ROTATION RATES OF SMALL MAGNETIC FEATURES FROM TWO- AND ONE-DIMENSIONAL CROSS-CORRELATION ANALYSES

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Abstract. We present results of an analysis of 628 high-resolution magnetograms taken daily with the NSO Vacuum Telescope on Kitt Peak from 1975 to 1991. Motions in longitude on the solar surface are determined by a two-dimensional cross-correlation analysis of consecutive day pairs. We find that the measured rotation rate of small magnetic features, i.e., excluding active regions, is in excellent agreement with the results of the previous one-dimensional analysis of the same data (Komm, Howard, and Harvey, 1993). The polynomial fits show magnetic torsional oscillations, i.e., a more rigid rotation during cycle maximum and a more differential rotation during cycle minimum, but with smaller amplitudes than the one-dimensional analysis. The full width at half maximum of the cross-correlations is almost constant over latitude which shows that the active regions are effectively excluded. The agreement between the one- and two-dimensional cross-correlation analyses shows that the two different techniques are consistent and that the large-scale motions can be divided into rotational and meridional components that are not affected by each other.

1. Introduction

We continue our investigation of large-scale motions on the solar surface by tracking small magnetic features observed on daily NSO/Kitt Peak magnetograms. Using a one-dimensional cross-correlation analysis, we previously derived the rotation rate of small magnetic features and the variation of the rotation with the solar cycle, revealing magnetic 'torsional oscillations' (Komm, Howard, and Harvey, 1993).

Now we extend the analysis to two dimensions, so that motions other than rotation can be investigated. Details of the technique of the two-dimensional cross-correlation analysis have been described by Howard, Harvey, and Forgach (1990).

Here we focus on the rotation rates derived by using the two-dimensional cross-correlations. We compare the results of the two-dimensional with the previous one-dimensional analysis, thereby studying the reliability and consistency of the two different techniques. This allows us to build confidence for the investigation of meridional and other motions and also to confirm the earlier results concerning solar rotation. Furthermore, the one-dimensional cross-correlation technique measures only one component of the flow field on the solar surface, averaging over the other, in contrast to the two-dimensional cross-correlation. Thus, the interpretation of the results derived by the two techniques as *the* differential rotation assumes that, over the time period considered, there are no large systematic motions or correlations between the flow components. Our comparison provides an immediate check of this assumption.

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2. Data and Analysis

In this study we have used full disk magnetograms which are obtained daily with the Vacuum Telescope of the National Solar Observatory (NSO), located on Kitt Peak. The data and the reduction procedures have been described in detail by Howard, Harvey, and Forgach (1990). For each year of the whole available interval from 1975 to 1991 we have studied the magnetograms of the two months (typically May and October) which show the longest uninterrupted sequence of consecutive day pairs. The total number of daily pair cross-correlations used for the whole 17-year interval was 628. For this investigation we used 'small maps', which are made by remapping the magnetograms in latitude and central meridian distance (CMD) for a zone of $\pm 60°$ in latitude and $\pm 60°$ in CMD into maps of 1024×1024 pixels.

The effect of active regions was eliminated in each small map by a program that masked out values which exceeded 10 G on a temporary copy of the small map with an angular resolution degraded to $3.^{\circ}75 \times 3.^{\circ}75$ in such a way as to minimize the correlation signal produced by the masks themselves, as is discussed in the earlier paper (Howard, Harvey, and Forgach, 1990). During cycle maximum, when the level of activity is high, the masking process eliminates a large fraction of each observation. In the cross-correlation array of the active day pair 19–20 April, 1981, for example, 30% of the array is masked. However, averaging over many day pairs leads to a cross-correlation array without 'holes' due to masking.

To cross-correlate the small maps of a day pair in two dimensions, the field of the first day's small map was divided into square, non-overlapping regions with a dimension of $7.^{\circ}5 \times 7.^{\circ}5$ (64×64 pixels), which are large compared to the size of the correlated features, therefore avoiding the problems due to a small masking window as discussed by November and Simon (1988). Thus the first small map was divided into 15 regions in latitude and 14 regions in CMD (the western-most regions are discarded because of lack of overlapping data on the next day). Each of these areas is then cross-correlated in latitude and in longitude with an area ($11.^{\circ}25 \times 11.^{\circ}25$ or 96×96 pixels) on the next day's map, shifted by an amount corresponding to an assumed rotation for the latitude of the center of the area. Therefore, the maximum lag is $\pm 1.^{\circ}875$ (16 pixels) in both directions. Note that we assume at this stage that the rotation is constant with latitude within each $7.^{\circ}5 \times 7.^{\circ}5$ area.

The cross-correlation arrays were scaled to the same time interval (one day) so that lags were converted to rotation rate units. This enabled us to combine them to create average cross-correlations for one-year intervals. The 15 by 14 regions are each weighted according to the amount of data going into the region.

The correlations were averaged over the 14 regions in CMD to provide an estimate of the rotation rate for each of the 15 latitude ranges. We fit the highest peak in the two-dimensional correlations in each of the 15 ranges to a two-dimensional Gaussian function. These fits use measured correlation values around the peaks down to an adjustable threshold, t, which is normally set to $\frac{1}{4}$ of the peak amplitude.

The resulting rotation rates were fitted to Legendre polynomials in latitude, then for the purpose of comparison with our previous results (Komm, Howard, and Harvey, 1993), we converted the Legendre coefficients to the coefficients in the expansion

$$\omega(\phi) = A + B \sin^2 \phi + C \sin^4 \phi \,, \tag{1}$$

where ϕ is the latitude. The errors given for the three coefficients correspond to ± 1 standard deviation of the means. We note that errors given in this paper are overestimated because the data consist of strings of consecutive days. In that case errors are not independent, but propagate from one day pair to the next, as discussed by Howard (1992).

3. Results

Figure 1 shows the average differential rotation derived from the whole data set 1975–1991 and compares it with the results of the one-dimensional analysis (Komm,

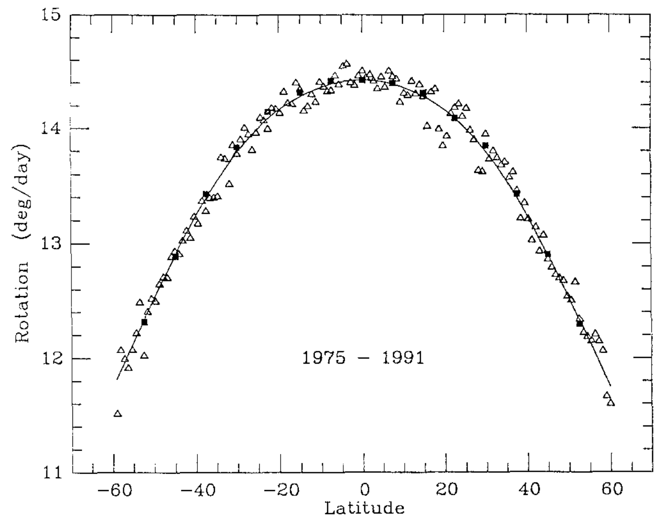


Fig. 1. Average solar rotation rate (sidereal deg day^{-1}) derived from 628 day pairs (■) as a function of latitude (deg) compared with the measured rotation (△) and the polynomial fit (solid line) of the one-dimensional analysis.

Howard, and Harvey, 1993). The size of the symbols is about ± 1 standard deviation. We find that the measured, two-dimensional rotation result is in excellent agreement with the measured rotation and the polynomial fit of the one-dimensional analysis. The average difference between the two-dimensional rotation result and the one-dimensional fit is 0.018 ($\pm$ 0.027) deg day^{-1} or 2.5 ($\pm$ 3.8) m s^{-1} cos(latitude), while the average standard deviation of the measured rotation derived from the two-dimensional analysis is $\pm$ 0.022 ($\pm$ 0.005) deg day^{-1}.

Figure 2 shows the two-dimensional results for three different cross-correlation peak fitting thresholds (t = 0.10, 0.25, 0.50). The threshold defines which correlation points

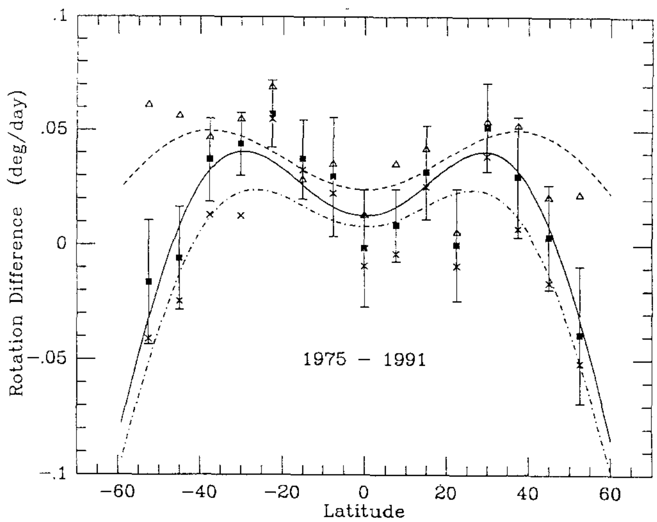


Fig. 2. The measured rotation rates and the polynomial fits of the 2-d analysis for three different thresholds minus the polynomial fit of the 1-d analysis as a function of latitude. t = 0.50: △, dashed line; t = 0.25: ■, solid line; and t = 0.10: ×, dot-dashed line.

around the peak are used in determining the position of the peak in order to reduce the influence of noise. To make the differences more visible, we subtract the polynomial fit of the one-dimensional analysis from the two-dimensional results. The measured rotation with $t = 0.25$, is already shown in Figure 1. We find that increasing the threshold leads to a slightly faster rotation. However, the effect on the measured rotation rates is small, about 0.1% (within one standard deviation), except at higher latitudes, where the signal-to-noise ratio decreases and the largest threshold ($t = 0.50$) leads to a faster rotation. The polynomial fits for the three different thresholds exhibit the same tendencies, which means they are very similar for latitudes equatorward of 40° and different for more poleward latitudes in both hemispheres. In particular, the extrapolation of the fit for $t = 0.50$ to latitudes between 52°5 and 60° in both hemispheres leads to a faster rotation than the fit of the one-dimensional analysis, while the extrapolations of the other two fits are slower by about 0.5%. This shows that the correlation functions are slightly asymmetric, which could result from different causes. For example, features with different sizes rotate differently, the signal-to-noise ratio decreases with latitude, and thus varies over the area being correlated.

Figure 3 shows the same comparison as in Figure 1 for the maximum 1980–1982 and for the minimum 1984–1986. In both cases the measured two-dimensional rotation results agree very well with the one-dimensional results.

Table I shows the fit parameters and compares them with the one-dimensional results. We find a more rigid rotation during cycle maximum than on average, and a more differential rotation during cycle minimum, i.e., 'torsional oscillations'. The difference between maximum and minimum at 52°5, for example, is 0.081 deg day^{-1} (6.9 m s^{-1})

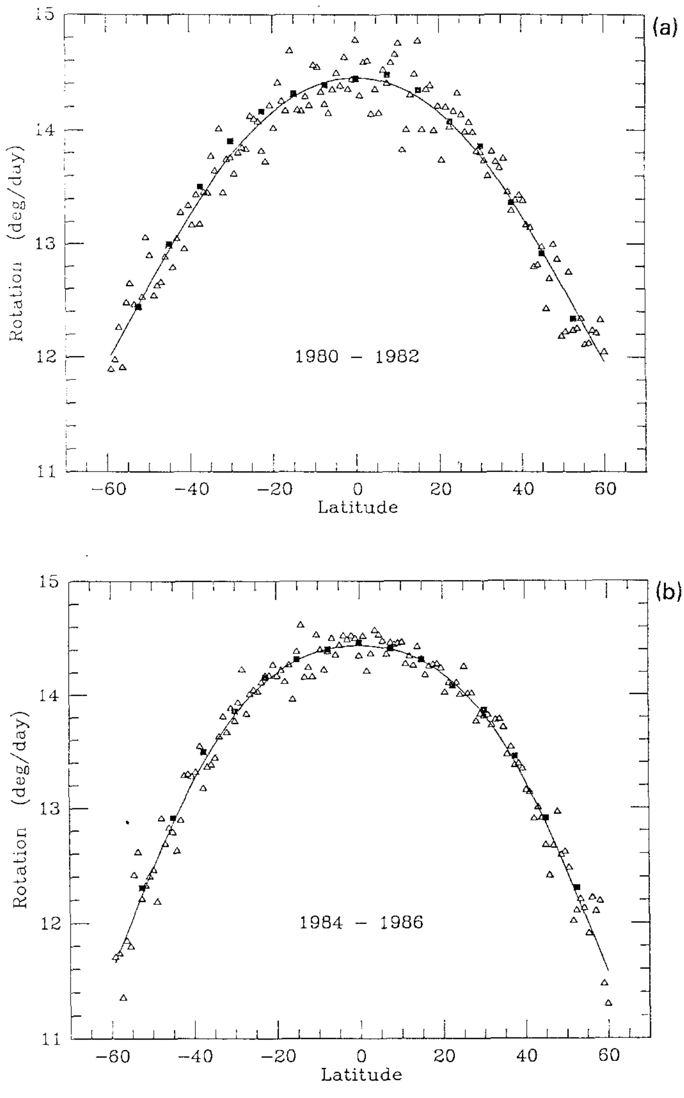


Fig. 3. Differential rotation (sidereal deg day^{-1}) of the two-dimensional analysis (■) as a function of latitude (deg) shown for (a) the maximum 1980–1982 (129 day pairs) and (b) the minimum 1984–1986 (142 day pairs) compared with the measured rotation (△) and the polynomial fit (solid line) of the one-dimensional analysis.

compared with 0.211 deg day^{-1} (18.1 m s^{-1}) in the one-dimensional analysis. In other words, the amplitude of the torsional oscillations is smaller when derived from the two-dimensional fit rates. The standard deviations are systematically smaller for two-dimensional results, which is most likely a by-product of averaging over more data points in the fitting process.

Figure 4 shows the rotation fit parameters as a function of time. As mentioned in Section 2, the error bars are overestimated. The equatorial parameter A is always in

TABLE I

The fit parameters in Equation (1) are given in μrad s^{-1} and in sidereal deg day^{-1} derived from the masked data

Time period	*A*	*B*	*C*
		(a) 2-d analysis	
1975–1991	2.915 ± 0.003	− 0.357 ± 0.020	− 0.521 ± 0.022
	14.43	− 1.77	− 2.58
1980–1982	2.920 ± 0.007	− 0.376 ± 0.046	− 0.459 ± 0.051
(Max.)	14.45	− 1.86	− 2.27
1984–1986	2.916 ± 0.004	− 0.327 ± 0.028	− 0.568 ± 0.031
(Min.)	14.43	− 1.62	− 2.81
		(b) 1-d analysis	
1975–1991	2.913 ± 0.004	− 0.405 ± 0.027	− 0.422 ± 0.030
	14.42	− 2.00	− 2.09
1980–1982	2.919 ± 0.007	− 0.483 ± 0.048	− 0.251 ± 0.033
(Max.)	14.45	− 2.39	− 1.24
1984–1986	2.917 ± 0.006	− 0.365 ± 0.037	− 0.541 ± 0.041
(Min.)	14.44	− 1.81	− 2.68

good agreement (within ~ 0.2%) with the one-dimensional result (Figure 4(a)). The mid- and high-latitude parameters *B* and *C* agree well overall with the one-dimensional results (Figure 4(b) and 4(c)), except for a discrepancy during the time period 1979–1982. In this interval the two-dimensional result indicates more differential rotation than the one-dimensional result. While this difference is smaller than two standard deviations for the parameter *B*, it is larger for the parameter *C*. This discrepancy can be explained by looking again at Figure 3(a), where the measured rotation rates at

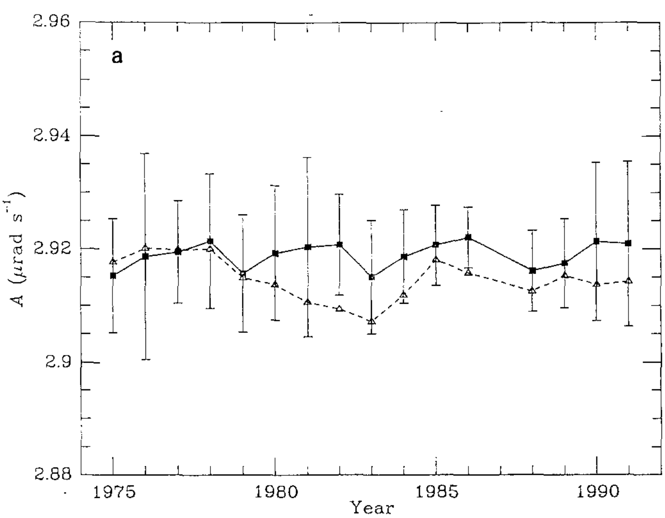
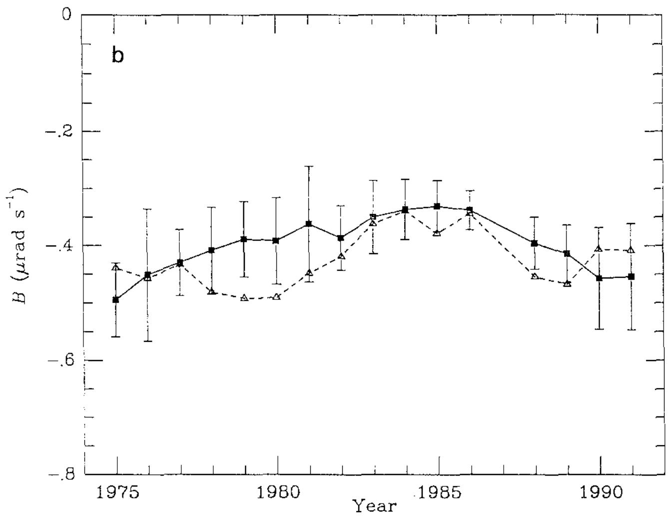


Fig. 4a.



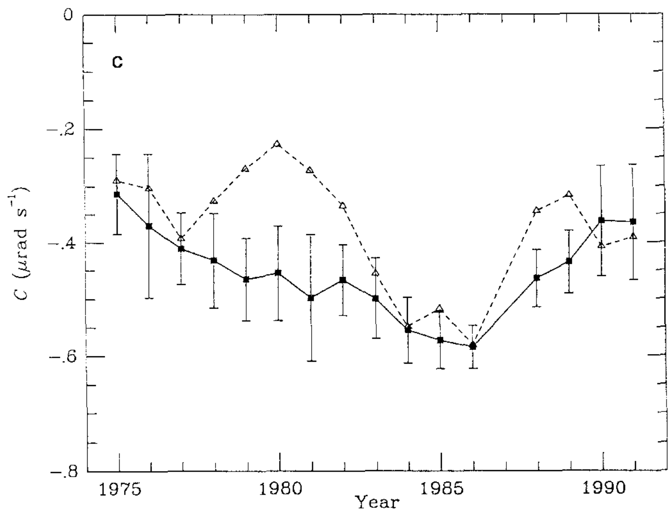


Fig. 4b, c.

Fig. 4. The rotation parameters A, B, and C and their variation with the solar cycle. The annual averages are smoothed by a 3-year running mean. Two-dimensional analysis – solid line; one-dimensional analysis – dashed line.

$-22^\circ\!.5$ and -30° in the southern hemisphere are slightly faster than the one-dimensional results. These two measurements cause the absolute value of B to be smaller than the corresponding one-dimensional value. The determination of the high-latitude parameter C is, in general, less accurate because the largest effective poleward latitude is only $52^\circ\!.5$. Also B and C are not independent of each other and so in the time period 1979–1982 this forces the absolute value of C to compensate for B and thus to be larger than the corresponding one-dimensional value.

Figure 5 shows the full width at half maximum (FWHM) of the longitude component of the Gaussian fit of the cross-correlations for the grand average 1975–1991, the maximum 1980–1982, and the minimum 1984–1986 as functions of latitude. The three

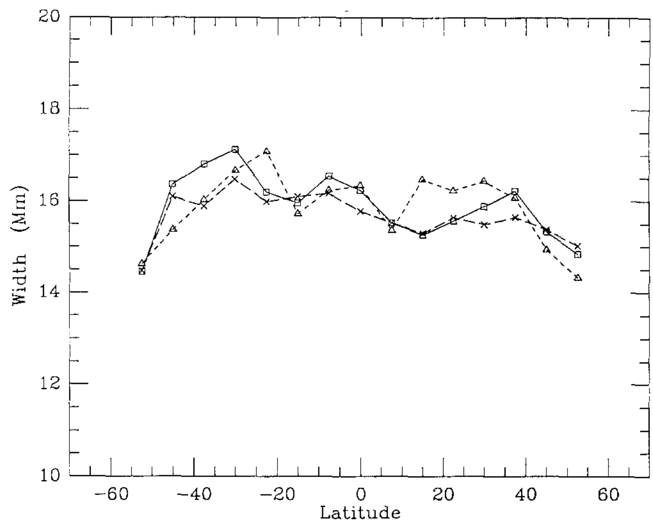


Fig. 5. The full width at half maximum of the cross-correlations as function of latitude. □, 1975–1991 (628 day pairs); △, maximum 1980–1982 (129 day pairs); and ×, minimum 1984–1986 (142 day pairs).

curves are almost the same, which shows that there is no variation with the solar cycle, and they are nearly constant with latitude. The average value for 1975–1991 is 15.9 ± 0.9 Mm, which is smaller than the 23.7 Mm of the one-dimensional analysis. However, the value of the FWHM depends on the threshold used. A larger threshold leads to a smaller FWHM, for example, $t = 0.50$ leads to an average value of 12.3 ± 0.6 Mm, while a smaller threshold leads to a larger one, for example, $t = 0.10$ leads to an average of 17.9 ± 1.2 Mm.

Figure 5 also shows a small trend of the FWHMs with latitude; the FWHMs in the northern hemisphere are smaller than in the southern. This trend is -7.43 km deg^{-1}, compared to -8.91 km deg^{-1} for the one-dimensional cross-correlation widths. We note here that the average FWHM in the north–south (latitudinal) direction is 17.0 ± 0.4 Mm (for $t = 0.25$).

4. Discussion

The measured two-dimensional rotation rates agree with the measured one-dimensional rates and also with the one-dimensional polynomial fits within the error bars. Figure 1 demonstrates this for the grand average rotation, and Figure 3 shows this agreement for different parts of the solar cycle. Figure 2 shows that the two-dimensional results are robust, since a change in the threshold, which changes the Gaussian fit, has only little effect.

We note here that the analysis of small magnetic features is not restricted to lower latitudes and thus we are able to derive a significant value of the parameter C which represents the rotation at higher latitudes. Tracers which are restricted in latitude, for example sunspots (equatorward of 35°), do not lead to a significant value of C as pointed out by Balthasar, Vázquez, and Wöhl (1986), who analyzed the rotation of sunspot groups. A comparison with other observations, especially Doppler measurements, can be found in the previous paper (Komm, Howard, and Harvey, 1993).

The shortcomings of the two-dimensional analysis are the polynomial fits, as can be seen in Figure 4 and in Table I. The polynomial fit is not able to reproduce the one-dimensional rotation fit at mid- and higher latitudes (poleward of about 40°) during cycle maximum 1980–1982 within one standard deviation. This is due to the fact that the largest effective poleward latitude which can be used in the two-dimensional analysis is only 52°.5, compared to 60° in the one-dimensional analysis, and that the latitude resolution is greater in the one-dimensional study. Thus, we find magnetic torsional oscillations but with a smaller amplitude than the one-dimensional investigation. The equatorial rotation rate as expressed in the parameter A (Figure 4(a)) is not affected and agrees with the one-dimensional result within the error bars.

The FWHMs of the two-dimensional cross-correlations are independent of latitude (Figure 5) and do not vary during the solar cycle which shows that the masking technique works properly in two dimensions. A slight north–south trend in the FWHMs, which is the same in both the one and two-dimensional analyses, probably results from the observational procedure. Every day the observing run starts in the northern hemisphere and the seeing generally degrades slightly during the 40-min observation. As a simple test we selected day pairs with average and with relatively bad seeing and derived the FWHMs of their cross-correlations. The FWHMs, averaged over latitude, are larger by up to 5%, when the seeing is bad, which is of the same amount as the north–south trend in the FWHMs, and thus supports the explanation given above. In addition, since there are slightly more observations during May than during October, the different inclination of the solar axis during these two months might also contribute slightly to this trend. The two-dimensional FWHMs are comparable to the one-dimensional results but are somewhat smaller. This comes as no surprise since they are estimated in a different way. The FWHMs are calculated using the Gaussian fits of the two-dimensional cross-correlations, while in the one-dimensional analysis the FWHMs are derived directly from the cross-correlations.

We conclude that the one- and the two-dimensional analyses lead to essentially the same results as far as the differential rotation is concerned. The large-scale motions can thus be divided into rotational and meridional components that are not affected by each other as a result of our analysis. The main purpose of this paper has been to demonstrate that the two-dimensional technique gives valid results. Having done so, we are confident that we can use this technique to study nonrotational motions in the future.

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