

(Ω_{y_e} is gyrofrequency in B_y field), and we can get the estimate for the marginal normal velocity $v_\perp^*$: $v_\perp^* = \Omega_{y_e} \varepsilon_e^{1/2} L = v_{T_e} b_y \varepsilon_e^{-1/2}$. Consequently, this group of particles will be magnetized by B_y field and their weight in the whole distribution is equal to

$$G_< = \frac{2}{v_{T_e}^2} = \frac{2}{v_{T_e}^2} \int_0^{v_\perp^*} v_\perp \exp(-v_\perp^2/v_{T_e}^2)\, dv_\perp = 1 - e^{-b_y^2/\varepsilon_e} \,. \tag{25}$$

Naturally, for $b_y \ll \varepsilon_e^{1/2}$ the fraction of these particles is small $G_< \sim b_y^2/\varepsilon_e$. On the other hand, for $b_y \gg \varepsilon_e^{1/2}$, $G_< \simeq 1$. Accordingly, the fraction of unmagnetized particles in the whole distribution is given by

$$G_> = \frac{2}{v_{T_e}^2} \int_{v_\perp^*}^{\infty} v_\perp \exp(-v_\perp^2/v_{T_e}^2)\, dv_\perp = e^{-b_y^2/\varepsilon_e} \,. \tag{26}$$

Therefore, calculting the contribution of particles in the dispersion relation (Equation (11)), we must take into account the contribution of both groups of particles with corresponding weights (formally, we should do so when matching in collisionless limit $\bar{\nu}_e < \varepsilon_e^{3/2}$ too, but here the growth rates match automatically).

Thus for these composite dispersion relation we get

$$\frac{1 - m^2}{m} = G_> J_0 + G_< J_{sc} = \bar{\gamma} \varepsilon_e^{-3/2} \exp(-b_y^2/\varepsilon_e) +$$

$$+ \bar{\gamma}^{3/2} \bar{\nu}_e^{-1/2} \varepsilon_e^{-2} b_y \{1 - \exp(-b_y^2/\varepsilon_e)\} \,. \tag{27}$$

Solving this cubic equation ($\bar{\gamma}^{1/2} \equiv x$) we obtain a generalized expression for the growth rate of the instability in the intermediate region $b_y \sim \varepsilon_e^{1/2}$ which gives $\bar{\gamma}_0$ and $\bar{\gamma}_{sc}$ in the asymptotical cases $b_y \ll \varepsilon_e^{1/2}$ and $b_y \gg \varepsilon_e^{1/2}$, respectively (see Figure 1).

5. General Picture of the Regimes

Now we can sketch the general picture of the different regimes of the tearing instability dependent on collision frequency ν_e and shear value b_y. On the diagram (ν_e, b_y) shown on Figure 2 we plotted the regions which the parameters of laboratory (experimental devices: Altyntsev *et al.*, 1977; Frank, 1974; tokamaks: Muchovatov, 1980) and space

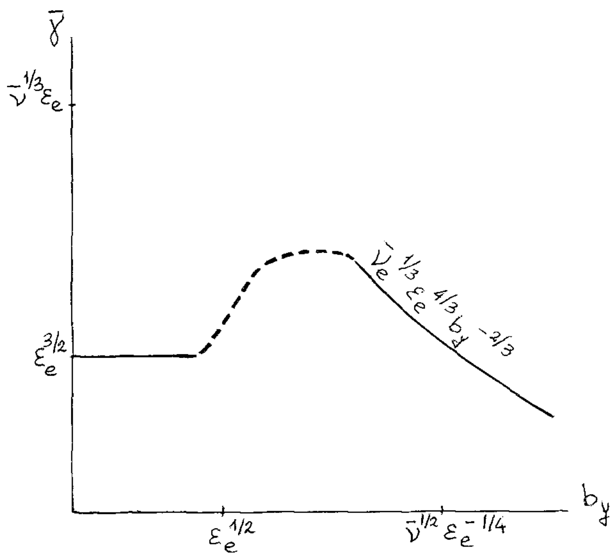

Fig. 1. The dependence of growth rate on b_y near the marginal region of collisionless and semi-collisional regimes.

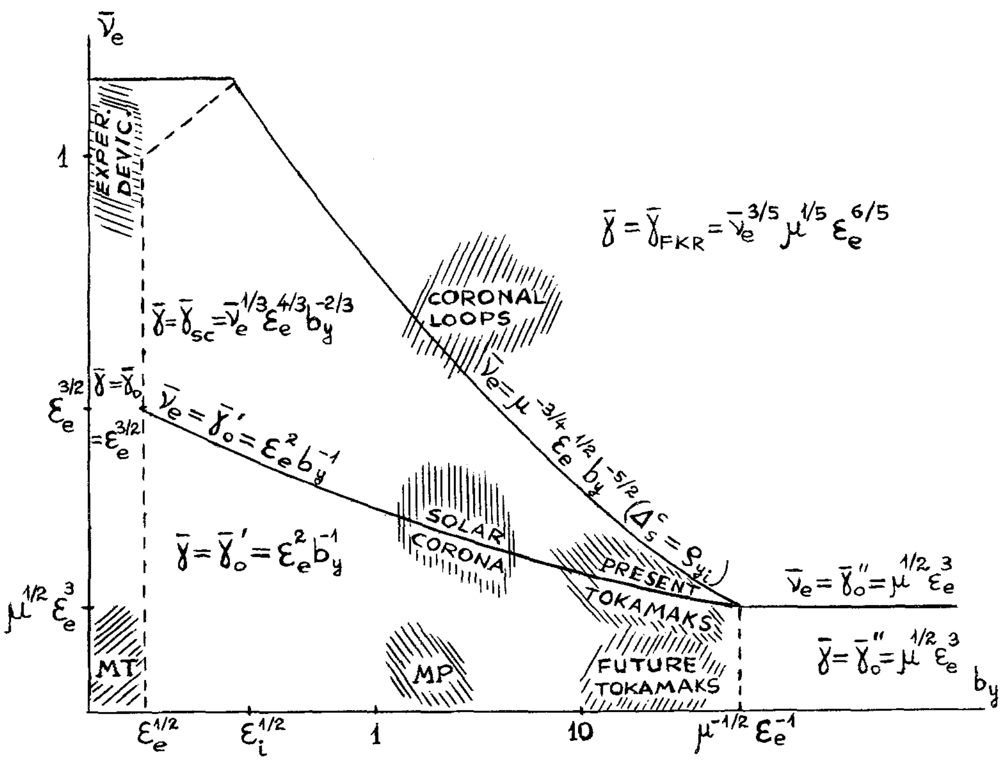

Fig. 2. Regimes of tearing-mode development. MT: magnetospheric tail, MP: magnetopause.

(solar coronal loops and solar corona: Švestka, 1976; the Earth's magnetopause: Berhem and Russell, 1982) current layers occupy.

It is clear from Figure 2 that solar flare (coronal) loops (particle density $n \sim 10^{11}$–10^{12} cm^{-3}, temperature $T \sim 10^6$ K), occupy the marginal position between semi-collisional (Equation (22)) and resistive MHD (Equation (23)) regimes. The parameter b_y here is evaluated as the ratio of longitudinal magnetic field $B_{\|}$ to the azimuthal one B_θ in a loop (magnetic tube). It characterizes the degree of warping of the flux tube: the greater B_θ, the more the magnetic field spirals, and more energy is stored in the loop. One can get the evaluation of b_y from the condition of stability of magnetic tube by the Kruskal–Shafranov criterium which gives the reasonable result consistent with experimental data $b_y \sim 2$–10.

If we assume that coronal loops posess filamentary structure – i.e., consist of rather thin filaments with the characteristic scale of magnetic field variation $L \sim 10^4$–10^5 cm, then the evaluation of appropriate time-scale of the instability development in this region gives the reasonable value $\tau \sim 10^3$ s. One can see from Figure 2 that the collisionless MHD regime of Coppi (1965) can occur only for unrealistically large values of shear length $b_y^* \simeq \mu^{-1/2} \varepsilon_e^{-1} \gtrsim 2 \times 10^3$ and is not met in reality.

In experimental devices (Altyntsev *et al.*, 1977; Frank, 1974) as well as in the Earth's magnetospheric tail the turning of field occurs at the angle that is close to 180°, and the value of b_y is small, $b_y \ll \varepsilon_e^{1/2}$. According to their collision frequencies $\bar{\nu}_e$, experimental devices and the magnetospheric tail fall mainly into the collisionless regime of Coppi *et al.* (1966) $\bar{\gamma} \simeq \varepsilon_e^{3/2}$. In the magnetospheric tail $\bar{\nu}_e$ varies within the limits $0 < \bar{\nu}_e < \bar{\nu}_{\text{eff}} \gtrsim 1$, where $\bar{\nu}_{\text{eff}}$ can be caused, e.g., by the development of low-hybrid turbulence in the inhomogeneous magnetospheric plasma. For the collisionless plasma of the magnetopause $b_y \sim 1$ and the instability always occur in the kinetic inertial regime $\bar{\gamma}_0' \simeq \varepsilon_e^2/b_y$ which occupies very wide range of parameters $\sqrt{\varepsilon_e} < b_y < \mu^{-1/2} \varepsilon_e^{-1}$.

As for tokamak devices, they generally fall outside the range of the MHD approach (see Figure 2). The role of b_y here plays the ratio of toroidal (B_T) to azimuthal (B_φ) fields, which is of the order of 12–20 (Muchovatov, 1980). The present tokamaks fall into the semi-collisional region, while the tokamaks of the next generation with higher temperatures will be approaching to the collisional inertial tearing-mode region $\bar{\gamma}_0' \simeq \varepsilon_e^2/b_y$.

5. Summary

Summarizing our results we conclude that there exist following regimes of tearing instability: (1) the kinetic collisionless resonant (Coppi *et al.*, 1966); (2) the kinetic collisionless inertial (Drake and Lee, 1977); (3) the kinetic resistive (Zeleny and Taktakishvili, 1981); (4) the kinetic semi-collisional (Drake and Lee, 1977); (5) the MHD collisionless inertial (Coppi, 1965); and (6) the MHD resistive (Furth *et al.*, 1963), all shown on Figure 2.

The general kinetic description of the tearing instability gives the possibility to obtain all the regimes of the instability and define the particular mechanism responsible for the development of the spontaneous reconnection in different cases.

As was mentioned above, we must be very careful when using the MHD approach, for the very thin width of the singular layer can make the MHD approximation inapplicable. Although, at a first glance, it looks a little strange, but this may remove many problems because – as we have shown above – the kinetic modes are in some sense simpler than the MHD ones (for their description they need only one differential equation, instead of a system of coupled equations in the MHD case).

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